CONSEQUENCES OF OBSERVATIONAL UNCERTAINTIES ON THE DETECTION OF COSMIC TOPOLOGY

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It is well known that as a consequence of local nature of general relativity, the global topology of space-time remains undetermined by the Einstein's field equations. This coupled with the enormous recent increase in high resolution cosmological observations has led to a great deal of interest in the possibility that the universe may possess compact spatial sections with a non-trivial topology (see for example Refs. [2](#page-2-0) and [3\)](#page-3-0). These observations have also shown that the spatial curvature is very small (and possibly 0). [8](#page-3-1) Whatever the nature of cosmic topology may turn out to be, the issue of its detectability is of fundamental importance.

Motivated by these observational results, a study was recently made of the question of detectability of the cosmic topology in nearly flat universes. It was demonstrated that as $\Omega_0 \to 1$ increasing families of possible topologies become undetectable by methods based on image (or pattern) repetitions (see Refs. [4](#page-3-2) – [6\)](#page-3-3). However, measurements of the density parameters unavoidably involve observational uncertainties, and therefore any study of the detectability of the cosmic topology should take such uncertainties into account.

In a recent paper, $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ we studied the sensitivity of the detectability of cosmic topology to the uncertainties in the density parameters, using two complementary methods. Here we briefly summarise some of those results.

As in standard cosmology we assume the universe is modelled by a 4-manifold $\mathcal{M} = \mathcal{R} \times M$, with a locally isotropic and homogeneous Robertson-Walker (RW) metric, with a matter-energy content well approximated by dust (of density ρ_m) plus a cosmological constant Λ , with associated fractional densities $\Omega_m = 8\pi G \rho_m / (3H^2)$ and $\Omega_{\Lambda} \equiv \Lambda c^2/(3H^2)$, and $\Omega_0 = \Omega_m + \Omega_{\Lambda}$. We also assume a small but non-zero curvature, since a flat universe has no preferred length scale, and therefore the

cosmological parameters impose no constraints on observable candidate manifolds. The redshift-(comoving)-distance relation in units of curvature radius then takes the form

$$
\chi(z) = \sqrt{|1 - \Omega_0|} \int_0^z \left[(1 + x)^3 \Omega_{m0} + \Omega_{\Lambda 0} - (1 + x)^2 (\Omega_0 - 1) \right]^{-\frac{1}{2}} dx , \quad (1)
$$

To study the topology of the spatial sections M of the universe, we need a topological invariant length that could be put into correspondence with depth of surveys. We employ the injectivity radius r_{inj} , the radius of the smallest sphere 'inscribable' in M , which is defined as half the length ℓ_M of the smallest closed geodesics, $r_{inj} = \frac{\ell_M}{2}$ (for details see ⁴). A manifold M is then detectable in principle in a survey of depth z if the density parameters are such that $\chi(z) > r_{inj}$. If $\chi(z) \leq r_{inj}$ then the topology is undetectable by any pattern repetition method.

This question can be restated in terms of countour lines in the $\Omega_{m0} - \Omega_{\Lambda 0}$ parametric plane. For any given manifold M with injectivity radius r_{inj}^M and fixed survey depth z_{obs} , we can define the contour curve $\chi(z_{obs}, \Omega_{m0}, \Omega_{\Lambda 0}) = r_{inj}^M$. This curve lies in either of two regions: the positive curvature $(\Omega_0 > 1)$ or the negative curvature $(\Omega_0 < 1)$ semi-planes, depending on whether the manifold is respectively spherical or hyperbolic in nature. The contour curve further subdivides its semiplane in a region where the topology is undetectable $(\chi_{obs} < r_{inj}^M)$, and a region where the topology is detectable in principle $(\chi_{obs} \geq r_{inj}^M)$. Therefore, given this curve, it would be possible to determine the (un)detectability of any given nonflat manifold for a range of density parameters. The question then becomes how to determine these countour curves.

In a recent work¹, we developed two complementary linear approximations for countour curves, one that necessarily overestimates detectability (by approximating the countour curve by its tangent line, and therefore named the tangent line method), and one that underestimates detectability (the secant line method, which approximates the countour curve by the line connecting its the extremes, $(\Omega_{m0}, 0)$ and $(0, \Omega_{\Lambda 0})$. We shall not describe either method in detail here, but see Figure 1 for a qualitative description.

We found the numerical results from both methods to be in good agreement, for density values compatible with current observations, $\frac{8}{3}$ $\frac{8}{3}$ $\frac{8}{3}$ thus demonstrating that they provide good approximations to the contour curve. Furthermore, the equation for the secant line can be obtained analytically in the limit $z \to \infty$. With this equation one can write the following inequalities, which is useful to state the conditions for (un)detectability of cosmic topology:

$$
\cosh^2\left(\frac{r_{inj}^M}{2}\right)\Omega_{m0} + \Omega_{\Lambda 0} > 1, \quad \text{for} \quad \Omega_0 < 1,
$$

$$
\cos^2\left(\frac{r_{inj}^M}{2}\right)\Omega_{m0} + \Omega_{\Lambda 0} < 1, \quad \text{for} \quad \Omega_0 > 1.
$$
 (2)

This last result is of particular interest, because it allows the study of the detectability of topology not only in individual manifolds, but also in whole classes of

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manifolds^{[1](#page-2-1)}

Figure 1. A schematic representation of the secant line (SL) and tangent line (TL) methods. The convexity of the contour curve for $\Omega_0 > 1$ can be proven analytically. The topology is shown to be detectable in principle in region I by the TL method, and undetectable in region IV by the SL method. Regions II and III are respectively detectable and undetectable, but are not discriminated by either methods. But II and III are only a very small fraction of the uncertainty region U for manifolds whose contour curves intersect the uncertainty region.

A consequence of these results is that the closed form inequalities [\(2\)](#page-1-0) can be seen, to a very good approximation, as establishing conditions for detectability in principle as well, as can be shown by comparison with numerical values obtained from both methods for $z = 1100$. For high redshifts we can therefore use [\(2\)](#page-1-0) to separate the parameter plane into undetectable and detectable sub-regions with great accuracy. The closed form of the inequalities makes its application quite straightforward and potentially more useful.

Finally even though we have used a ΛCDM framework, similar methods could be developed for other cosmological models [7](#page-3-4) with different redshift-distance relations in order to obtain conditions for undetectabilty of cosmic topology.

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