Phase Transitions in Nucleonic Matter and Neutron-Star Cooling

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A new scenario for neutron-star cooling is proposed, based on the correspondence between pion condensation, occurring in neutron matter due to critical spin-isospin fluctuations, and the metal-insulator phase transition in a two-dimensional electron gas. Beyond the threshold density for pion condensation, where neutron-star matter loses its spatial homogeneity, the neutron single-particle spectrum acquires an insulating gap that quenches neutron contributions to neutrino-production reactions and to the star's specific heat. In the liquid phase at densities below the transition point, spin-isospin fluctuations are found to play dual roles. On the one hand, they lead to a multi-sheeted neutron Fermi surface that extends to low momenta, thereby activating the normally forbidden direct-Urca cooling mechanism; on the other, they amplify the nodeless P-wave neutron superfluid gap while suppressing S-wave pairing. In this picture, lighter stars without a pion-condensed core experience slow cooling, while enhanced cooling occurs in heavier stars through direct-Urca emission from a narrow shell of the interior.

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Observations designed to detect and measure thermal radiation from neutron stars with ages between 10^2 and 10^5 years can provide valuable information on the internal temperatures T of these stars at the stage when cooling proceeds via neutrino emission from their neutrino-transparent cores [1]. Such information may be used to constrain theories of hadronic matter at high density. The relevant neutrino reactions are (i) the so-called direct Urca (DU) process, in which beta decay and inverse beta decay operate in tandem on thermally activated nucleons and electrons, (ii) the modified Urca (MU) process, involving a nucleon spectator in the Urca reactions, (iii) neutrino bremsstrahlung from nucleon-nucleon collisions, and (iv) neutrino emission due to Cooper pairing in the superfluid phase [2, 3]. Of these, only the DU mechanism can produce rapid cooling, as the data seems to require for some stars. However, its operation is normally forbidden by the large mismatch of neutron and proton Fermi momenta, which can be compensated by spectators in the MU process generally associated with slow cooling. The importance of the Cooper-pairing mechanism (iv) is highly sensitive to the proton and neutron gaps, whose values in turn depend crucially on the internal composition and structure of the star.

A new perspective on neutron-star interiors is suggested by laboratory studies of (i) the metal-insulator transition in two-dimensional (2D) silicon samples of low disorder [4], and (ii) the liquid-solid phase transition in 2D ^{3}He [5]. Quite far from the transition point, the electron and ^{3}He systems involved may be described as normal Fermi liquids obeying Landau theory. Beyond the transition, these systems become inhomogeneous, with an insulating gap in the single-particle (sp) spectrum. Similar behavior of the sp spectra has been observed in the "normal" states of high-T_c superconductors [6]. Cold neutron matter also behaves as a normal (or superfluid) Fermi liquid at relatively low densities. With increasing depth in the star and coincident increase in baryon density ρ, spin-isospin fluctuations grow in importance. At a critical density ρ_c, a condensate of spin-isospin excitations, specified by critical wave number $q_c \sim p_{Fn}$, forms in the channel with π^0 quantum numbers [7, 8]. State-of-the-art microscopic calculations [9] predict that π^0 condensation (PC) sets in at a density comparable to the equilibrium density $\rho_0 = 0.16$ fm^{-3} of symmetrical nuclear matter, while earlier phenomenological estimates put ρ_c in the range $0.3-0.4$ fm^{-3} [7, 8] . Comparing these values to the central density $\rho(0) \simeq (0.5-1.0)$ fm^{-3} [9] of a typical neutron star, we infer that at low T, a significant part of the stellar bulk exists in the inhomogeneous PC phase, rather than as a Fermi liquid.

Accordingly, in contrast to the pionic cooling scenario reviewed in Ref. [10], we propose that in the domain occupied by the PC, the neutron spectrum acquires an insulating gap exceeding in value any expected superfluid gaps in neutron matter. It follows that in this region of the star all cooling processes requiring the participation of neutrons are strongly inhibited. The neutron contribution to the specific heat $C(T)$ is suppressed as well, since its leading term $\sim T^3$ now derives from the phonon spectrum rather than from the gapped sp excitations. The properties of the embedded proton subsystem are not affected so dramatically. Analogously to the electronic subsystem in solids, the protons form a "conductivity band" with a sp spectrum of unaltered shape, specified by an effective mass, and the behavior of their contribution $C_p(T) \sim T$ to the specific heat remains unchanged. Thus, in the new cooling scenario, the large region of the star in which the π^0 condensate holds sway becomes irrelevant to the cooling process, except for the effects of neutrino-generating reactions involving protons as the only nucleonic participants, along with the proton contribution to the stellar specific heat.

As the internal temperature drops, the proton subsystem undergoes a superconducting phase transition, which we assume to originate from phonon-induced attraction. A rough estimate of the critical temperature, $T_c^p \simeq 20$ keV, is then obtained in terms of the proton Fermi energy in the same way as for ordinary superconductors.

Having addressed the organization of neutron-star matter at moderately high densities, let us now analyze what is transpiring in the domain $0.5\,\rho_0 < \rho < \rho_c$ corresponding to the outer core, where the stellar material is a neutron liquid with fluid admixtures of protons and neutralizing leptons. We argue that in this region, spin-isospin fluctuations have a strong impact on neutron pairing, suppressing it in the S-wave channel and enhancing it in P-waves. This situation is familiar from the physics of superfluid ^{3}He, where spin fluctuations play the key role in promoting P-pairing over S-pairing [11]. To estimate the neutron triplet-P gap, we adopt the BCS formalism and write the gap equation as

$$\hat{\Delta}(\mathbf{p}) = -\int \left[\mathcal{V}(\mathbf{p},\mathbf{p}')+\mathcal{V}^{\pi}\left(\mathbf{p}-\mathbf{p}';\omega=E(\mathbf{p}')\right)\right] \frac{\hat{\Delta}(\mathbf{p}')}{2E(\mathbf{p}')} d\tau' \,, \tag{1}$$

with $d\tau = d^3p/(2\pi)^3$. In the triplet P-wave channel, the gap function has the form $\hat{\Delta}(\mathbf{p}) = i d_{ik} p_k \sigma_i \sigma_2$, with coefficients d_{ik} yet to be determined. The quasiparticle energy $E(\mathbf{p})$ is given by $E(\mathbf{p}) = \left[\xi^2(p) + \mathrm{Tr}(\hat{\Delta}(\mathbf{p})\hat{\Delta}^{\dagger}(\mathbf{p}))\right]^{1/2}$, where $\xi(p)$ is the sp spectrum of the normal Fermi liquid relative to the chemical potential μ, and $\mathrm{Tr}(\hat{\Delta}(\mathbf{p})\hat{\Delta}^{+}(\mathbf{p})) = \Delta^2 + \sum_m a_m Y_{2m}(\mathbf{n})$. In triplet P-pairing, the regular interaction $\mathcal{V}$ is nearly exhausted by the frequency- and density-independent spin-orbit component of the scattering amplitude. The fluctuation contribution to the pairing interaction is written as [7] $\mathcal{V}^{\pi}_{\alpha\beta,\gamma\delta}(q,\omega) = \lambda_n^2(q)(\boldsymbol{\sigma}_{\alpha\gamma}\cdot\mathbf{q})\,\mathrm{Re}\,D(q,\omega)(\boldsymbol{\sigma}_{\beta\delta}\cdot\mathbf{q})/m_\pi^2$, where λ_n is an effective charge accounting for the renormalization of the associated vertex part. The propagator D is given by the Migdal formula $-D^{-1}(q,\omega) = q^2 + m_\pi^2 + \Pi_{NI}(q,\omega=0) + \Pi_{NN}(q,\omega)$, made up of the ordinary part $\Pi_{NN}(q,\omega) = -Bq^2 - iq|\omega|M^2/(2\pi m_\pi^2)$ of the pion polarization operator Π, along with the term $\Pi_{NI}(q,\omega=0) = -q^2\rho/\rho_I(1+q^2/q_I^2)$ arising from pion conversion into a Δ-isobar and neutron hole. In the domain of critical fluctuations, we have [7]

$$-D^{-1}(q\to q_c;\rho\to\rho_c;\omega{=}0) = \gamma^2\frac{(q^2-q_c^2)^2}{q_I^2} + \eta\kappa^2 q_I^2 \,, \tag{2}$$

where $\eta = (\rho_c-\rho)/\rho_c$. The parameters γ and κ are determined from the obvious relations $(1-B)q_c^2 + m_\pi^2 - q_c^2 r_c\zeta_c = 0$ and $1 - B - r_c\zeta_c^2 = 0$, with $r_c = \rho_c/\rho_I$ and $\zeta_c = (1+q_c^2/q_I^2)^{-1}$. Simple algebra leads to $q_c = (m_\pi q_I)^{1/2}(1-B)^{-1/4}$, $r_c = \left(m_\pi/q_I + \sqrt{1-B}\right)^2$, $\gamma^2 = r_c\zeta_c^3$, and $\kappa^2 = r_c\zeta_c q_c^2/q_I^2$. Employing the parameter set $q_I^2 = 5\,m_\pi^2, \rho_I \simeq 1.8\,\rho_0$, and $B \simeq 0.7$ from Ref. [7], one arrives at $q_c \simeq 1.9\,m_\pi$, $\rho_c \simeq 2\,\rho_0$, $\gamma \simeq 0.4$, and $\kappa \simeq 0.7$. As will be seen, the contribution to the gap value from spin-orbit forces is insignificant; with its neglect Eq. (1) is finally recast as

$$\sum_k d_{ik}p_k = -\int \frac{\lambda_n^2(q)}{m_\pi^2}\left(\sum_k d_{ik}q^2p_k' - 2\sum_{k,l} d_{kl}q_iq_kp_l'\right) \times \frac{\mathrm{Re}D(q,|\xi(p')|)}{2E(\mathbf{p}')}\,d\tau' \,, \tag{3}$$

where $\mathbf{q} = \mathbf{p} - \mathbf{p}'$. In arriving at this result, we have made use of the relations $\sum_\delta(\sigma_l\sigma_i\sigma_2)_{\alpha\delta}(\sigma_m)_{\beta\delta} = (\sigma_l\sigma_i\sigma_2\sigma_m^+)_{\alpha\beta} = -(\sigma_l\sigma_i\sigma_m\sigma_2)_{\alpha\beta}$.

Unfortunately, the pairing problem (3) still defies full solution. In neutron matter, only the spectrum of solutions restricted to the $^3P_2$–$^3F_2$ channel has been explicated [12]. The solution set contains both nodeless and nodal combinations of the different basis states. At $T \ll T_c$, a nodeless solution wins the energetic competition, and we anticipate that this feature, also inherent in superfluid ^{3}He, is present here as well. For nodeless solutions, an adequate approximation to the gap in the sp spectrum can be obtained by retaining only the Δ^2 term in $\mathrm{Tr}(\hat{\Delta}(\mathbf{p})\hat{\Delta}^{+}(\mathbf{p}))$. The matrix d_{ik} then becomes proportional to δ_{ik} and the angle integration is obviated, giving rise to the expression $q^2\,(\mathbf{p}\cdot\mathbf{p}') - 2(\mathbf{q}\cdot\mathbf{p})(\mathbf{q}\cdot\mathbf{p}')$ in the numerator of the r.h.s. of Eq. (3). Furthermore, the system (3) becomes decoupled, and we are left with a single integral equation to solve. Exploiting the fact that the propagator $D(q,\omega=0)$ is peaked at $q = q_c < 2p_{Fn}$, this equation is simplified to

$$1 = \frac{\lambda_n^2 q_c^2 M}{8\pi\gamma\kappa m_\pi^2 p_F}\int_0^\infty \mathrm{Re}\,\frac{d\xi}{\left[(\eta + iM^2\xi/2\pi\kappa^2 m_\pi^2 q_c)\,(\xi^2+\Delta^2)\right]^{1/2}} \,, \tag{4}$$

where $\lambda_n^2 \equiv \lambda_n^2(q_c)$. In deriving this formula, we have made the replacements $\mathbf{p}\cdot\mathbf{p}' = (p^2 + (p')^2 - q^2)/2 \to p_F^2 - q_c^2/2$, $\mathbf{q}\cdot\mathbf{p} = p^2 - \mathbf{p}\cdot\mathbf{p}' \to q_c^2/2$, and $\mathbf{q}\cdot\mathbf{p}' = \mathbf{p}\cdot\mathbf{p}' - (p')^2 \to -q_c^2/2$; their validity has been confirmed in numerical calculations. Since the parameter λ_n is as yet uncertain, numerical calculations of the gap $\Delta(\rho)$ have been performed for three different values, with the results shown in Fig. 1. We see that the P-wave neutron gap is dramatically magnified in comparison with standard estimates of S-wave and P-wave gaps [13] that ignore spin-isospin fluctuations, justifying our neglect of the regular interaction.

Next we assess the inhibitory effect of spin-isospin fluctuations on proton S-wave pairing. To estimate the suppression factor, we appeal to the property

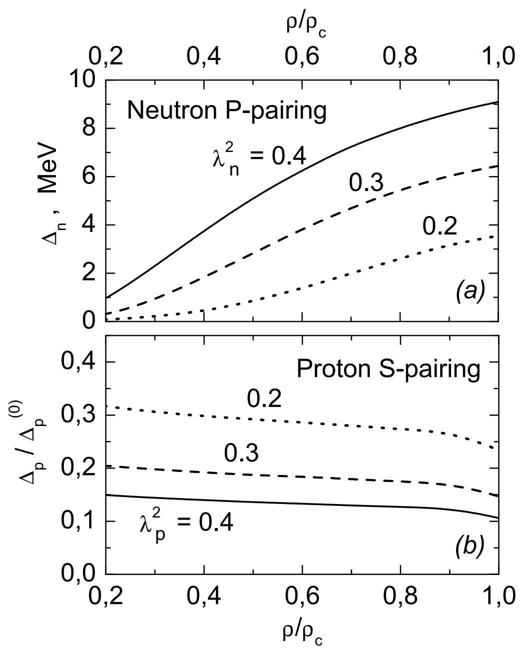

FIG. 1: Neutron P-wave gap Δ_n at the Fermi surface (in MeV) (panel (a)) and suppression factor $\Delta_p/\Delta_p^{(0)}$ for proton S-wave gap (panel (b)), versus baryon density ρ in units of ρ_c. (The proton fraction is taken as 0.06.) Values of λ_n^2 and λ_p^2 are indicated by numbers near the corresponding curves.

$\langle S{=}0|\sigma_i^1\sigma_k^2|S{=}0\rangle = -\delta_{ik}$ and multiply both sides of Eq. (1), rewritten for the full proton S-wave gap function Δ_p, by $\Delta_p^{(0)}/2E_0(p)$, where $E_0(p){=}\left[\xi^2(p)+(\Delta_p^{(0)})^2\right]^{1/2}$ and $\Delta_p^{(0)}$ is the gap value found [13] when critical fluctuations are neglected. Integration over the intermediate proton momentum and simple manipulations utilizing the gap equation for $\Delta_p^{(0)}$ lead to

$$\int\left(\frac{1}{E_0(\xi)}-\frac{1}{E(\xi)}\right)d\xi=\iint\frac{I(\xi,\xi_1)}{E_0(\xi)E(\xi_1)}d\xi d\xi_1\ , \tag{5}$$

where

$$I(\xi,\xi_1)=\frac{\lambda_p^2 M}{16\pi^2 m_\pi^2 p_F}\int\limits_{|p-p_1|}^{p+p_1} q^3\,D(q;|\xi_1|)dq \tag{6}$$

with $p\equiv p(\xi)$ determined from the formula for the sp spectrum $\xi(p)$. Results of numerical calculations of the ratio $\Delta/\Delta^{(0)}$ based on Eq. (5) are also presented in Fig. 1. Proton S-pairing in the liquid phase is suppressed over a wide density range.

The above results suggest that neutron P-pairing in the liquid domain of the core is enhanced so strongly by spin-isospin fluctuations that the neutrino emissivity due to neutron Cooper pairing, which behaves as [3] $Q_{\rm nCp}\sim(\Delta/T)^6\exp(-2\Delta/T)$, is completely suppressed by the exponential factor. At the same time, recent microscopic calculations [14] indicate that the $^1S_0$ proton gap in neutron-star matter depends crucially on the density ρ, falling off rapidly when ρ exceeds ρ_0. In view of the suppression factor from spin-isospin fluctuations found here and plotted in Fig. 1, we conclude that proton pairing is irrelevant to the neutrino-cooling stage.

We now turn to the role of the direct Urca process. Despite their limitations, the available experimental data on the surface temperatures T_s of neutron stars give evidence for the existence of slow and rapid cooling tracks. It is generally presumed that DU reactions are somehow involved in the rapid cooling process. Yet if one adopts the best available equations of state of neutron matter [9] derived from first principles, this highly efficient cooling mechanism is precluded in all but the most massive neutron stars, because of the large difference between neutron and proton Fermi momenta. However, a novel route to the DU process in dense matter is opened by a rearrangement of the neutron quasiparticle distribution $n(p)$ at a critical density $\rho_r<\rho_c$. The rearrangement is precipitated by critical spin-isospin fluctuations seething in the neutron liquid near its inner boundary with the PC domain [15].

To elucidate this phenomenon, we employ the Landau relation for the sp spectrum $\epsilon(p)$ in the specific form

$$\frac{\partial\epsilon(p)}{\partial\mathbf{p}}=\frac{\partial\epsilon_0(p)}{\partial\mathbf{p}}-\frac{1}{2}\int\mathcal{V}^\pi(\mathbf{p}-\mathbf{p}_1,\omega=0)\frac{\partial n(\mathbf{p}_1)}{\partial\mathbf{p}_1}d\tau_1\ , \tag{7}$$

where $\epsilon_0(p)=p^2/2M_0^*$ is the regular part of the neutron spectrum with $M_0^*\simeq 0.7M$, the customary value of the neutron effective mass in the absence of spin-isospin fluctuations. Upon straightforward momentum integration, relation (7) yields a closed RPA-like equation

$$\epsilon(p)=\epsilon_0(p)-\frac{1}{2m_\pi^2}\iint\limits_{|p-p_1|}^{p+p_1}\lambda_n^2(q)\frac{q^3}{p}\,D(q,0)\,n(p_1)\,\frac{p_1dp_1\,dq}{(2\pi)^2}\ , \tag{8}$$

well suited to investigation of the rearrangement of the Landau state as the density climbs to the critical value ρ_c. Results of numerical calculations based on Eq. (8), depicted in Fig. 2, indicate that the neutron Fermi surface becomes doubly connected at $\eta_r=(\rho_c-\rho_r)/\rho_c\simeq 0.065$.

With the redistribution of quasiparticles in momentum space, an inner neutron Fermi surface emerges at a low momentum p_i, enabling momentum/energy conservation in the DU reactions and unleashing rapid cooling. When the neutron liquid becomes superfluid under decreasing temperature, a triplet pairing gap Δ_i forms on the inner Fermi surface and applies the brakes to DU emission. Even so, we find that this novel opportunity for DU cooling [15] is not destroyed. To gauge the neutron gap value Δ_i at the inner Fermi surface, we again employ Eq. (1). Numerical computation yields

$$\Delta_i\simeq\Delta(p_F)\,p_i/p_F\ . \tag{9}$$

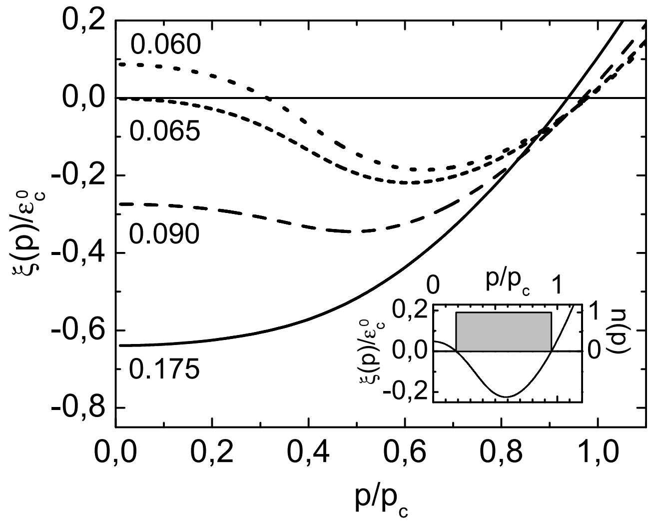

FIG. 2: Neutron quasiparticle spectrum $\xi(p)$ in units of $\epsilon_c^0 = p_c^2/2M = (3\pi^2\rho_c)^{1/3}/2M$, plotted for different densities. Numbers near the curves give the corresponding values of $\eta = (\rho_c - \rho)/\rho_c$. Inset shows the quasiparticle spectrum $\xi(p)$ evaluated for $\eta = 0.062$, together with the associated quasiparticle momentum distribution $n(p)$.

The DU process may operate with full force only if the gap value Δ_n is markedly less than 100 keV. According to Eq. (9), this constraint is met at the inner Fermi surface if $p_i(\rho) \leq 0.02 p_F$. The numerical calculation underlying Fig. 2 indicates that this inequality does hold in a thin shell of stellar material where $\rho - \rho_r \sim 5 \times 10^{-4}\rho_c$.

Now consider, in outline, how our model explains experimental cooling data. We assume that neutron stars such as RX J0822−4300 and PSR B1055−52 have relatively low masses, such that pion condensation is not triggered in their cores. We then attribute the high values of their surface temperatures to the absence of cooling mechanisms other than neutrino bremsstrahlung from pp collisions, emission processes involving neutrons being switched off by substantial neutron P-pairing.

In explanation of the enhanced cooling rates of the Vela, Geminga, and 3C58 pulsars, we suggest that their central densities exceed some 0.5 fm^{-3}, and hence that a pion condensate occupies a significant portion of their interiors. Rearrangement of the neutron quasiparticle distribution in a small region adjacent to the boundary between liquid and solid phases lifts the ban on the DU process. However, the DU cooling rate, proportional to the volume where this mechanism operates vigorously, is suppressed due to the restriction of the process to a narrow shell. Consequently, the corresponding T_s values need not lie far below those of the first group of neutron stars. One feature of our model that warrants further examination is the fast transition from slow to enhanced cooling under increasing stellar mass.

Restriction of DU reactions to a narrow shell does not apply for the most massive neutron stars, in which the internal pressure becomes high enough to melt the pion-condensate lattice. In the dense liquid core, the DU reactions are allowed to proceed apace even if the standard microscopic equation of state [9] is employed. As yet, no observational evidence exists for such an unrestrained DU cooling mechanism.

In this letter, we have proposed a new picture of the interior of neutron stars based on the similarity between pion condensation and the metal-insulator phase transition in the 2D strongly-correlated Fermi systems. We have demonstrated that the incorporation of spin-isospin fluctuations has dramatic effects on the neutron and proton gaps in the liquid part of the stellar interior and induces a rearrangement of the neutron Fermi surface that triggers the direct-Urca reaction. The associated cooling scenario provides for slow and accelerated cooling tracks that do not conflict with observational data.

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