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A Theoretical Model for Mars Crater-Size Frequency Distribution

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ABSTRACT

We present a theoretical and analytical curve with reproduce essential features of the frequency distributions vs. diameter, of the 42,000 crater contained in the Barlow Mars Catalog. The model is derived using reasonable simple assumptions that allow us to relate the present craters population with the craters population at each particular epoch. The model takes into consideration the reduction of the number of craters as a function of time caused by their erosion and obliteration, and this provides a simple and natural explanation for the presence of different slopes in the empirical log-log plot of number of craters (N) vs. diameter (D).

1. Introduction

The present impact crater size frequency distribution, N is the result, on one hand, of a rate of crater formation, ϕ, and, on the other hand, the elimination of craters, as time goes by, due to effects like erosion and obliteration. Therefore if we want to understand the crater formation history we will need to know how these forming and erasing factors combine to create N. Thus, in this work the above problem is analyzed, and in section 2 we find that N can be expressed in terms of ϕ and the fractional reduction of craters per unit of time, C. Then, a simple model is discussed that describe the crater size distribution in Mars data, collected by Barlow (Barlow 1988), where it is assumed that ϕ is independent of time. The above model is realistic, since according to several investigations ϕ has remained nearly constant for the last 3 to 3.5 billion years (Hartmann 1966b; Neukum 2001, 1983; Ryder 1990). The simplest interpretation of this model implies that ϕ and C are given as the following inverse power of the diameter, D, of the crater: $\phi \propto \frac{1}{D^{4.3}}$, $C \propto \frac{1}{D^{2.5}}$. In section 3 the model is applied to craters data on Earths, and it is concluded that also in our planet $C \propto \frac{1}{D^{2.5}}$. This result is interpreted to mean that on Mars and Earth we have $C \sim \frac{1}{\text{Volume}}$, or equivalently the crater mean life $\equiv \frac{1}{C} \propto$ Volume $\propto D^2\, h$, with $h \propto D^{0.5}$. Investigations of geometric properties of Martian impact craters reflect values of the average height $h(D)$ consistent with the above conclusion.

2. Theoretical Models for the Observed Data

In what follows we will present theoretical and analytical curves which will reproduce the essential features of the martian crater-size frequency distribution empirical curves (Figure 1), based on Barlow's (1987) database of about 42,000 impact craters. The models will be derived using reasonable simple assumptions, that will allow us to relate the present crater population with the crater population at each particular epoch.

To this end, let $\Delta N(D,\tau)$ represents the number of craters of diameter $D \pm \frac{\Delta D}{2}$ formed during the epoch $\tau \pm \frac{\Delta\tau}{2}$, where we are assuming that ΔD and $\Delta\tau$ are sufficiently large that is justified treating ΔN as a statistical continuous function, but, on the other hand, they should be sufficiently small ($\frac{\Delta\tau}{\tau} \ll 1, \frac{\Delta D}{D} \ll 1$) to be able to treat them as differentials in the following discussion. This initial population will change as time goes on due to climatic and geological erosion, and the obliteration of old craters by the formation of new ones. Then, we expect that the change in ΔN during a time interval dt will be proportional to itself and dt:

$$d(\Delta N) \;=\; -C\,\Delta N dt, \tag{1}$$

where C is the factor that takes into account the depletion of the craters, and should be a function of the diameter, since the smaller a crater is the most likely it will disappear. Furthermore, C could also depend on time however, we will ignore such changes here, which we believe is a good starting approximation to the general problem. It is easy to integrate Equation (1) in time to obtain:

$$\Delta N(D,t) \;=\; \Delta N(D,t_n)\,\mathrm{Exp}\left[-C\,\tau_n\right], \tag{2}$$

$$\tau_n \;=\; t - t_n. \tag{3}$$

Equation (2) gives the number of craters, as a function of D, observed at time t, that were produced at the time interval $t_n \pm \frac{\Delta\tau}{2}$. Therefore the total contribution to the present (t=0) population due to all the epochs t_n is:

$$N(D) \;=\; \sum_n \Delta N(D,t_n)\,\mathrm{Exp}\left[-C\tau_n\right], \tag{4}$$

or in the continuous limit $\Delta\tau\rightarrow 0$,

$$N(D) \;=\; \int_0^{\tau_f} \phi(D,\tau)\,\mathrm{Exp}\left[-C\tau\right]\,d\tau, \tag{5}$$

where

$$\phi(D,\tau) \;\equiv\; \mathrm{lim}_{\Delta\tau\rightarrow 0}\,\frac{\Delta N(D,\tau_n)}{\Delta\tau}. \tag{6}$$

$\phi(D,\tau)$ is the rate of crater formation of diameter D at the epoch τ, and τ_f is the total time of crater formation.

In the next section we will determine the function $C(D)$ and ϕ for a model where we assumed that the rate of crater formation, ϕ, is independent of τ.

3. ϕ Independent of τ

Investigations of the time dependance of cratering rate of meteorites have concluded (Hartmann 1966b; Neukum 2001, 1983; Ryder 1990) that the impact rate went through a heavy bombardment era that decayed exponentially until about 3 to 3.5 Gy, and since then has remained nearly constant until the present. Therefore, for surfaces that are younger than 3 to 3.5 Gy we can reasonably assume that ϕ is independent of τ, and hence from Equation (5) immediately obtain

$$N(D) = \frac{\phi(D)}{C(D)}\left[1 - Exp\left\{-C\tau_f\right\}\right]. \tag{7}$$

We then find that the simplest model that essentially reproduces the data in Figure 1, for $D \geq 6$ km, is given by Equations (8)and (9):

$$\phi(D) \;=\; \frac{3.55\times 10^{9}}{D^{4.3}\,\tau_f}, \tag{8}$$

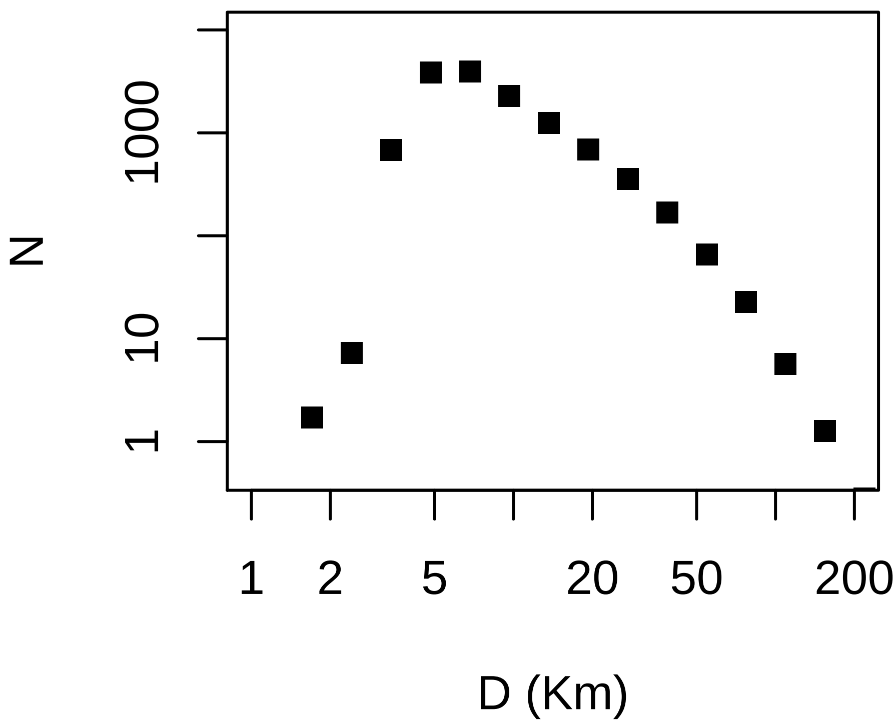

Fig. 1.— Log-log plot of number of craters (N) vs. diameter (D) in Mars. Following (Neukum 2001) the number of craters per kilometer squared were calculated for craters in the diameter $D_L < D < D_R$, where D_L and D_R are the left and right bin boundary and the standard bin width is $D_R/D_L = 2^{1/2}$.

$$C(D) \;=\; \frac{2.48\times 10^4}{D^{2.5}\,\tau_f}. \tag{9}$$

We see that the theoretical curve (7), shown in Figure(2), differs significantly from the observed curves for D less than about 6 km. However, according to Barlow (Barlow 1988) the empirical data is undercounting the actual crater population for D less than 8 km, and therefore no meaningful comparison is then possible between models and data for this region of small craters.

Equation (2) implies that the fraction of craters of diameter D formed at each epoch τ that still survive at the present time $\tau = 0$ is given by:

$$\frac{\Delta N(D,0)}{\Delta N(D,\tau)} \;=\; \mathrm{Exp}\,[-C\tau] \;\approx\; \mathrm{Exp}\left[-\left(\frac{57}{D}\right)^{2.5}\frac{\tau}{\tau_f}\right] \tag{10}$$

and thus we have that the mean life for craters of diameter D, $\tau_{\rm mean}$, is

$$\tau_{\rm mean} \;=\; \frac{1}{C} \approx \left(\frac{D}{57}\right)^{2.5}\tau_f. \tag{11}$$

Hence, craters with $D\,\approx 57$ km have $\tau_{\rm mean}\,\sim\,\tau_f$, while

$$\begin{aligned} \tau_{\rm mean}\; &>>\; \tau_f \quad , \quad D\; >>\; 57\,\mathrm{km},\\ \tau_{\rm mean}\; &<<\; \tau_f \quad , \quad D\; <<\; 57\,\mathrm{km}. \end{aligned} \tag{12}$$

The region $D\;>>\;57$ km is approximately described by the limit of Equation (7) when $D\;\rightarrow\;\infty$:

$$\lim_{D\to\infty} N \;=\; \phi\,\tau_f \;=\; \frac{3.55\times 10^9}{D^{4.3}}, \tag{13}$$

which corresponds to a straight line of slope -4.3 in a Log N vs Log D plot, and that would be the form of Equation (7) in the absence of erosion and obliterations ($C\approx 0$). Hence,we have that the bending of the empirical curve (Figure 1) for $D < 57$ km is explained in this model as the result of the elimination of smaller craters as they get older. We also see from Equations(13) that when the effect of C can be ignored we have $N\;=\;\phi\,(D)\;\tau_f$, and therefore the actual crater density N is proportional to the age of the underlying surface τ_f. On the other hand, when for smaller craters $\mathrm{Exp}\,[-C\,\tau_f]\;<<\;1$ we will have from (7) that

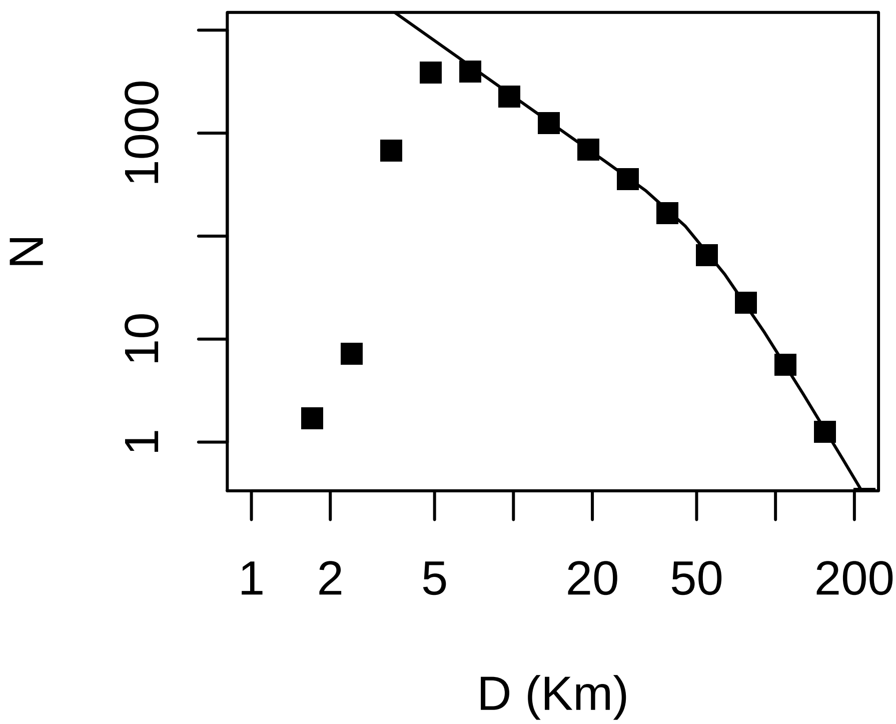

Fig. 2.— Comparison of theoretical model with the empirical log-log plot of number of craters (N) vs. diameter (D) in Mars.

$$N(D) \approx \frac{\phi\,(D)}{C\,(D)} = \phi\,(D)\,\tau_{\rm mean}, \qquad (14)$$

and in this limit the crater density $N(D)$ is proportional to the survival mean life, $\tau_{\rm mean}$, of the craters of size D. Thus, when saturation occurs and hence N is independent of τ_f, we have, instead, that N is proportional to $\tau_{\rm mean}$. This feature is called by Hartmann (Hartmann 2002)"Crater retention age", and in Mars this effect shows, according to this model, in craters smaller than about 57 km.

4. Application to Earth

The model given by Equations (7), (8), and (9) assumed a simple polynomial form for ϕ and C, however, alternative models can be also considered. For instance, by assuming that

$$N = \phi\,\tau_f = \frac{1.43\times10^5}{D^{1.8}}\left[1 - {\rm Exp}\left\{\frac{-2.48\times10^4}{D^{2.5}}\right\}\right], \qquad (15)$$

we will reproduce the Mars crater data, exactly as in model given by Equations (7),(8),(9) but now with $C = 0$, and the change in slope in Figure (1) around $D \approx 57$ km will now be interpreted as intrinsic behavior of $\phi(D)$ rather than due to the erosion and obliteration of smaller craters. How can we then discriminate between these two alternative views?. We see that in the model given by Equation (7) the fraction of craters of a given diameter, D, produced at a time τ, decreases with time according to Equation (10) as

$$\frac{\Delta N(D,0)}{\Delta N(D,\tau)} = {\rm Exp}\,[-C\tau], \qquad (16)$$

while in the model of Equation (15) this fraction is independent of time. Therefore we can put to test the validity of Equation (16) by studying crater size frequency distributions as a function of time. This is possible to do in our planet, and in this section we will investigate the consistency of the hypothesis (16) with the Earth craters data.

Thus consider the average diameter of craters observed today that were formed during a given time $\tau \pm \frac{d\tau}{2}$, which is given, according to Equation (16), by

$$\bar{D} = \frac{\int_0^\infty D\,\phi\,e^{-c\tau}\,dD}{\int_0^\infty \phi\,e^{-c\tau}\,dD}. \qquad (17)$$

Assuming that C and ϕ behave in the form

$$\phi \,=\, \frac{A}{D^m}\,, \quad A \,=\, \text{const}, \;\, m \,=\, \text{const}, \tag{18}$$

$$C \,=\, \frac{B}{D^l}\,, \quad B \,=\, \text{const}, \;\, l \,=\, \text{const}, \tag{19}$$

we can rewrite Equation (17) in the form (Appendix)

$$\bar{D} \,=\, B^{\frac{1}{l}}\,\alpha\,\tau^{\frac{1}{l}}, \tag{20}$$

where

$$\alpha \,\equiv\, \frac{\Gamma\left(\frac{m-2}{l}\right)}{\Gamma\left(\frac{m-1}{l}\right)}, \tag{21}$$

and

$$\Gamma(n) \,\equiv\, \int_0^\infty U^{n-1}\,e^{-U}\,dU \tag{22}$$

is the Gamma function. Equation (20) can be rewritten as

$$\text{Log}\,\bar{D} \,=\, \frac{1}{l}\,\text{Log}\,\tau + \text{Log}\,\text{B}^{\frac{1}{l}}\,\alpha, \tag{23}$$

which represents a linear relation between $\text{Log}\,\bar{D}$ and $\text{Log}\,\tau$ with slope $\frac{1}{l}$. In Figure (3) we plot $\text{Log}\,\bar{D}$ vs $\text{Log}\,\tau$ from data of crater size vs τ on Earth, and the straight line best fitting gives $l \,=\, 2.5$, which is the value determined for model (7) for Mars. This result is interpreted as follows. If we assume that, as expected, τ_{mean} is a function of the volume of the crater, V, that decreases with decreasing V, then it is reasonable to expand it in terms of powers of V, and thus we will have

$$\tau \,=\, \frac{1}{C} \,=\, a_1 V + a_2 V^2 + a_3 V^3 + ...,. \tag{24}$$

Furthermore, for sufficiently small volumes we would have, as a good approximation to C, that

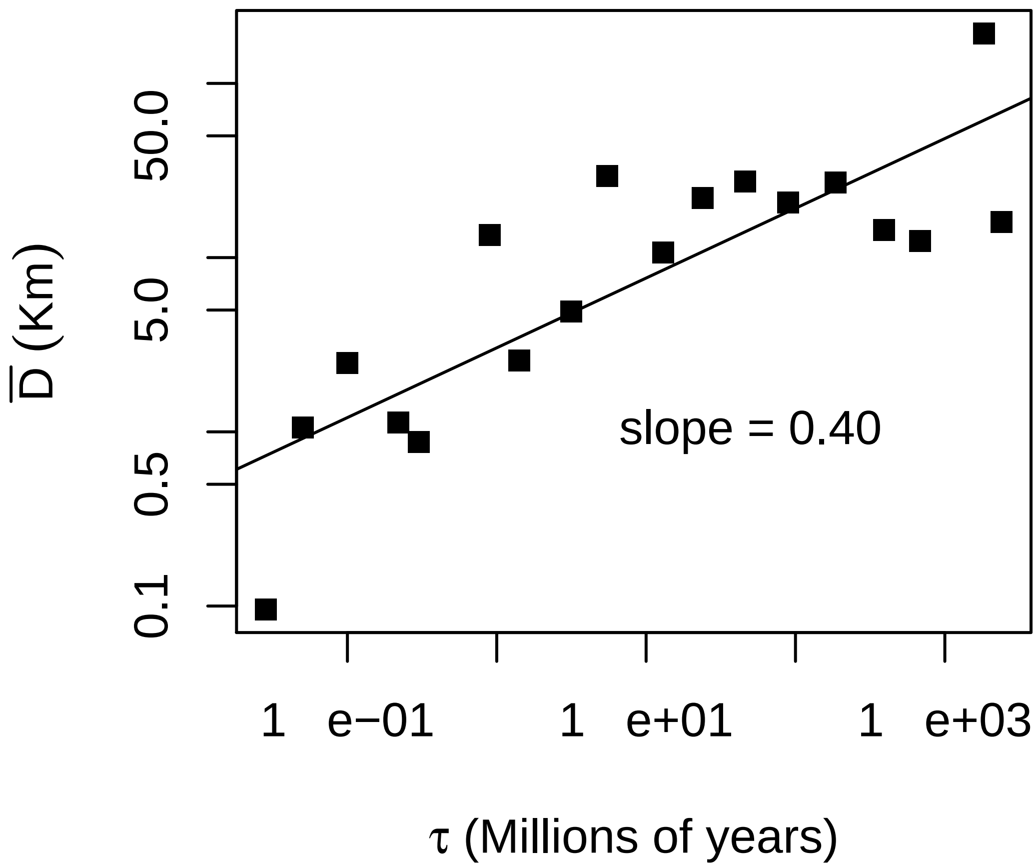

Fig. 3.— Average diameter ($\bar{D}$) vs. average age ($\bar{\tau}$) for terrestrial craters. The bin size increases as $2^{\frac{n}{2}}$. The slope of the straight line best fit (0.40) correspond to $l = 2.5$.

$$\frac{1}{C} \approx a_1 V \; = \; a_1 \, D^2 h, \tag{25}$$

where we are writing

$$V \; = \; D^2 \, h, \tag{26}$$

with h as the average height of the crater of size D. The comparison of Equation (25) with Equations (19), with $l = 2.5$, imply that

$$h \; \sim \; \mathrm{Const} D^{\frac{1}{2}}, \tag{27}$$

which is a prediction that can be investigated, and we have found that indeed Equation (27) is consistent with results from studies of impact crater geometric properties on the surface of Mars, by J.B. Garrin (Garrin 2002).

Therefore it appears that the age distribution of craters on Earth favor the simple model considered for Mars, where there is an erosion and obliteration factor C with the approximate form

$$C \; \approx \; \frac{\mathrm{Const}}{D^{2.5}}. \tag{28}$$

It is also suggested here that the above behavior for C follows from a relation of the form

$$C \; \approx \; \frac{\mathrm{Const}}{V} \; = \; \frac{\mathrm{Const}}{D^2 \, h}; \tag{29}$$

with

$$h \; \propto \; D^{\frac{1}{2}} \tag{30}$$

Further investigations and observations of the crater data on the terrestrial planets, the moon and the asteroids are necessary for additional tests of the validity of the model (7) and its interpretation.

5. Appendix

Lets define

$$U \;=\; \frac{B}{D^l}\,\tau, \tag{31}$$

or, equivalently

$$D \;=\; \left(\frac{B\tau}{U}\right)^{\frac{1}{l}}. \tag{32}$$

Then we have

$$dD \;=\; \frac{-\,(B\tau)^{\frac{1}{l}}\,dU}{l\,U^{1+\frac{1}{l}}}, \tag{33}$$

and therefore Equation (17) becomes

$$\bar{D} \;=\; (B\tau)^{\frac{1}{l}}\;\frac{\int_0^\infty U^{\frac{m-2}{l}-1}\,e^{-U}dU}{\int_0^\infty U^{\frac{m-1}{l}-1}\,e^{-U}dU} \;\equiv\; (B\tau)^{\frac{1}{l}}\;\alpha, \tag{34}$$

where α is the ratio of the following gamma functions:

$$\alpha \;\equiv\; \frac{\int_0^\infty U^{\frac{m-2}{l}-1}e^{-U}dU}{\int_0^\infty U^{\frac{m-1}{p}-1}e^{-U}dU} \;\equiv\; \frac{\Gamma\left(\frac{m-2}{l}\right)}{\Gamma\left(\frac{m-1}{l}\right)}. \tag{35}$$

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