

PROBING DARK ENERGY USING ITS DENSITY INSTEAD OF ITS EQUATION OF STATE

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ABSTRACT

The variation of dark energy density with redshift, $\rho_X(z)$, provides a critical clue to the nature of dark energy. Since $\rho_X(z)$ depends on the dark energy equation of state $w_X(z)$ through an integral, $\rho_X(z)$ can be constrained more tightly than $w_X(z)$ given the same observational data. We demonstrate this explicitly using current type Ia supernova (SN Ia) data [the Tonry/Barris sample], together with the Cosmic Microwave Background (CMB) shift parameter from CMB data (WMAP, CBI, and ACBAR), and the large scale structure (LSS) growth factor from 2dF galaxy survey data. We assume a flat universe, and use Markov Chain Monte Carlo (MCMC) technique in our analysis. We find that, while $w_X(z)$ extracted from current data is consistent with a cosmological constant at 68% C.L., $\rho_X(z)$ (which has far smaller uncertainties) is not. Our results clearly show the advantage of using $\rho_X(z)$, instead of $w_X(z)$, to probe dark energy.

Subject headings: cosmology:observations – distance scale – supernovae:general

1. INTRODUCTION

Recent observations of type Ia Supernovae (Riess et al. 1998; Perlmutter et al. 1999) indicate that the universe is accelerating. A fundamental quest in physics and cosmology is to identify the nature of the “dark energy” driving this acceleration. Possibilities include: (1) a cosmological constant, (2) a time dependent vacuum energy, or a scalar field known as “quintessence” that evolves dynamically with time (Freese et al. 1987; Peebles & Ratra 1988; Wetterich 1988; Frieman et al. 1995; Caldwell, Dave, & Steinhardt 1998; Zlatev, Wang, & Steinhardt 1999)³ or (3) modified Friedmann equation, e.g. the Cardassian models (Freese & Lewis 2002; Freese 2003; Gondolo & Freese 2003; Wang et al. 2003), that could result as a consequence of our observable universe living as a 3-dimensional brane in a higher dimensional universe. Other proposed modifications to the Friedmann equation include Parker & Raval (1999); Deffayet (2001); Bilic, Tupper, & Viollier (2002); Ahmed et al. (2002); Capozziello (Carloni); Carroll et al. (2003); Meng & Wang (2003); Puetzfeld & Chen (2004). The various dark energy models produce dark energy densities $\rho_X(z)$ with different redshift dependences. Hence, in order to differentiate between dark energy models, it is important that we allow the dark energy density to be an arbitrary function of redshift z (Wang & Garnavich 2001; Wang & Lovelace 2001; Wang et al. 2003).

A powerful probe of dark energy is type Ia supernovae (SNe Ia), which can be used as cosmological standard candles to measure how distance depends on redshift in our universe. The luminosity distance $d_L(z) = (1+z)r(z)$, with the comoving distance $r(z)$ given by

$$r(z) = cH_0^{-1} \int_0^z \frac{dz'}{E(z')}, \quad (1)$$

with

$$E(z) \equiv [\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_X \rho_X(z)/\rho_X(0)]^{1/2}, \quad (2)$$

where $\Omega_k \equiv 1 - \Omega_m - \Omega_X$, and $\rho_X(z)$ is the dark energy density.

The dark energy equation of state, $w_X(z)$, is related to $\rho_X(z)$ as follows (Wang & Garnavich 2001):

$$w_X(z) = \frac{1}{3}(1+z) \frac{\rho'_X(z)}{\rho_X(z)} - 1, \quad (3)$$

so that

$$\frac{\rho_X(z)}{\rho_X(0)} = \exp \left\{ \int_0^z \frac{3[1+w_X(z)]}{1+z} dz \right\}. \quad (4)$$

One can see that it is easier to extract $\rho_X(z)$ from the data than to extract $w_X(z)$. To obtain the dark energy density directly, one need only take a single derivative of the luminosity distance, whereas to extract $w_X(z)$, one needs to take a second derivative as well; from Eq.(3) one can see that $w_X(z)$ is on the same footing as $\rho'_X(z)$. Specifically, Wang & Garnavich (2001) argued that $\rho_X(z)$ should be preferred since it suffers less from the smearing effect (due to the multiple integrals that relate $w_X(z)$ to $d_L(z)$) that makes constraining $w_X(z)$ extremely difficult (Maor, Brustein, & Steinhardt 2001; Barger & Marfatia 2001). Tegmark (2002) came to the same conclusion. However, researchers have generally chosen to parametrize dark energy using its equation of state $w_X(z)$. Some have used $H(z) = H_0 E(z)$ (for example, see Kujat et al. (2002); Daly & Djorgovski (2003); Nesseris & Perivolaropoulos (2004), and references therein), which is similar to $\rho_X(z)$, but measurements of which are not as straightforward to interpret, since $E(z)$ depends on Ω_m (see Eq.(2)).

In this paper, we explicitly demonstrate the advantage of using $\rho_X(z)$, instead of $w_X(z)$, to probe dark energy. Sec.2 contains a comparison of $w_X(z)$ and $\rho_X(z)$ parametrizations using current SN Ia, CMB, and LSS data. We give a recipe for parametrizing dark energy using $\rho_X(z)$ in Sec.3. Sec.4 contains a summary and discussions.

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³ See Padmanabhan (2003) and Peebles & Ratra (2003) for reviews with more complete lists of references).

2. DARK ENERGY EQUATION OF STATE VERSUS DARK ENERGY DENSITY

From SN, CMB, and LSS data, we independently reconstruct first the dark energy equation of state and then the dark energy density directly. We use (1) current SN Ia data from Tonry et al. 2003 and Barris et al. 2003 [the Tonry/Barris sample], (2) the Cosmic Microwave Background (CMB) shift parameter (Bond, Efstathiou, & Tegmark 1997) from CMB data [WMAP (Bennett et al. 2003; Spergel et al. 2003), CBI (Pearson et al. 2003), and ACBAR (Kuo et al. 2002)], and (3) the large scale structure (LSS) growth factor from 2dF (Percival et al. 2002; Verde et al. 2002; Hawkins et al. 2003) galaxy survey data. (Wang & Mukherjee 2004)

We parametrize the data first in terms of the dark energy equation of state, and then in terms of the dark energy density, to see which parametrization produces the reconstruction with the least uncertainty.

To find the function $w_X(z)$ which best fits the data, we consider a five-dimensional parameter space: the function evaluated at three discrete redshift intervals, the value of Ω_m , and the value of the Hubble constant. We assume a flat universe. It is our goal to find a set of values for these five parameters that fits the data. We use the Markov Chain Monte Carlo (MCMC) technique (Neal 1993; Lewis & Bridle 2002), which selects randomly from the 5D parameter space, evaluates χ^2 , and create a large number of sets of parameter values (each set is a MCMC sample); we use 10^6 MCMC samples⁴. For the current data, we consider the function $w_X(z)$ at three redshift values: $z=0$, $z_{max}/2$, and z_{max} (where z_{max} is the maximum redshift of SNe Ia)⁵. We find the values (that fit the data) at these points and interpolate at all intermediate redshifts. The parameters estimated from data are $w_X(0)$, $w_X(z_{max}/2)$, $w_X(z_{max})$, Ω_m , and a dimensionless Hubble constant h .

Fig.1a shows the $w_X(z)$ reconstructed from 192 SNe Ia from the Tonry/Barris sample,⁶ combined with CMB (shift parameter $\mathcal{R}_0 = 1.716 \pm 0.062$) and LSS data (growth parameter $f_0 \equiv f(z = 0.15) = 0.51 \pm 0.11$) (Wang & Mukherjee 2004). The regions inside the solid and dashed lines correspond to 68.3% and 95% confidence levels respectively; the 68.3% confidence level (C.L.) region is also shaded. The circles indicate the mean values at the three redshift points, $z=0$, $z_{max}/2$, and z_{max} . The other simultaneously estimated parameters (mean, 68.3% and 95% confidence ranges) are: $\Omega_m = .39[.29, .50][.21, .57]$ and $h = .658[.642, .674][.627, .689]$.⁷ Clearly, the equation of state is consistent with a constant $w_X(z) = -1$ for all redshifts at 95% confidence level (C.L.). At 68.3% C.L., it is consistent with a constant for $0 \lesssim z \lesssim 0.5$ and marginally consistent with $w_X(z) = -1$ for $0.5 \lesssim z \lesssim 1$.

Fig.1b shows the $\rho_X(z)$ directly reconstructed from the same data as Fig. 1. The same technique of discretizing the function $\rho_X(z)$ has been used. The solid lines

and dashed lines indicate the 68.3% and 95% confidence levels respectively (Wang & Mukherjee 2004). The other simultaneously estimated parameters (mean, 68.3% and 95% confidence ranges) are: $\Omega_m = .33[.27, .39][.22, .46]$, $h = .660[.644, .673][.630, .688]$. One can see that the uncertainties in Fig.1b on $\rho_X(z)$ obtained from the data are smaller than those on $w_X(z)$ obtained from the data. We see that the time dependence of the dark energy density deviates from a constant at 68.3% C.L. (a similar statement could not be made from the $w_X(z)$ reconstruction). With more data in the future, the statistical significance of this discrepancy will become more clear.

For comparison, in Fig.2 we have also plotted $\rho_X(z)$ obtained in a more indirect way: by first obtaining $w_X(z)$ from the data, as described above, and then integrating over redshift as in Eq.(4). Clearly the uncertainties obtained in this way are far larger than if one obtains the dark energy directly from the data. While the results from $w_X(z)$ parametrization and $\rho_X(z)$ parametrization (Wang & Mukherjee 2004) are consistent with one another, the $w_X(z)$ parametrization results have uncertainties that are several times larger. One can also obtain $w_X(z)$ indirectly from $\rho_X(z)$ in Fig.1b (similar to what was done by Alam et al. (2003)). However, doing so would require taking the derivative of the polynomial used in the interpolation, thus making the resultant $w_X(z)$ dependent on the interpolation technique used.

Our main result is that one can learn more information by reconstructing $\rho_X(z)$ rather than $w_X(z)$ from the data. At 95% C.L., both the $w_X(z)$ and $\rho_X(z)$ reconstructions are consistent with a cosmological constant. However at 68.3% C.L., the $\rho_X(z)$ reconstruction has smaller uncertainties and hence shows more information than the $w_X(z)$ reconstruction: the $\rho_X(z)$ reconstruction is *not* consistent with a time-independent dark energy. Even with the $\rho_X(z)$ parametrization, a significant number of SNe Ia at $z > 1$ from a deep SN survey on a dedicated telescope (Wang 2000a) will be required to place robust constraints on the time-dependence of $\rho_X(z)$.

3. A RECIPE FOR PARAMETRIZING DARK ENERGY USING ITS DENSITY

For the convenient application of our methodology by others, we now present a recipe for parametrizing dark energy using $\rho_X(z)$ as an arbitrary continuous function.

(1) Choose the number of redshift bins, N (the number of parameters for $\rho_X(z)$). N needs to be sufficiently large to probe the time-variation of $\rho_X(z)$. However, if N is too large, the uncertainties on all the estimated parameters will increase, leading to less stringent constraints. $N = 2$ is appropriate for current (sparse) data.

(2) The values of the dimensionless dark energy density $f_i \equiv \rho(z_i)/\rho_X(0)$ ($i = 1, 2, \dots, N$) are the independent variables to be estimated from data. Note that $z_N = z_{max}$ (the maximum redshift of SNe Ia in the data).

⁴ Here χ^2 is only used to move around efficiently in the entire parameter space (based on entropy considerations), such that for sufficient sampling, the resultant parameter distributions converge to the true probability distribution functions (pdf's). This leads to smooth pdf's since they receive contributions from all MCMC samples.

⁵ This is the largest number of values one can get out of the current (sparse) data.

⁶ Flux averaging has been performed to reduce the bias in estimated parameters due to weak gravitational lensing. See Wang (2000b) and Wang & Mukherjee (2004) for details.

⁷ The uncertainty on h are statistical error only, not including the contribution from the much larger SN Ia absolute magnitude error of $\sigma_h^{int} \simeq 0.05$. (Wang & Mukherjee 2004)

(3) Parametrize $\rho_X(z)/\rho_X(0)$ as a continuous function, given by interpolating its amplitudes at equally spaced z values in the redshift range covered by SN Ia data ($0 \leq z \leq z_{max}$), and a constant at larger z ($z > z_{max}$, where $\rho_X(z)$ is only weakly constrained by CMB data). The results should not be sensitive to the interpolation method used. Polynomial interpolation was used in Wang & Mukherjee (2004). For $N = 2$, this gives

$$\frac{\rho_X(z)}{\rho_X(0)} = 1 + (4f_1 - f_2 - 3) \frac{z}{z_{max}} + (f_2 - 2f_1 + 1) \frac{2z^2}{z_{max}^2}, \quad (5)$$

where $f_1 = \rho(z_{max}/2)/\rho_X(0)$, and $f_2 = \rho(z_{max})/\rho_X(0)$.

(4) Use Eq.(5) or its equivalent (if the interpolation method or N differs) in all equations where the factor $E(z)$ from Eq.(2) appears.

Caution: It is important to note that we are using a polynomial to interpolate $\rho_X(z)$ between equally spaced z values; the independent variables are the values of $\rho_X(z)$ at these z values, as in Eq.(5). In this case, the errors on the reconstructed $\rho_X(z)$ are tied to how the quality of the data varies with z (sparse data lead to large errors). Changing the interpolation method from polynomial interpolation to a different method should have negligible effect on the reconstructed $\rho_X(z)$. On the other hand, if a polynomial is used as a global fit function with its coefficients being the independent variables; the errors on the reconstructed $\rho_X(z)$ will not correlate with how the quality of data varies with z .

4. SUMMARY AND DISCUSSION

The critical first step in solving the mystery of dark energy is to determine whether the dark energy density $\rho_X(z)$ varies with time.(Wang & Garnavich 2001) A definitive an-

swer to this question can have profound implications for particle physics and cosmology.

Our main result is that one can learn more information by reconstructing $\rho_X(z)$ rather than $w_X(z)$ from the data. The two quantities are related by an integral, which in the case of $w_X(z)$ smears out much of the information one could otherwise learn. We show this explicitly by using a combination of SN Ia data from the Tonry/Barris sample as well as CMB (WMAP, CBI, and ACBAR) and large scale structure (2dF) data. At 95% CL, both the $w_X(z)$ and $\rho_X(z)$ reconstructions are consistent with a cosmological constant. However at 68% CL, the $\rho_X(z)$ reconstruction has smaller uncertainties and hence shows information that the $w_X(z)$ reconstruction cannot: the $\rho_X(z)$ reconstruction is *not* consistent with a time-independent dark energy, and the dark energy density appears to be increasing with redshift. Future data will be required to resolve this question.

We have shown definitively the advantage of the $\rho_X(z)$ parametrization over the $w_X(z)$ parametrization in determining the time-variation of $\rho_X(z)$. To help others apply the $\rho_X(z)$ parametrization, we have given a recipe for using the $\rho_X(z)$ parametrization in data analysis to probe dark energy (see Sec.3). Our methodology should be very useful in all data analysis aiming at unraveling the nature of dark energy.

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REFERENCES

- Alam, U., Sahni, V., Saini, T.D., and Starobinsky, A.A. 2003, astro-ph/0311364
 Ahmed, M., Dodelson, S., Greene, P.B., and Sorkin, R. 2002, astro-ph/0209227;
 Armendariz-Picon, C., Mukhanov, V., Steinhardt, P.J. 2000, Phys. Rev. Lett. 85, 4438
 Barger, V., and Marfatia, D. 2001, Phys. Lett. B498, 67-73
 Barris, B.J., et al. 2003, astro-ph/0310843, ApJ, in press
 Bennett, C., et al. 2003, ApJ, Suppl. 148, 1
 Bilic, N., Tupper, G.B, and Viollier, R., Phys.Lett. **B535** 17 (2002);
 Bond, J.R.; Efstathiou, G.; & Tegmark, M. 1997, MNRAS, 291, L33
 Caldwell, R., Dave, R., Steinhardt, P.J. 1998, Phys. Rev. Lett., 80, 1582
 Capozziello, S., Carloni, S., and Troisi, A., astro-ph/0303041;
 Carroll, S.M.; Hoffman, M., & Trodden, M. 2003, Phys. Rev. D68, 023509
 Carroll, S., Duvvuri, V., Trodden, M., and Turner, M., astro-ph/0306438.
 Daly, R.A., & Djorgovski, S.G. 2003, ApJ, 597, 9
 Defayet, C. 2001, Phys. Lett. B502, 199
 Freese, K., Adams, F.C., Frieman, J.A., and Mottola, E. 1987, Nucl. Phys. B287, 797
 Freese, K., and Lewis, M., 2002, Phys. Lett., B540, 1
 Freese, K., Nuclear Physics B (Proc. Suppl.) **124**, 50 (2003)
 Frieman, J., Hill, J., Stebbins, A., and Waga, I. 1995, Phys. Rev. Lett., 75, 2077
 Gondolo, P., and Freese, K. 2003, Phys.Rev. **D68** 063509
 Hawkins, E. et al. 2003, astro-ph/0212375, MNRAS in press
 Knop, R. A., et al. 2003, astro-ph/0309368, ApJ, in press
 Kujat, J.; Linn, A.M.; Scherrer, R.J.; & Weinberg, D.H. 2002, ApJ, 572, 1
 Kuo, C.L., et al. 2002, submitted to ApJ, astro-ph/0212289
 Lewis, A., & Bridle, S. 2002, Phys. Rev. D, 66, 103511, astro-ph/0205436
 Maor, I., Brustein, R., & Steinhardt, P.J. 2001, Phys. Rev. Lett., 86, 6; Erratum-ibid. 87 (2001) 049901
 Meng, X., & Wang, P. (2003), Class.Quant.Grav. 20, 4949
 Neal, R.M. 1993, Technical Report CRG-TR-93-1, ftp://ftp.cs.utoronto.ca/pub/radford/review.ps.gz
 Nesseris, S., & Perivolaropoulos, L. 2004, astro-ph/0401556
 Padmanabhan, T. 2003, Physics Reports 380, 235-320
 Parker, L., and Raval, A. 1999, Phys. Rev. D **60**, 063512
 Peebles, P. J. E.; Ratra, B. 1988, ApJ, 325L, 17
 Peebles, P. J. E.; Ratra, B. 2003, Rev.Mod.Phys. 75, 559-606
 Pearson, T.J., et al. 2003, ApJ, 591, 556-574
 Percival, W.J. et al. 2002, MNRAS, 337, 1068
 Perlmutter, S., et al. 1999, ApJ, 517, 565
 Phillips, M.M., ApJ, 413, L105 (1993)
 Puetzfeld, D., and Chen, X. 2004, gr-qc/0402026
 Riess, A. G., et al. 1998, AJ, 116, 1009
 Riess, A.G., Press, W.H., and Kirshner, R.P., ApJ, 438, L17 (1995)
 Riess, A.G., et al. 1999, AJ, 118, 2675
 Spergel, D.N., et al. 2003, astro-ph/0302209
 Tegmark, M. 2002, Phys. Rev. D66, 103507
 Tonry, J.L., et al. 2003, ApJ, 594, 1-24
 Verde, L. et al. 2002, MNRAS, 335, 432
 Wang, Y. 2000a, ApJ, 531, 676
 Wang, Y. 2000b, ApJ, 536, 531
 Wang, Y., and Garnavich, P. 2001, ApJ, 552, 445
 Wang, Y., and Lovelace, G. 2001, ApJ, 562, L115
 Wang, Y.; Freese, K.; Gondolo, P.; & Lewis, M. 2003, ApJ, 594, 25
 Wang, Y., & Mukherjee, P. 2004, astro-ph/0312192, ApJ submitted
 Wetterich, C. 1988, Nucl. Phys. **B302**, 668
 Zlatev, I., Wang, L., and Steinhardt, P.J. 1999, Phys. Rev. Lett. **82**, 896

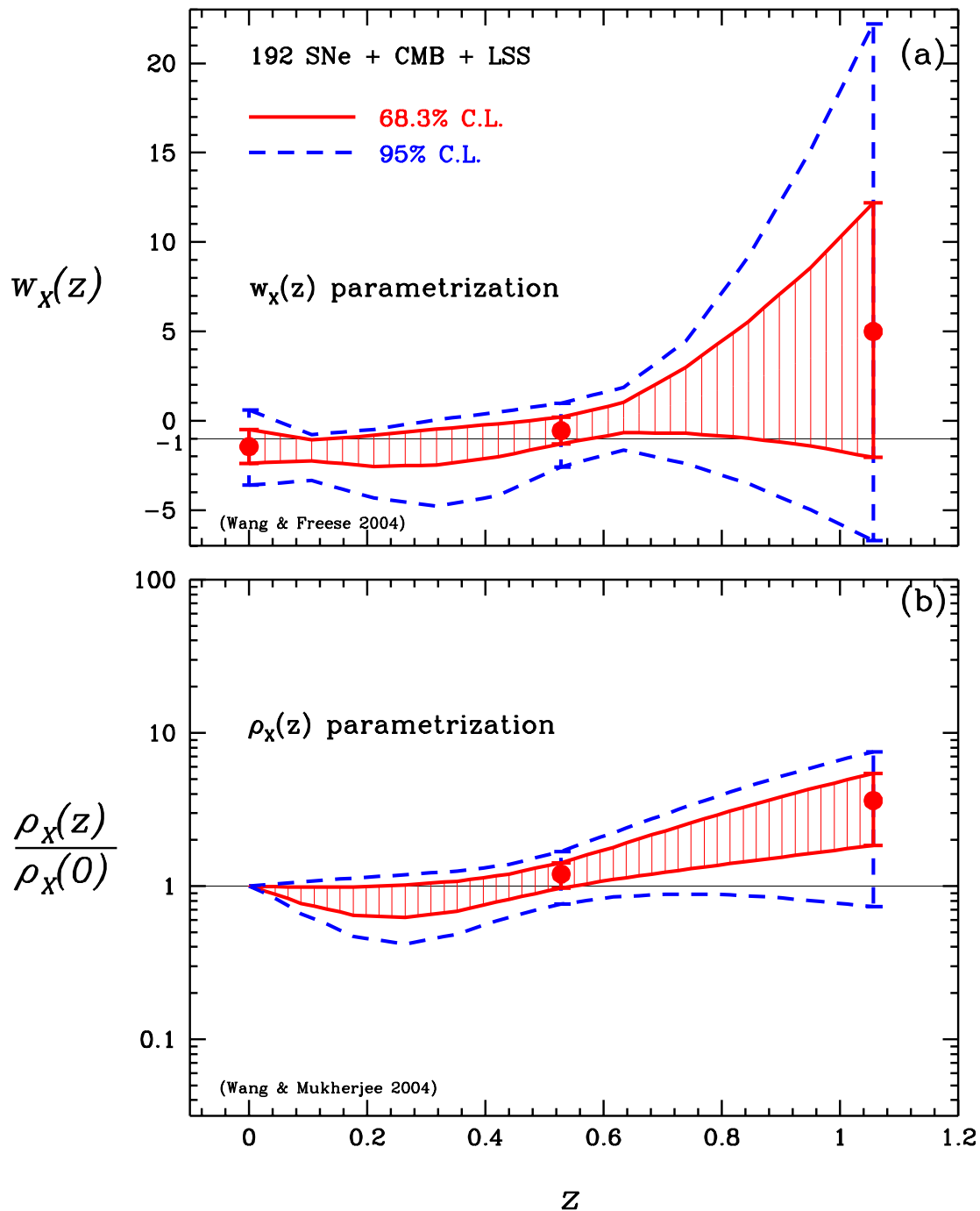


FIG. 1.— (a) The $w_X(z)$ reconstructed from 192 SNe Ia from the Tonry/Barris sample, flux-averaged, and combined with CMB and LSS data. The regions inside the solid and dashed lines correspond to 68.3% and 95% confidence levels respectively; the 68.3% confidence level region is also shaded. Circles indicate the mean values of the regions. (b) The $\rho_X(z)$ reconstructed from the same data as Fig.1(a), with the same shading and line types (Wang & Mukherjee 2004). Whereas the $w_X(z)$ reconstruction is consistent at 68.3% C.L. with a cosmological constant, the $\rho_X(z)$ reconstruction is not.

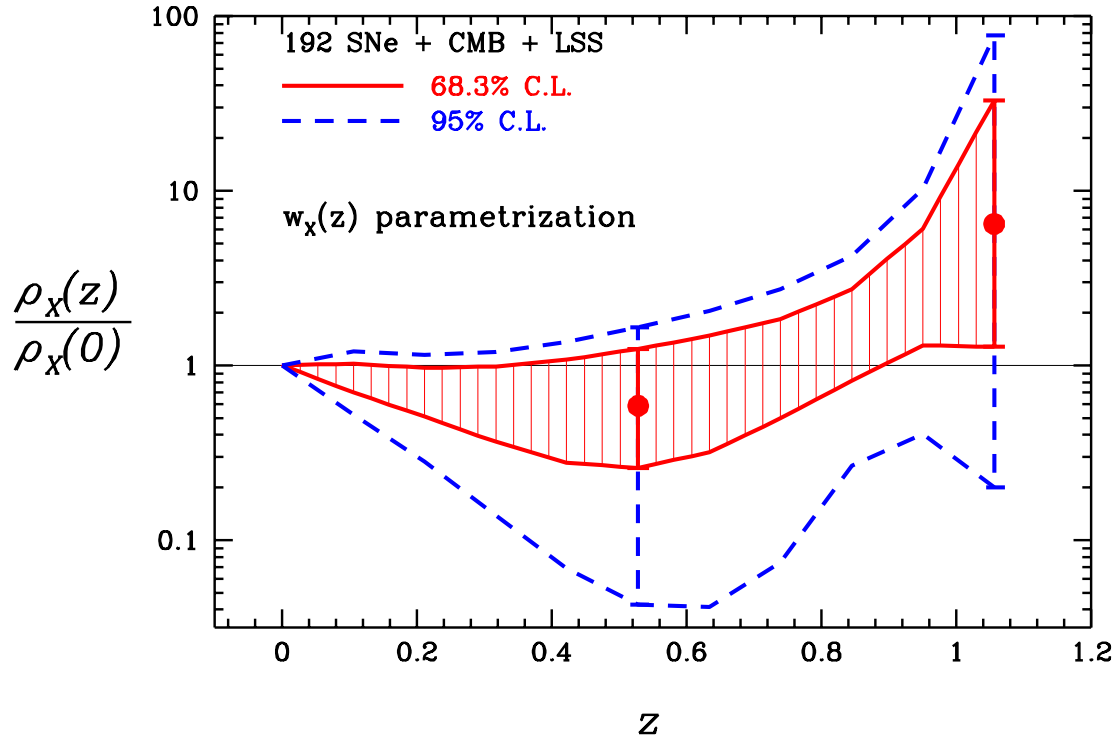


FIG. 2.— The $\rho_X(z)$ reconstructed by taking the integral in Eq.(4) of the $w_X(z)$ plotted in Fig.1(a). Again, we use the same data, with the same shading and line types. Clearly the uncertainties in this method are much greater than if one obtains $\rho_X(z)$ directly from the data as in Fig.1(b).