

# Are the WMAP angular magnification measurements consistent with an inhomogeneous critical density Universe?

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## ABSTRACT

The propagation of light through a Universe of (a) isothermal mass spheres amidst (b) a homogeneous matter component, is considered. We demonstrate by an analytical proof that as long as a small light bundle passes *through* sufficient number of (a) at various impact parameters - a criterion of great importance - its average convergence will exactly compensate the divergence within (b). The net effect on the light is statistically the same as if all the matter in (a) is ‘fully homogenized’. When applying the above ideas towards understanding the angular size of the primary acoustic peaks of the microwave background, however, caution is needed. The reason is that most (by mass) of (a) are in galaxies - their full mass profiles are not sampled by passing light - at least the inner 20 kpc regions of these systems are missed by the majority of rays, while the rest of the rays would map back to unresolvable but magnified, randomly located spots to compensate for the loss in angular size. Therefore, a scanning pair of WMAP beams finds most frequently that the largest temperature difference occurs when each beam is placed at diametrically opposite points of the Dyer-Roeder collapsed sections. This is the *mode* magnification, which corresponds to the acoustic *peaks*, and is less than the mean (or the homogeneous pre-clumping angular size). Since space was seen to be Euclidean without taking the said adjustment into account, the true density of the Universe should be supercritical. Our analysis gives  $\Omega_m = 0.278 \pm 0.040$  and  $\Omega_\Lambda = 0.782 \pm 0.040$ .

## 1. Introduction

The propagation of light through the inhomogeneous near Universe is an intriguing phenomenon, especially from the viewpoint of the cosmic microwave background (CMB), because the subject is sufficiently unfathomable that a large number of papers appeared in the literature (triggered by the tautological ‘flux conservation’ argument of Weinberg 1976). Some of the earlier works were cited in section 9.2 of the review of Bartelmann & Schneider

(2001). The recent controversy persists over whether, in a critical density Universe, the convergence of rays by mass concentrations is balanced by the divergence in between, so that the average behavior of the light continues to reflect zero curvature space (see e.g. Holz & Wald 1998, Claudel 2000, and Rose 2001).

The observational status of the near Universe is that it comprises a smooth component which harbors  $\sim 35\%$  (Fukugita 2003; Fukugita, Hogan, & Peebles 1998) of the  $\Omega_m = 0.27$  total matter density (Bennett et al 2003), plus mass clumps with (to zeroth order) a limited isothermal sphere density profile  $\rho \propto 1/r^2$  for  $r \leq$  some cutoff radius  $R$ , which are the galaxies, groups, and clusters. In the present work we demonstrate that the problem concerning the mean convergence of light in a Universe of isothermal spheres placed within an otherwise homogeneous space can be solved analytically.

## 2. Cross section evolution of a light bundle from the Sach's optical equations

Let us express the normalized matter density parameter for the near Universe as  $\Omega_m = \Omega_h + \Omega_g$ , where  $\Omega_h$  represents the homogeneous component and  $\Omega_g$  an ensemble of uniformly but randomly placed isothermal spheres.

The framework of our treatment is the Friedmann-Robertson-Walker (FRW) space-time as shaped by the homogeneous component of the matter distribution. Upon this metric we envision a null geodesic directed along the backward light cone from the spatial origin at the observer, whose clock keeps the present time. The standard solution of the geodesic equation (Eq. (14.40), Peebles 1993) gives

$$\dot{t} = -\frac{a_0}{a} = -(1+z), \quad (1)$$

where the dot derivative is w.r.t. the affine parameter  $\lambda$ ,  $a(t)$  is the (Hubble) expansion parameter at world time  $t$ , and the initial condition  $\dot{t} = -1$  at  $z = 0$  was applied along with the notational shorthand  $c = 1$ .

Now consider small excursions of the actual light path from a given radial null geodesic. It is convenient to introduce transverse coordinates  $\vec{l} = (l_\alpha)_{\alpha=1,2}$ , such that

$$d\vec{l}^2 = dl_\alpha dl^\alpha = a^2 r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2). \quad (2)$$

Take in particular a pair of backward null geodesics starting from the origin at the present time. Let their separation at affine distance  $\lambda$  be  $\delta\vec{l}(\lambda)$ . The rate of change of this separation is governed indirectly by the Sachs scalar optical equations, which in the (reasonable) limit of vanishing Weyl tensor read:

$$\dot{\theta} = -\theta^2 - \frac{1}{2}w^2 - \frac{1}{2}R_{\mu\nu}u^\mu u^\nu, \quad (3)$$

$$\dot{w}_{\alpha\beta} = -2\theta w_{\alpha\beta}. \quad (4)$$

where

$$u^\mu = \dot{x}^\mu = (\dot{t}, 0, 0, \dot{r}), \quad (5)$$

and  $w^2 = w_{\alpha\beta}w^{\alpha\beta}$ . Here  $\theta$  is the expansion and  $w_{\alpha\beta}$  is the shear (the latter is a symmetric and traceless tensor) - they are quantities which determine the evolution of  $\delta\vec{l}(\lambda)$  via the null Raychaudhuri equation

$$\delta\dot{l}_\alpha = \theta\delta l_\alpha + w_{\alpha\beta}\delta l_\beta. \quad (6)$$

In Eq. (3) the Ricci tensor  $R_{\mu\nu}$  is obtained from the Einstein's field equations with the stress energy tensor  $T_{\mu\nu}$  having  $T_{00} = \rho$  as the only non-vanishing entry. The solution,  $R_{00} = R_{33} = 4\pi G\rho$ , is well known, and may be coupled with Eqs. (5) and (1) to yield an expression for the Ricci focussing source term as  $R_{\mu\nu}u^\mu u^\nu = 8\pi G\rho(1+z)^2$ . Hence Eq. (3) may be written as

$$\dot{\theta} = -\theta^2 - \frac{1}{2}w^2 - \frac{3}{2}H_0^2\Omega_h(1+z)^5 \quad (7)$$

For the present purpose it is only necessary to work with the two scalar variables  $\theta$  and  $w^2$ , the evolution of the latter is according to the equation

$$\frac{d}{d\lambda}(w^2) = -4\theta w^2. \quad (8)$$

which is obtainable from Eq. (4).

In the next step, we suppose that light from a source at affine distance  $\lambda_s$  passes through one single isothermal sphere of mass  $M$ , centered at  $\lambda_l$ , and with an impact parameter  $b$  (a physical distance measured at the lensing epoch). The angle of deflection  $\psi(b)$  is given by:

$$\psi(b) = \frac{4GM}{R} \left[ \arccos\left(\frac{b}{R}\right) + \frac{R - \sqrt{R^2 - b^2}}{b} \right] \quad (b \leq R), \quad (9)$$

and

$$\psi(b) = \frac{4GM}{b} \quad (b > R). \quad (10)$$

From Eq. (6) we see that the changes in  $\theta$  and  $w_{\alpha\beta}$  due to the presence of the lensing mass are of the form:

$$\delta(\theta + w_{\rho\rho}) = -(1+z)\frac{d\psi(b)}{db}, \quad (11)$$

$$\delta(\theta + w_{\phi\phi}) = -(1+z)\frac{\psi(b)}{b}, \quad (12)$$

where  $z$  is the redshift for the epoch of interaction. The factor of  $(1+z)$  arises because of the relation between  $\delta\lambda$  and the proper distance. It follows that

$$\delta\theta = -\frac{1+z}{2} \left[ \frac{\psi(b)}{b} + \frac{d\psi(b)}{db} \right], \quad (13)$$

Substituting Eqs. (9) and (10) into Eq. (13), we obtain

$$\delta\theta = 0 \text{ for } b > R; \quad \delta\theta = -\frac{(1+z)4GM}{2R} \arccos\left(\frac{b}{R}\right) \text{ for } b \leq R. \quad (14)$$

For the shear  $w^2$ , the calculations are more complicated. Yet the quantity is easily shown to assume importance only in the strong lensing limit, i.e. in the present context we can ignore it.

We have to compute the average effect of all the mass inhomogeneities. The number density of the isothermal spheres (neglecting evolution and assuming uniform distribution in *total* density FRW space) is

$$n = n_0(1+z)^3, \quad n_0 = \frac{3H_0^2\Omega_g}{8\pi GM}. \quad (15)$$

The probability of finding a clump with center at the position  $(\lambda, b)$  to within small ranges  $d\lambda, db$  is

$$P(\lambda, b) d\lambda db = 2\pi n_0(1+z)^4 d\lambda b db. \quad (16)$$

Since the expansion is *additive*, the globally averaged change of  $\theta$  with  $\lambda$  is, from Eqs. (14) and (16),

$$\left\langle \frac{d\theta}{d\lambda} \right\rangle_g = -\frac{3H_0^2\Omega_g}{4GM}(1+z)^5 \int_{b_{\min}}^R \frac{2GM}{Rb} \arccos\left(\frac{b}{R}\right) b db = -\frac{3}{2}H_0^2\Omega_g(1+z)^5 \quad (17)$$

where at the last step the integral was evaluated with  $b_{\min} \ll R$  in mind.

Putting together the effects of both smooth and clumped matter, we find for  $\theta$ , from Eqs. (7) and (17) the equation

$$\langle \dot{\theta} \rangle = -\theta^2 - \frac{1}{2}w^2 - \frac{3}{2}H_0^2\Omega_h(1+z)^5 - \frac{3}{2}H_0^2\Omega_g(1+z)^5. \quad (18)$$

The contribution from the  $w^2$  (shear) term may be estimated by noting that, from Eq. (11) and (12),

$$\delta(w^2) = \frac{(1+z)^2}{2} \left[ \frac{\psi(b)}{b} - \frac{d\psi(b)}{db} \right]^2.$$

Hence, using Eqs. (9), (10) and the fact that, because of the random orientation of  $\vec{l}$ ,  $w^2$  is additive, we arrive after integration w.r.t.  $\lambda$  at

$$\langle w^2 \rangle \sim H_0^2\Omega_g \frac{GMx_s}{R^2}. \quad (19)$$

where  $x_s$  is the Euclidean distance to the source as measured at  $z = 0$ . When compared with the last term of Eq. (18), however, we see that the shear from the mass concentrations remains relatively unimportant until  $GMx_s/R^2 \geq 1$ , i.e. violation of the weak lensing criterion (also the value of  $w^2$  for homogeneous matter is zero). As long as the lensing is weak, then, one may ignore the 2nd term on the right side of Eq. (18). The outcome is that the expansion of a light bundle depends only on the total matter content, and *not* on the degree of homogeneity of space. If space is Euclidean the solution of Eq. (18) is

$$\theta = -(1+z)H(z) + \frac{(1+z)^2}{a_0 r} = \frac{1}{ar} \frac{d(ar)}{d\lambda}. \quad (20)$$

This is in full agreement with the expected value of the angular diameter distance at zero curvature, viz.  $a(t)r$ .

The chief conclusion of this section may also be obtained by first directly integrating each percentage angular magnification  $\eta = \psi(x_s - x)x/[2(1+z)x_s b]$  over  $dP = 2\pi n_0[1+z(x)]^2 b db dx$ , the latter because of randomly located lenses in an inhomogeneous critical density Universe. Here  $x$  and  $x_s$  are respectively critical density FRW physical distances at the present epoch (the same meaning as  $a(t)r$  in the line element of Eq. (2) with  $t = t_0$ ), to a lens and the source. We find

$$\langle \eta \rangle = \int \eta dP = \frac{3}{2} \Omega_g H_0^2 \int_0^{x_f} dx [1+z(x)] \frac{(x_s - x)x}{x_s}, \quad (21)$$

where  $x_f$  is the distance to the furthest lens, beyond which space is smooth. Then, it has been shown (Lieu & Mittaz 2005) that, irrespective of the lensing strength,  $\langle \eta \rangle$  is *exactly equal to the demagnification due to the Dyer-Roeder (DR) beam divergence* in between these encounters, Dyer & Roeder (1972). This method of proof, though no less valid, is not as elegant in that the two counteracting effects have to be calculated separately, and subtracted from each other afterwards.

### 3. Interpretation of the results, flux conservation; comparison with observations

We must now understand what the result of section 2 means. In particular, we need to know when the integration over a cylindrical probability element like that of Eq. (16) corresponds to observational reality. Among the isothermal spheres of different scales, viz. galaxies, groups, and clusters, the first encompasses by far the lion's share of the matter budget at low  $z$ , with  $\approx 50\%$  of the entire matter content clumped into this kind of large scale structures, i.e.

$$\Omega_g \approx \frac{\Omega_m}{2} \text{ for galaxies} \quad (22)$$

(Fukugita 2003, and Fukugita, Hogan, & Peebles 1998). Thus, in order to secure the precarious balance between beam convergence and divergence, a light signal must pass through

sufficient numbers of galaxies - sampling the full range of impact parameters - larger systems like clusters have too small an associated  $\Omega$  to play a significant role. From the observed density of galaxies

$$n_0 = 0.17h^{-3} \text{ Mpc}^{-3} = 0.06 \text{ Mpc}^{-3} \text{ for } h = 0.71 \quad (23)$$

(Ramella et al 1999) one estimates that throughout the 3 Gpc distance between  $z = 0$  and  $z = 1$  a typical light ray is within  $\approx 40$  kpc from only one galaxy. If these isothermal spheres cutoff at  $R \approx 20$  kpc (which implies a circular velocity  $\sim 250 \text{ km s}^{-1}$  using the value of  $M$  deduced from Eqs. (22) and (23)), then for a separation  $\Delta \geq 40$  kpc the only ‘clump’ contribution to the beam expansion will come from the shear term  $w^2$  of Eq. (19) with  $R$  replaced by  $\Delta \approx 40$  kpc. This calls for an insignificant correction to  $\theta$ .

The conclusion is that despite the euphoria arising from Eqs. (18) and (20), most light rays experience to lowest order only the gravity of homogeneous matter. What are the ramifications? Specifically what will the appearance of features be on a large scale? Consider a sequence of small and contiguous emission pixels on the outlining contour of an emitting source. If the ray bundles connecting them and the observer miss the clumps, their expansion will evolve according to Eq. (18) without the last term and with a negligible second term. This is precisely the ‘partially loaded’ DR beam. It implies demagnification of the pixels in question. By Liouville theorem, the pixels remain adjoint, so to prevent the entire segment from shrinking they must be tangentially sheared and pushed back outwards by the clumps within. In other words, when the bulk of a *randomly* located source boundary is shrunk, it can be restored to original shape only if the enclosed foreground matter acts as a systematic gravitational lens. Yet this is clearly an absurd scenario. In fact, given that the clumps are uniformly distributed on either side of any boundary ray, there is no preferential deflection of the ray, i.e. concerning most of the boundary which is demagnified by the DR beam, the pixels involved should not on average be mobilized radially inwards or outwards by shear - this is consistent with the smallness of the  $w^2$  term. The situation is quite unlike a more homogeneous Universe where each ray passes through enough representative matter and (from section 2) all pixels are magnified, thereby enlarging the boundary without any center of radial migration.

If under the ‘Poisson regime’ of clump distribution large sections of the main boundary of an extended source shorten without distortion how may this be reconciled with the expected source flux? Since lensing conserves surface brightness, a smaller source means less detected flux, yet from section 2 we saw that on balance the effects of lensing and the DR beam cancel, i.e. the flux (or source size) should be unchanged by clumping. The answer comes from that minor fraction of the boundary rays which *do* go through clumps. From the figures given in and after Eq. (22), we found that this is  $\approx 25$  %. The segments involved here are

substantially enlarged, leading to bulges on randomly located portions of the boundary, such that the perimeter now acquires sufficient total length to enclose a re-magnified area.

We apply the above development to the CMB observations, which have conventionally been modeled in terms of a critical density FRW Universe, even though during the ‘last leg’ of the light propagation, the matter at low redshift is anything but smooth. In a more accurate picture, we assume that within  $z = 1$  some of the matter is clumped into galaxies, which have properties as given in Eqs. (15), (22) and (23), since galaxies exhibit no evidence for significant evolution up to this redshift (Ofek, Rix, & Maoz 2003). At earlier epochs, the effect of mass clumping is completely ignored, i.e. the Universe is treated as homogeneous, and the possibility of masses missed by the propagating light above  $z = 1$  will not be taken into account. This understates the outcome of our analysis, which is: the angular scale of temperature variation like the CMB primary acoustic peaks (hereafter PAPs in short) must in the circumstance demagnify by the percentage expected from a ‘half-loaded’ DR beam between  $z = 0$  and  $z = 1$ . From the reasoning at the end of section 2, we see that the required quantity is  $\langle \eta \rangle$  of Eq. (21) with  $\Omega_g = \Omega_m/2$ ,  $x_f = 3.3$  Gpc ( $z_f = 1$ ), and  $x_s = 14.02$  Gpc. The percentage of shrinking is then  $\langle \eta \rangle = 10$  %.

When a pair of WMAP TT cross correlation beams surveys the CMB sky to measure temperature differences at some beam separation, it most frequently finds that the largest temperature difference occurs when each beam is placed at diametrically opposite points of the DR collapsed contour sections. This is the *mode* magnification, and corresponds directly to the acoustic *peaks*. Some of the time, however, the maximum temperature difference is seen at larger angular separations, when e.g. one beam is on a point of the DR demagnified section while the other is on a bulge (or lensed section). This makes the distribution skewed. As a result, the mean magnification remains at the pre-clumping value. Yet the mean is not relevant, for it is the peak position that determines the total density of the Universe. Although each beam cannot resolve the lensed and unlensed sections of the contour (e.g. the former is  $\approx 0.1$  arcmin in size for a galaxy lens of  $R \approx 20$  kpc, while the beam width is  $\sim 30$  arcmin for WMAP), that does not change the conclusion. All it means is that the skewed distribution becomes blurred after convolution with the beam, the mode stays at its DR demagnified position.

We determined, as is illustrated in Figure 1, the CMB parameters required to match the data from WMAP’s TT cross correlation power spectrum, after including the effect of a 10% systematic shift (towards smaller sizes relative to the pre-clumping value) in the spherical harmonic number of all structures within the harmonic range of the PAPs. Since the observed angular size is Euclidean, one now expects the best fit total density to be supercritical. They are found at  $\Omega_m = 0.278 \pm 0.040$  and  $\Omega_\Lambda = 0.782 \pm 0.040$ , i.e.  $\Omega = 1.06$ .

Given our estimate of the bias by which the PAP positions represent the true mean density of the Universe, we also fitted the WMAP data with a smaller bias,  $\langle\eta\rangle = 5\%$ . The figures so obtained are  $\Omega_m = 0.275 \pm 0.040$  and  $\Omega_\Lambda = 0.755 \pm 0.040$ , i.e.  $\Omega = 1.03$ . The other, perhaps even more interesting, point is that the skewness of the angular size distribution induced by galaxy lensing, which separates the peak from the mean at this  $\approx 10\%$  level, is not apparent in the WMAP data, because the PAPs are symmetric gaussians.

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**Note added in Proof** (though too late to appear in the ApJL article itself): Lyman Page was among several who questioned whether the galaxy lensing bias effect we discussed could still lead to an acceptable match between the  $\Omega = 1$  standard model and the WMAP data, if we are prepared to adjust the value of the Hubble constant  $H_0$ . The answer is no. In fact, such an undertaking yielded a minimum  $\chi^2$  of 401.8 for 28 degrees of freedom - a completely unacceptable fit. The best fit values of  $H_0$  and  $\sigma_8$  then become 72.6 and 0.845 respectively. discussed



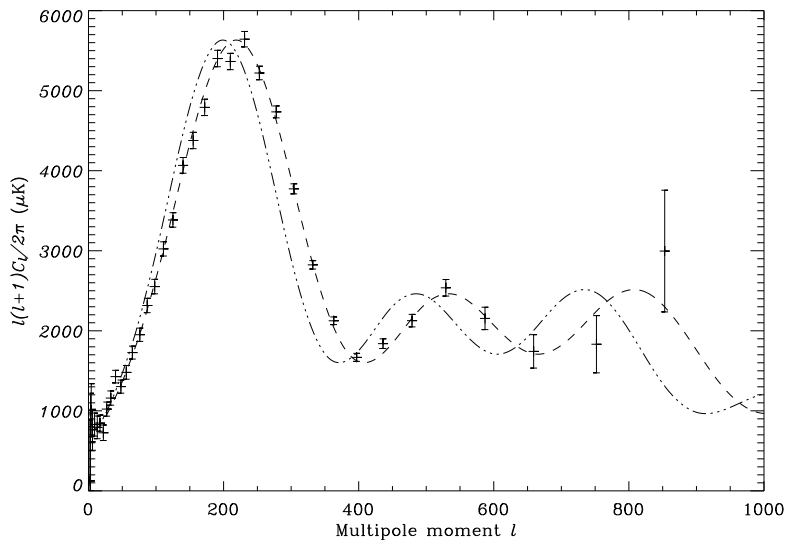


Fig. 1.— Re-interpreting the WMAP TT power spectrum, taking account of the fact that the angular size of large structures as determined with the cross correlation beam are underestimated by 10 % w.r.t. the homogeneous (pre-clumping) benchmark. Our approach involved using the CMBFAST code to generate a model spectrum, then shifting it by 10 % to the right (i.e. towards higher values of  $l$ , or smaller size) and adjusting the parameters so that the match with the data is secured. The new parameters are  $\Omega_m = 0.278 \pm 0.040$ ,  $\Omega_\Lambda = 0.782 \pm 0.040$ , spectral slope =  $0.975 \pm 0.030$ , and Hubble constant  $h = 0.72 \pm 0.03$ . Goodness of fit is  $\chi^2 = 41.7$  for 38 degrees of freedom. This model, when used to make predictions for a homogeneous Universe, is shown on the plot as the dot-dashed curve. When shifted by 10 % to account for the effect of inhomogeneities as discussed above, it becomes the dashed curve.