

2:3 Twin Quasi-Periodic Oscillations in Magnetohydrodynamic Accretion Flows

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Abstract

We study the radial and vertical oscillations in the three-dimensional magnetohydrodynamic (MHD) accretion flows around black holes. General relativistic effects are taken into account by using pseudo-Newtonian potential. We find that the structure of MHD flows is changed at $3.8 r_s \leq r \leq 6.3 r_s$ and the two pairs of quasi-periodic oscillations (twin QPOs) are excited in that region. The time evolution of the power spectrum density (PSD) indicates that these twin QPOs are most likely to be produced by the resonance between the Keplerian frequency ν_K and the epicyclic frequency ν_κ . The PSD shows that the lower peak frequency ν_l corresponds to ν_K , while the upper peak frequency ν_u corresponds to $\nu_K + \nu_\kappa$. The ratio of two peak frequencies is close to 2:3. The results provide the first direct evidence for the excitation of the resonant disk oscillation in the MHD accretion flows.

Key words: magnetohydrodynamics: MHD – accretion, accretion disks – black holes – quasi-periodic oscillations – relativity – X-rays: stars

1. Introduction

Quasi-periodic X-ray brightness oscillations (so-called QPOs) have been observed in some low-mass X-ray binaries (LMXBs) including neutron stars (NSs) and black hole candidates (BHCs). After the discovery of millisecond QPOs (so-called kHz QPOs) in NSs, the study of kHz QPOs have been paid more attention. Since their peak frequencies are close to the orbital frequency at the innermost circular orbit (ISCO) around NSs, the structures of both the disk and the magnetosphere close to NSs can be obtained by studying their origin (see van der Klis 1999 for a review).

In almost all kHz QPOs, the power spectrum density (PSD) shows twin peaks whose frequencies are correlated with their X-ray intensity and their peak separation are almost constant (e.g., Strohmayer et al. 1996; Wijnands et al. 1997; Ford et al. 1997; Zhang et al. 1998). In some NSs, a frequency separation of the twin peaks is comparable to a frequency of a third QPO, which is detected during type I X-ray bursts.

Since the third QPO is inferred to as a spin frequency of NSs, the excitation of twin QPOs are conventionally explained by the beat-frequency modulation between the Keplerian frequency of the disks and the spin frequency of the NSs (so-called beat-frequency model: see van der Klis 2000 for a review). However, in contrast to the beat-frequency model, the peak separations of some kHz QPOs are not constant (e.g. Sco X-1: van der Klis 1997, 4U 1608-52: Méndez et al. 1998).

QPOs are also found in several BHCs (e.g., for GRS

1915+105: Morgan, Remillard, and Greiner 1997, for XTE J1550-564: Remillard et al. 1999a, for GRO J1655-40: Remillard et al. 1999b). The PSD of QPOs in BHCs shows single peak and its frequency is relatively stable. In this regards, QPOs in NSs and BHCs have been thought to have different origins. However, the recent discoveries of twin peak QPOs in BHCs (e.g. Remillard et al. 2002) have cast some doubt on the distinction of QPOs in NSs and BHCs. Since the ratio of two peak frequencies is about 2:3 in both BHCs and NSs, the origin of twin QPOs may be identical to the accretion disk.

Interpretations of QPOs in the context of disk oscillations have been developed by many authors (e.g., trapped disk oscillations: Kato & Fukue 1980; Okazaki, Kato, & Fukue 1987; Nowak & Wagoner 1991, 1992, resonances of disk oscillations: Abramowicz & Kluzniak 2001; Kato 2003; Abramowicz et al. 2003). These works investigate a growth of unstable oscillation modes in geometrically thin disks around BHs in order to produce a large fluctuations of X-ray intensity (see Kato 2001 for a review).

The first hydrodynamic simulations in the context of disk oscillations were carried out by Matsumoto et al. (1988, 1989; see Kato, Fukue, & Mineshige 1998 for a review). They showed that acoustic oscillations are trapped in the innermost region of the relativistic disk, when a viscosity parameter α (Shakura & Sunyaev 1973) is large, typically $\alpha \geq 0.1$. As a result, a quasi-periodic oscillation with the maximum of epicyclic frequency is established in that region.

Later, many hydrodynamic simulations were published (e.g., for non-isothermal disks: Honma, Matsumoto, & Kato 1992; Chen & Taam 1995, for two-dimensional non-adiabatic disks: Milsom & Taam 1996, 1997). Although these studies successfully demonstrated the excitation of

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a trapped oscillation in the disk, the resonant oscillations have a bit advantage to explain 2:3 twin QPOs.

In the framework of the resonant oscillations in relativistic disks, Abramowicz et al. (2003) investigated the geodesic motion of particles around BHs. Later, Kato (2003) extended their idea into the disk fluid. They concluded that twin peak QPOs are excited by the resonance of the disk oscillations. They also asserted that the resonant disk oscillation model can determine the mass and the spin parameter of BHs by using the peak frequencies of twin QPOs (see also Abramowicz & Kluzniak 2001).

The study of the resonant oscillations in the disk have many precedents. Periodic brightness variations in dwarf nova were investigated in the context of the tidal instability as a result of the resonance in the disk (e.g., Whitehurst 1988; Hirose & Osaki 1990; Lubow 1991a). Hirose, Osaki, and Mineshige (1991) demonstrated the tidal deformation of the disk as a result of the resonance in the disk by using the three-dimensional smoothed particle hydrodynamics (SPH) simulations (see also Lubow 1991b).

The previous simulations of disk oscillation assumed the α -viscosity model. However, the MHD turbulence is now widely accepted to be the source of disk viscosity since the Maxwell stress coupled with the magnetorotational instability (MRI: Balbus & Hawley 1991) can effectively transport angular momentum of the disk (e.g., Stone & Pringle 2001; Hawley, Balbus, & Stone 2001; Sano et al. 2004). Therefore the dynamics of the accretion flow is controlled by MHD turbulence and the α -viscosity calculations may fail to represent the numerous aspects of the disk oscillations (see Ogilvie 2003).

Although many MHD disk simulations have been carried out so far (e.g., Hawley 2000; Machida, Hayashi, & Matsumoto 2000; Hawley & Krolik 2001; Hawley, Balbus, & Stone 2001; Armitage & Reynolds 2003; Machida & Matsumoto 2003), the numerical study of MHD disk oscillations has not been worked out yet.

In the present study, we perform three-dimensional MHD simulations of a weakly magnetized rotating torus with the same initial condition as that of Kato, Mineshige, & Shibata (2004, hereafter referred to as KMS04). The purpose of the present study is to examine the disk oscillations in MHD accretion flows plunging into BHs. To elucidate MHD disk oscillations, we focus on the time evolution of the radial structure of mass fluxes near the equatorial plane. Our procedure has an advantage to determine where QPOs are excited and what frequency they have all together. In §2, we describe methods of our study. We then present results in §3. The final section is devoted to discussion and summary.

2. Methods

We solve the basic equations of the resistive magnetohydrodynamics in the cylindrical coordinates, (r, ϕ, z) . General relativistic effects are incorporated by the pseudo-Newtonian potential (Paczynski & Wiita 1980), $\psi = -GM/(R - r_s)$, where $R(\equiv \sqrt{r^2 + z^2})$ is the distance from

the origin, and $r_s (\equiv 2GM/c^2)$ is the Schwarzschild radius (with M and c being the mass of a BH and the speed of light, respectively). The basic equations and the physical conditions in the present study are described in KMS04.

Hereafter we normalize all the lengths, velocities, and density by the Schwarzschild radius, r_s , the speed of light, c , and the maximum density of the initial torus, ρ_0 , respectively (see also KMS04). The unit of the time is $r_s/c = 10^{-4}(M/10M_\odot)[\text{sec}]$ when the mass of a BH is $10M_\odot$. For reader's convenience, the frequency is presented in the unit of $10M_\odot/M$ [Hz].

We start calculations with the same initial condition as that of model B in KMS04. Since model B does not present strong outflows, this model is appropriate to evaluate how disk oscillations are excited in the MHD accretion flows. Taking into consideration of the flows crossing the equatorial plane, we take away a symmetric boundary condition on the equatorial plane in the previous calculation. We impose the absorbing inner boundary condition at the sphere $R = \sqrt{r^2 + z^2} = 2$ (see KMS04 for more details). In the present calculations, we use $300 \times 32 \times 400$ non-uniform mesh points. The grid spacing is the same as KMS04. The entire computational box is $0 \leq r \leq 200$, $0 \leq \phi \leq 2\pi$, and $-50 \leq z \leq 50$ (in comparison with KMS04, who solved only $0 \leq z \leq 100$).

In order to examine disk oscillations, we focus on the structure of the flow near the equatorial plane. We calculate both radial and vertical mass fluxes as follows:

$$\dot{m}_r(r, t) = - \int_{-5}^5 dz \int_0^{2\pi} d\phi \int_r^{r+\Delta r} dr [\rho v_r], \quad (1)$$

$$\dot{m}_z(r, t) = \int_{-5}^5 dz \int_0^{2\pi} d\phi \int_r^{r+\Delta r} dr [\rho v_z]. \quad (2)$$

where ρ , v_r and v_z is the density, the radial and vertical velocity, respectively. Note that Δr is the grid spacing in the radial direction.

To quantify the oscillations of mass fluxes at different radii, we employ the following quantities which are related to the gravitational energy dissipated by the change of mass fluxes as functions of radius and time,

$$L_r(r, t) = \dot{m}_r(r, t) \delta\psi(r), \quad (3)$$

$$L_z(r, t) = \dot{m}_z(r, t) \delta\psi(r), \quad (4)$$

where $\delta\psi(r) = d\psi(r)/dr$ is the gradient of the gravitational potential $\psi(r)$ in the radial direction.

In the present study, we carry out the calculations until $\sim 3.2 \times 10^4 [r_s/c] = 3.2 [(M/10M_\odot) \text{ sec}]$ which correspond to more than 600 orbital time at the ISCO around $10M_\odot$ BH. We sample the mass fluxes in the following time interval:

$$\Delta t = 0.1[r_s/c] = \frac{10}{(10M_\odot/M)} [\mu\text{sec}]. \quad (5)$$

This sampling rate is sufficient for the purpose of our study.

3. Results

We first display the overall evolution of MHD accretion flows in figure 1, in which the space-time diagrams of both radial and vertical mass fluxes are shown in (a) and (b), respectively. Mass accretion takes place from the initial torus by the Maxwell-stress as a result of the growth of MRI inside the torus. After $t \sim 0.8$, the radial structure of the flow become quasi-steady and is consistent with a hot, thick, subthermal, and near Keplerian disk (KMS04; see also Hawley & Balbus 2002; Igumenshchev, Narayan, & Abramowicz 2003).

The initial torus has totally been destroyed by the MHD turbulence until $t \sim 1$. Subsequently, the inner torus is created around $r = 25$ and it oscillates quasi-periodically (as is indicated by orange color in the left panel). The amplitude of both radial and vertical mass fluxes seems to be increased after $t \sim 2$. We can divide the entire evolution of the MHD flows in the following two part: early phase $t = 1 - 2$ and late phase $t = 2 - 3$.

The most interesting feature in the present simulations is that the structures of the mass fluxes are changed at $r \sim 4 - 6$ in both early and late phase. In addition, the oscillations seems to be excited in that region and they propagate outwards.

We compute the PSD of such variabilities of mass fluxes at all radii by taking the Fourier transform of L_r and L_z as,

$$a(r, \nu) \approx \int_{t_0}^{t_1} L(r, t) \sin[2\pi\nu(t - t_0)] dt, \quad (6)$$

$$b(r, \nu) \approx \int_{t_0}^{t_1} L(r, t) \cos[2\pi\nu(t - t_0)] dt, \quad (7)$$

$$P(r, \nu) = \sqrt{[a(r, \nu)]^2 + [b(r, \nu)]^2}. \quad (8)$$

where t_0 (t_1) are the start (end) time of each phase, ν is the frequency ranging from 1 to 10^4 [Hz]. $L(r, t)$ represents the quantity of mass fluxes in either direction. $P(r, \nu)$ is the PSD as functions of the radius and the frequency.

Figure 2 shows the contour of PSD of the radial and vertical component in (a) and (b), respectively. The color indicates $\nu P(r, \nu)$ in logarithmic scale. A dashed curve indicates the Keplerian frequency $\nu_K = \Omega/2\pi$ where $\Omega \equiv 1/\sqrt{2r(r - r_s)^2}$. A solid curve indicates the epicyclic frequency $\nu_\kappa = \kappa/2\pi$ where $\kappa \equiv \sqrt{2\Omega[2\Omega + r(d\Omega/dr)]}$. Note that these expressions are in the pseudo-Newtonian potential. This figure is useful to check a propagation region of radial and vertical oscillations in the MHD flows.

In figure 2 (a), the most of PSD locates in the region $\nu \geq \nu_\kappa$ where axisymmetric acoustic waves can propagate. The radial oscillation whose frequency about 150 is distributed in the wide range of the disk. In addition, the radial oscillations with broad range of frequency seems to be distributed in the innermost region of the disk between the radius of the maximum epicyclic frequency and the ISCO. This component indicates that the radial oscillations are trapped in that region.

Interestingly, the strong vertical oscillations with broad frequency $5 \leq \nu \leq 200$ are distributed at $r \sim 6$ in figure 2 (b). Moreover, the vertical oscillations seems to be distributed in the series of radii (e.g. $r \sim 6, 13, 22, 35$) indicating that a pattern of PSD may be produced by the resonance in the disk.

To simulate the observed PSD of X-ray brightness variation, we integrate $P(r, \nu)$ among the entire disk (from $r_{\text{in}} = 2$ to $r_{\text{out}} = 200$), namely,

$$P(\nu) = \int_{r_{\text{in}}}^{r_{\text{out}}} P(r, \nu) dr. \quad (9)$$

where $P(\nu)$ is the integrated PSD of the radial (vertical) oscillations in the early phase is shown in figure 3 (a) [(b)], whereas that in the late phase is shown in (c) [(d)].

Roughly speaking, all of the integrated PSD show power-law relations, i.e., (a) $P(\nu) \propto \nu^0$ for $\nu < 40$, $\propto \nu^{-1}$ for $40 \leq \nu < 200$, $\propto \nu^{-2}$ for $\nu \geq 200$, (b) and (d) $P(\nu) \propto \nu^{-1}$ for all ν , (c) $P(\nu) \propto \nu^{-1}$ for $\nu < 200$, $\propto \nu^{-2}$ for $\nu \geq 200$. These features are similar to that in the notion of self-organized criticality (SOC: e.g., Mineshige, Takeuchi, and Nishimori 1994; Takeuchi, Mineshige, and Negoro 1995).

We find a pair of *transient* QPOs (as indicated by the arrows labeled as ν_{u1} and ν_{l1}) in early phase, while these QPOs disappear simultaneously in the late phase. The lower peak frequency ν_{l1} , which is the vertical oscillation, corresponds to the Keplerian frequency ν_{K1} at $r = 6.3$ (a *dashed line*). The upper peak frequency ν_{u1} , which is the radial oscillation, corresponds to the sum of the Keplerian frequency ν_{K1} and the epicyclic frequency $\nu_{\kappa1}$ (a *thin solid line*) at the same radius (a *thin dotted line*). The frequencies of the transient twin QPOs are

$$\nu_{l1} = 85 (10M_\odot/M)[\text{Hz}], \quad (10)$$

$$\nu_{u1} = 152 (10M_\odot/M)[\text{Hz}], \quad (11)$$

and the ratio of the peak frequencies is $\nu_{l1} : \nu_{u1} = 2 : 3.58$.

By comparing with (a) and (c), we find a pair of *persistent* QPOs that are the radial oscillations (as indicated by the arrows labeled as ν_{u2} and ν_{l2}). The lower peak frequency ν_{l2} corresponds to the Keplerian frequency ν_{K2} at $r = 3.8$ (a *dashed line*). The upper peak frequency ν_{u2} corresponds to the sum of the Keplerian frequency ν_{K2} and the epicyclic frequency $\nu_{\kappa2}$ (a *thin solid line*) at the same radius (a *thin dotted line*). The frequencies of the persistent twin QPOs are

$$\nu_{l2} = 188 (10M_\odot/M)[\text{Hz}], \quad (12)$$

$$\nu_{u2} = 316 (10M_\odot/M)[\text{Hz}], \quad (13)$$

and the ratio of the peak frequencies is $\nu_{l2} : \nu_{u2} = 2 : 3.36$.

All the twin QPOs are excited near the resonant points as reported by Kato (2004a). We find the following empirical relations between the upper frequency ν_u and the lower frequency ν_l in these twin QPOs,

$$\nu_u \approx \nu_K + \nu_\kappa, \quad (14)$$

$$\nu_l \approx \nu_K, \quad (15)$$

where ν_K and ν_κ is the Keplerian frequency and the epicyclic frequency at the resonant radii, respectively.

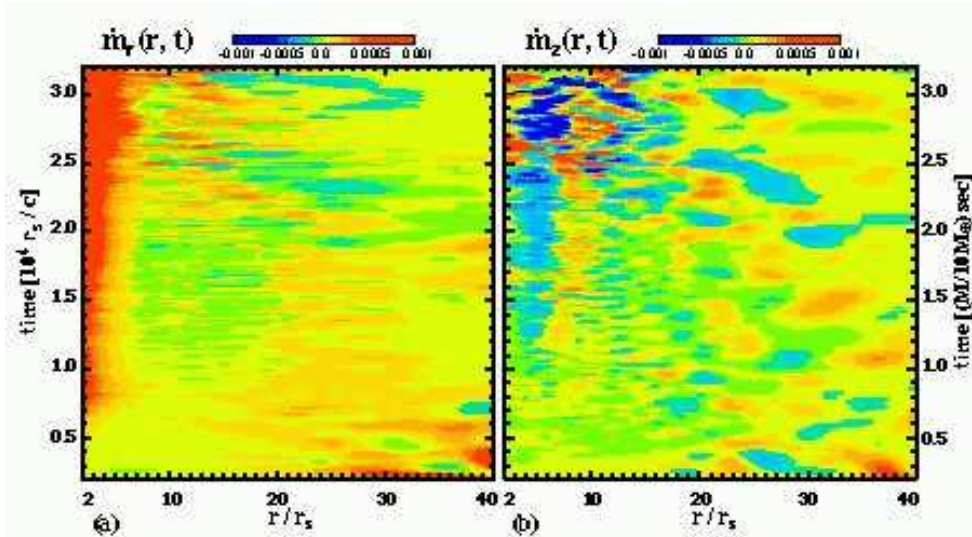


Fig. 1. The space-time diagrams of MHD accretion flows. (a) The time evolution of the radial mass-flux. The positive flux indicates the inflow. (b) The time evolution of the vertical mass-flux. The positive flux indicates the upward flow. The flow patterns are changed at $r \sim 4 - 6$. The fluctuations seems to be produced at this region and they propagate outwards.

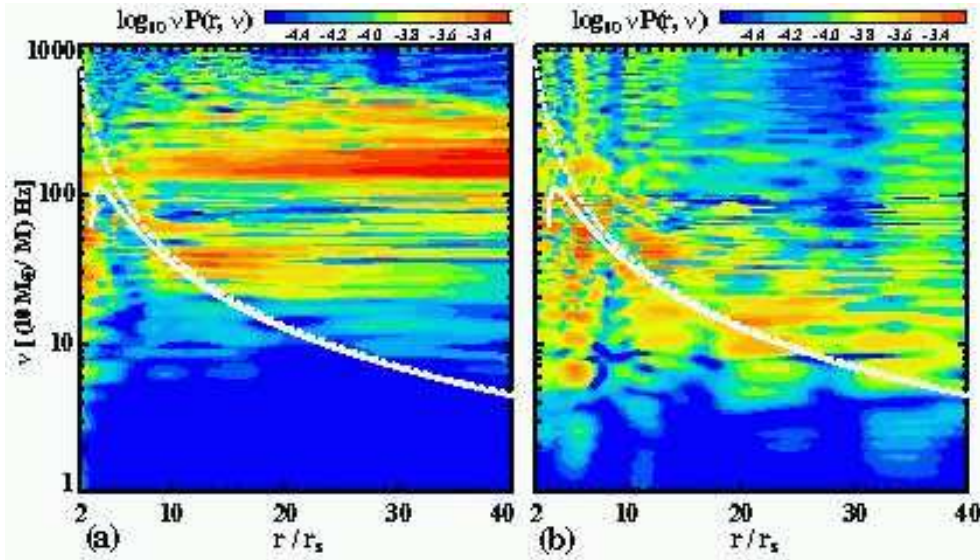


Fig. 2. The power spectral density (PSD) of (a) the radial oscillation, (b) the vertical oscillation in the early phase. The color-contour indicates the distribution of PSD in logarithmic scale. A solid curve indicates the epicyclic frequency ν_κ and a dashed curve indicates the Keplerian frequency ν_K . The radial oscillation, whose frequency is about 150, is distributed in the wide range of the disk. The vertical oscillations seems to be distributed at the series of radii (e.g. $r \approx 6, 13, 20, 35$). These patterns are most likely to be produced by the resonant disk oscillations.

4. Discussion

We performed three-dimensional MHD simulations of the radiatively inefficient accretion flows around BHs. We examined the time variation of mass fluxes near the equatorial plane and found two twin QPOs. The peak frequency of the lower QPO corresponds to the Keplerian frequency, whereas that of the upper QPO corresponds to the sum of the Keplerian frequency and the epicyclic frequency at the same radius. The ratio of these peak

frequencies is close to 2:3. In addition, the flow structure is changed at the radii where the oscillations are excited. Our results indicate that these twin QPOs are most likely to be produced by the resonance between the Keplerian frequency and the epicyclic frequency.

Kato (2004b) investigated that the excitation of the disk oscillations by warps at the resonant radii $r \sim 3, 3.63, 4, 6.46$ in the case of Schwarzschild metric. He reported that the resonant oscillation is excited at the radius $r \sim 4$, on the other hand, it is dumped at the radius

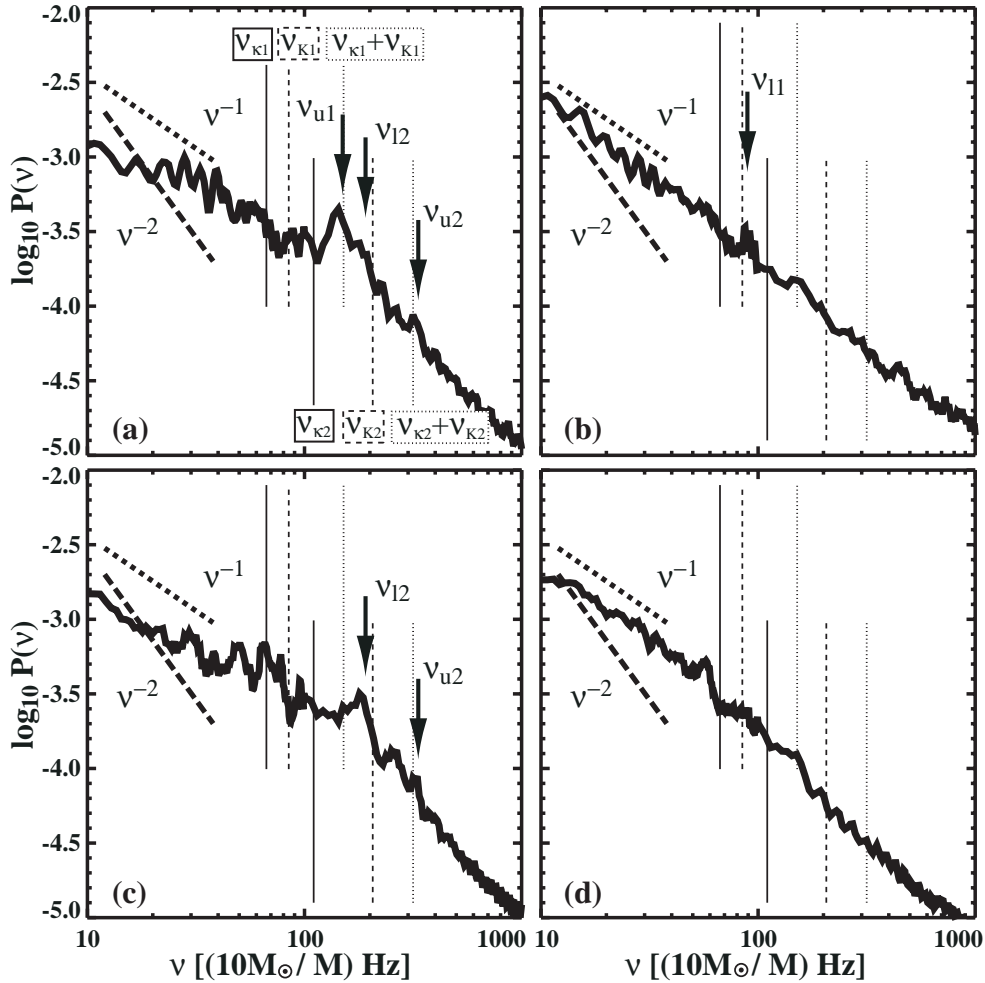


Fig. 3. The integrated PSD of (a) the radial oscillation, (b) the vertical oscillation in early phase, whereas (c) the radial oscillation, (d) the vertical oscillation in late phase. A thick dashed slope and a thick dotted slope indicate the frequency dependence of ν^{-2} and ν^{-1} , respectively. Thin solid, dashed, and dotted lines indicate the the epicyclic frequency ν_κ , the Keplerian frequency ν_K , the sum of them $\nu_\kappa + \nu_K$, respectively. Their higher (lower) frequencies are at $r = 3.8$ ($r = 6.3$). The QPOs are indicated by the arrows.

$r \sim 6.46$. He predicted that the twin QPOs is the resonant oscillations at the radius $r = 4$ where $2\nu_\kappa = \nu_K$ and their peak frequencies are equivalent to ν_K and $\nu_K + \nu_\kappa = 3\nu_\kappa$ at this radius (see also Abramowicz et al. 2003; Rebusco 2004).

Our numerical study in the pseudo-Newtonian potential has a good agreement with his results. In fact, the flow structure is changed at these resonant radii (see figure 1). We have a similar empirical relation of the peak frequencies of twin QPOs (see equations 14 and 15) and the ratio of lower and upper peak frequencies is close to 2:3. In addition, the twin QPO at $r = 6.3$ is dumped in late phase, while that at $r = 3.8$ is persisted for the entire calculation time (see figure 3).

In the present calculation, the peak frequencies of twin QPOs are slightly different from that in the resonant disk oscillation model. The deviation of their peak frequency ratio from 2:3 may be as a result of the pseudo-Newtonian approximation since the maximum epicyclic frequency is larger than that in the Schwarzschild metric by about a

factor of $\sqrt{2}$ (e.g., Okazaki, Kato, & Fukue 1987). For this reason, we expect that the peak frequency ratio is identical to 2:3 in general relativistic MHD calculations by using the Schwarzschild metric. Further numerical investigation of the detailed internal disk structure of the MHD flows, such as fluctuations of Maxwell stresses and magnetic reconnections would help clarify such discrepancies. Moreover, to elucidate the nature of several peaks in the integrated PSD of our study [see figure 3 (c)], detailed mode analysis in the magnetized disk is also important. The method described in this paper may provide a basis for investigating the excitation mechanism of the disk oscillations in the MHD flows.

We can estimate masses of the non-rotating BHs by using the relation of peak frequencies in the persistent twin QPOs [see equations (12) and (13)] as

$$M_{\text{BH},l} = (\nu_{l2}/\nu_{l,\text{obs}})10M_\odot \quad (16)$$

$$M_{\text{BH},u} = (\nu_{u2}/\nu_{u,\text{obs}})10M_\odot \quad (17)$$

where $\nu_{l,\text{obs}}$ ($\nu_{u,\text{obs}}$) and $M_{\text{BH},l}$ ($M_{\text{BH},u}$) is the observed

lower (upper) peak frequency and the mass of BHs estimated by using the lower (upper) QPO, respectively. The estimated masses of BHs in two microquasars (see Remillard et al. 2002) are shown in table 1. Since our calculation in the pseudo-Newtonian potential overestimates both the epicyclic frequency and the upper peak frequency, equation (17) indicates that $M_{\text{BH},u}$ should be larger than $M_{\text{BH},l}$. In addition, the difference between $M_{\text{BH},l}$ and $M_{\text{BH},u}$ is important to discriminate the spin of those BHs. In the case of the rotating BHs, we expect that the estimated masses of BHs should be larger than $M_{\text{BH},l}$ and they may be converged at a unique value.

Interestingly, the MHD flow structure is changed at these resonant radii (see figure 1). Hawley & Krolik (2001) also found a similar flow pattern at $3 \leq r \leq 15$ (see figure 9 in Hawley & Krolik 2001). Although they reported no evidence of QPOs in their PSD of the mass accretion rate at ISCO, we could find the persistent twin QPOs. This is because our method has an advantage to evaluate where QPOs are excited and what frequency they have simultaneously.

We also find that the radial oscillations are distributed in the region between the radius of the maximum epicyclic frequency and that of the inner boundary. Although this indicates that the radial oscillations are trapped in that region, we can find no strong oscillations with the maximum epicyclic frequency as reported by the previous studies (e.g., Matsumoto et al. 1988, 1989; Honma et al. 1992).

In comparison with the hydrodynamic study, we compute an effective viscosity parameter $\alpha_* \equiv B_r B_\phi / 4\pi p$ where B_r (B_ϕ) and p is the radial (toroidal) component of the magnetic field and the gas pressure. Since time-averaged $\alpha_* \leq 0.1$ in that region, the viscous pulsational instability may be marginally stable in the MHD flows (see Kato, Honma, & Matsumoto 1988a, 1988b).

The power-law relations in the PSD of the present study (see figure 3) is very similar to that of the aperiodic X-ray fluctuations with $1/\nu$ -like PSD (where ν is a frequency) in some BHCs. Takeuchi et al. (1995) constructed an accretion disk model based on a cellular automaton model (e.g., Mineshige et al. 1994) and successfully reproduced similar profiles of observed PSD. They asserted that the sporadic magnetic flares in the accretion disks may be responsible to produce avalanche mass accretions in order to satisfy their assumption of critical behavior. Kawaguchi et al. (2000) showed that the time fluctuations of the joule heating term in the 3-D MHD simulation of Machida et al. (2000) produced a similar power-law profile. However, a direct connection between the magnetic flares and the mass accretions has not been cleared yet. Since the variability of mass fluxes in our calculation may depends on the magnetic stress induced locally in the disk, these may reflect the spontaneous evolution of the MHD turbulence. To elucidate their connection, we need more careful studies.

The present paper assumes that the emissivity at various regions in the disk is related to the gravitational energy dissipated by the fluctuation of the mass fluxes. Although the PSD in the present study does not directly

represent that of observed X-ray brightness oscillations, our study confirms that a pair of frequencies can be obtained by the resonance in the MHD disk. The direct comparison with observations is left as future work.

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Table 1. The estimated masses of BHs in two microquasars (Remillard et al. 2002)

Microquasars	$\nu_{l,obs}$ [Hz]	$\nu_{u,obs}$ [Hz]	$M_{BH,l}$ [M_{\odot}]	$M_{BH,u}$ [M_{\odot}]
GRO J1655-40	300	450	6.26	7.02
XTE J1550-564	184	276	10.2	11.4

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