

Magnetic domain walls of relic fermions as Dark Energy

Urjit A. Yajnik

Department of Physics, Indian Institute of Technology, Bombay, Mumbai 400076 INDIA

Abstract. We show that relic fermions of the Big Bang can enter a ferromagnetic state if they possess a magnetic moment and satisfy the requirements of Stoner theory of itinerant ferromagnetism. The domain walls of this ferromagnetism can successfully simulate Dark Energy over the observable epoch spanning ~ 10 billion years. We obtain conditions on the anomalous magnetic moment of such fermions and their masses. Known neutrinos fail to satisfy the requirements thus pointing to the possibility of a new ultralight sector in Particle Physics.

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INTRODUCTION

The recent strong evidence [1] [2] for the presence of a Cosmological Constant in the Universe, contributing energy density $\approx (0.03eV)^4$ (in units $\hbar = c = 1$), accounting for close to 70% of its contents presents a new challenge to fundamental physics. The enigma of the discovery of such a small value, or indeed that of possible exact zero value for it has been discussed in [3]. A less stringent alternative is to explore the possibility of a substance capable of mimicking the equation of state $p = w\rho$ with w close to -1 at present epoch as appropriate to Cosmological Constant, but by contrast, capable of undergoing dynamical evolution. A large number of proposals have emerged along these lines, some attempting to connect the explanation to other enigmas of Cosmology, while some with a connection to theories beyond the Planck scale. In this paper we consider the possibility of a more conventional explanation based known phenomena. It has been argued in [4] that the equation of state obeyed by this form of energy could be well fitted by a network of domain walls which obey an effective equation of state $p = (-2/3)\rho$, and called Solid Dark Matter (SDM). This possibility has been further examined in [5][6] and shown to be still consistent with more recent data. In this paper we suggest that the domain walls could be of the same kind that occur in ferromagnets. The domain wall complex obeys the Kibble law [7], its energy density contribution scaling as $1/S(t)$, (S being the Friedmann-Robertson-Walker (FRW) scale factor) thus providing a candidate SDM.

Approaches to ferromagnetism based on the well known Heisenberg hamiltonian rely on interaction between spins localised in space at lattice sites. Such models are inadequate in explaining the large spontaneous ferromagnetism of iron, nickel and cobalt. A more fruitful approach is provided by Stoner theory [8] which concerns delocalised or itinerant electrons. In this approach

the cooperative phenomenon is visualised in two parts, first fermionic correlations, equivalently the Exclusion Principle causes a deficit of fermions of the same spin in a given region of space. This enhanced average separation of electrons leads to reduction in energy since the screened coulomb interaction is repulsive. In turn same orientation of the fermions is favoured energetically. A crucial ingredient of implementation of this ansatz is the excess interaction energy per particle, and in turn the density of states at the Fermi surface.

In the cosmological setting, the density of states will simply be the number density of a free gas at the Fermi surface. Further, we assume neutral fermions with mutual interaction of purely magnetic origin, which is repulsive for same spin fermions. With these hypotheses we obtain a relation constraining the values of the anomalous magnetic moment and mass of the fermions. The suggested mass range is $\lesssim 10^{-5}eV$, tantalisingly close to the small values suggested by neutrino oscillations [9]. However, the anomalous magnetic moment required is far in excess of the experimental and astrophysical bounds on magnetic moments of the three known neutrino species, thus ruling them out as viable candidates. Further, as discussed in conclusion, the gauge interaction proposed may not be Electromagnetism. Thus we have an explanation of Dark Energy within the framework of known physical phenomena which leads to the hypothesis of a new class of extremely light particles with large magnetic moment and also the corresponding new gauge force.

COSMOLOGICAL SETTING

The cosmological epoch we focus on is the one when the energy density in Dark Energy became comparable to that in the form of non-relativistic matter, an epoch indicated to be about 7 billion years in the past. Assuming

the energy density of walls scales as $1/S(t)$ and using the law $1/S^3(t)$ for the matter component, we can determine the time t_1 when the two contribute equally to the energy density. Using values of density fractions $\Omega_m \approx 0.3$, for matter and $\Omega_\Lambda \approx 0.7$ for the Dark Energy gives $(S_1/S_0)^2 = 3/7$ where 0 refers to current epoch. Photon temperature at this epoch is $T_1 = 4.18K = 5.0 \times 10^{-4}eV$.

Let us refer to the particle species generically as F and assume that it was relativistic at the time of nucleosynthesis, ie $m_F < 0.1MeV$. There are two possibilities for their abundance. One is that like neutrinos their abundance is similar to that of photons, $n_F(t_1) = (S_0/S_1)^3 120cm^{-3}$ [10]. A less restrictive possibility which we would like to exploit is that there may be excess abundance of these particles, characterised by a factor Υ relative to photons. These possibilities together, in the units $\hbar = c = 1$ become

$$n_F(t_1) \approx 3.2 \times 10^{-12} \Upsilon (eV)^3 \quad (1)$$

Excess abundance of a relativistic species conflicts with nucleosynthesis unless it occurs only after the latter is complete. However, nontrivial Υ can arise from late decay of a weakly interacting heavy particle. At present epoch t_0 this abundance is constrained by the requirement that the density fraction Ω_F of a potential hot dark matter member must remain less than 0.003 [2] [11]. Using the value for $\rho_{crit} = 3.6 \times 10^{-6}(eV)^4$,

$$\Omega_F = \frac{m_F n_F(t_1)}{\rho_{crit}} = 2.25 \times 10^{-7} \times \Upsilon \left(\frac{m_F}{eV} \right) < 0.003 \quad (2)$$

Thus if the species F is saturating this bound, $\Upsilon(m_F/eV) \sim 10^4$. These considerations apply to any one independent species. For more than one species participating in ferromagnetism, appropriate modifications can be incorporated. Finally, we use the expression for the Fermi energy

$$E_F = (p_F^2 + m_F^2)^{1/2} - m_F \quad (3)$$

which accords with the non-relativistic expression, and where the zero-temperature Fermi momentum is $p_F = (3\pi^2 n_F)^{1/3}$ for number density n_F .

THE STONER CRITERION AND COSMIC FERROMAGNETISM

We now turn to collective magnetic properties of this gas. We assume that individual particles have an effective intrinsic magnetic moment

$$\mu_F \equiv g_F \frac{e\hbar}{2m_F} = g_F \mu_B \frac{m_e}{m_F} \quad (4)$$

where m_e is the electron mass, μ_B is the Bohr magneton, and g_F is the gyromagnetic ratio which must be entirely anomalous since we are assuming the fermions to be neutral. For neutrinos the radiatively induced magnetic moment is expected to be small [12], or $\mu_F/\mu_B < 10^{-15}$ as derived in [13] under certain reasonable assumptions. In a more general setting, g_F can be order unity as in the case of the neutron. No such particle is expected from terrestrial experiments, however most of the Universe seems to be composed of particles not suggested by any terrestrial experiments and there is no reason to forbid their existence. Also most unified theories such as $E_8 \otimes E'_8$ and likewise the gauge mediated supersymmetry breaking scheme invoke an unobserved gauge sector and this may be a manifestation of such a sector.

The Pauli paramagnetic susceptibility χ_F of a spin gas is usually small. Large susceptibility and spontaneous magnetization arise according to the Stoner ansatz [8] if there is an additional shift in single particle energies, proportional to the difference between the spin up (N_\uparrow) and the spin down (N_\downarrow) populations. A parameter I is introduced to incorporate this, the single-particle energy spectrum being

$$E_{\uparrow,\downarrow}(\mathbf{k}) = E(\mathbf{k}) - I \frac{N_{\uparrow,\downarrow}}{N} \quad (5)$$

Using this it is shown [14][15][16] that the ferromagnetic susceptibility is

$$\chi = \frac{\chi_P}{1 - I \frac{3}{4E_F}} \quad (6)$$

The condition for spontaneous magnetization is negative χ , which is ensured provided the second term in the denominator dominates. A sufficient condition for the gas to be spontaneously magnetised at zero temperature is the Stoner criterion,

$$I > \frac{4E_F}{3} \quad (7)$$

Further, it can be shown that concordance with the Curie-Weiss law suggests a critical temperature for the ferromagnetic phase transition as $T_c = I/4$. Thus, if the zero temperature condition is satisfied and temperature is less than T_c , ferromagnetic state is possible.

We now turn to the origin of the ferromagnetism. Firstly we note that in the vicinity of a given fermion there is a deficiency of other fermions of same spin, the so called "exchange hole" which is shown to exist in a standard derivation for which we refer the reader to [17][15]. By averaging over the Fermi sphere and integrating over the relative positions this density deficiency can be estimated to be

$$\Delta n_F = -0.86 n_V \quad (8)$$

The upshot of this is that in principle a local population deficit of order unity is easy to obtain. Now consider a long range two particle interaction γ^2 which is repulsive. Compared to absence of interaction and compared to a classical gas, the density deficit causes a reduction in total energy density. This energy reduction should be proportional to Δn_F . To retain the significance of two particle interaction energy γ^2 , we stipulate the relation

$$I = \gamma^2 \frac{|\Delta n_F|}{n_F} \quad (9)$$

We now make the assumption that for the fermions under consideration, this coupling arises from magnetic dipole-dipole interaction, which is repulsive between same spins. The resulting increase in single particle energy can be estimated as (with μ_0 the magnetic coupling in MKS units)

$$\gamma^2 = \kappa \mu_0 \mu_F^2 |\Delta n_F| \quad (10)$$

being the mean field magnetic field due to the deficit density of the magnetic moments, times the magnetic moment μ_F of the single particle. Here κ is an unknown factor expected to be of order unity. Note that the dipole interaction energy goes as inverse third power of interparticle separation and hence correctly scales as $|\Delta n_F|$. Now we use (4), (8), (9), (10), and (3) in the Stoner criterion (7), and assume $|\Delta n_F| \approx n_F$, and $\kappa = 1$ for simplicity. The resulting regions in $\log m_F - \log g_F$ parameter space permitting ferromagnetic state to occur are shown in fig. 1 for $\Upsilon = 1$ and for the value of Υ which saturates the bound in eq. (2). For $g_F \sim O(1)$, the allowed mass range of the species is $\leq 10^{-6} eV$ for $\Upsilon = 1$, and relaxes to $\leq 10^{-3} eV$ for largest allowable Υ .

The current upper bounds [18] [19] on the neutrino magnetic moments are 10^{-10} for ν_e and ν_μ and 4×10^{-7} for ν_τ . These values are smaller by many orders of magnitude, $\sim (m_e/m_F) \times 10^{10}$ ($\times 10^7$ for ν_τ) compared to what is required here. We must conclude that neutrinos cannot participate in such a mechanism. We also note that the excess abundance factor for neutrinos has been recently well constrained to be small, as deduced from neutrino oscillations [20][21].

DOMAIN WALLS

We now assume that at the phase transition when the ferromagnetic state becomes favourable a domain structure sets in due to finite correlations in the system. These domain walls are not expected to be topologically stable. This is because the underlying symmetry is $SU(2)$ of spin, which permits rotations within the vacuum manifold for the defect to disentangle. However the situation is analogous to the case of global internal symmetries

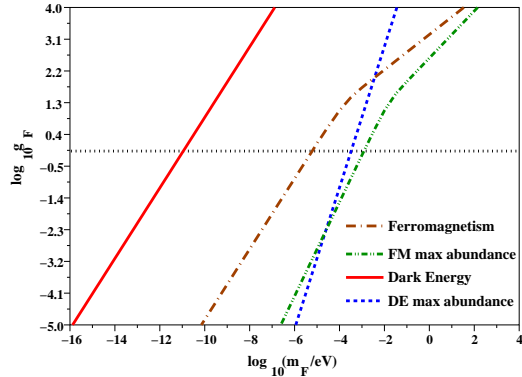


FIGURE 1. Permitted regions in parameter space $\log m_F - \log g_F$ are to the left of the curves in all cases. Regions which satisfy Stoner criteria are labelled Ferromagnetism (FM), with-out or with maximum permissible abundance. The requirement of simulating Dark Energy (DE) restricts the regions further as shown. The horizontal dotted line corresponds to $g_F = 1$.

and the decay of the walls is governed by tunneling processes as detailed later. We assume that the domain wall dynamics can be described by a Landau-Ginzburg effective lagrangian[22] for a vector order parameter \mathbf{S} with a symmetry breaking self-interaction $\lambda(\mathbf{S} \cdot \mathbf{S} - \sigma^2)^2$, where σ determines the magnitude of the magnetization and can be related to I and γ^2 introduced above. From standard solitonic calculation [23] the domain walls have a width $w \sim (\sqrt{\lambda} \sigma)^{-1}$ and energy per unit area $E/A \sim \sqrt{\lambda} \sigma^3$.

We can now estimate the energy trapped in the domain wall structure and require that it must account for half of the total energy density of the Universe at the epoch t_1 . Let the domain wall structure be characterise by length scale L . Equivalently, there is one wall passing through a cubical volume of size L^3 on the average. The energy density contained in such a wall is E/wL^2 , while its average contribution to the total density is E/L^3 . If the walls are sufficiently distinguished as a structure, we expect $w/L \ll 1$.

$$\rho_{walls} = \frac{E}{AwL} \geq \frac{1}{2} \rho_{crit} \left(\frac{T_1}{T_0} \right)^4 \quad (11)$$

For the generic case with no excess abundance this places a more stringent requirement on the allowed values of g_F and m_F as shown in fig. 1, where $w/L = 0.1$ is assumed. The restriction to smaller mass values enhances the magnetic energy stored in the walls, however the mass values are in the range $\lesssim 10^{-12} eV$, much smaller than known mass scales. If excess abundance is permitted this situation is considerably relaxed. In this case a future problem would be to understand the mechanism of the excess

abundance.

One of the main sources of wall depletion is mutual collisions. However the walls become non-relativistic soon after formation and the free energy available for bulk motion reduces. The walls can also spontaneously decay as was discussed in [24]. However the decay rate would be governed by an exponential factor $\exp(-B/\lambda)$, with B the Euclidean action of the "bounce" [25] of order unity. Stability of this complex for several billion years, compared to intrinsic time scales of microscopic physics in the range $(0.03eV)^{-1}$, requires the suppression factor to be 10^{-30} which is natural for $\lambda \sim 0.01$.

Finally we face the important question whether the gauge force responsible for this magnetism is Electromagnetism, $U(1)_{EM}$. Further investigation is required to determine the extent to which known phenomena such as [19] constrain the magnetic properties of such particles. If magnetic moment of purely electromagnetic origin is too tightly constrained, the gauge group may be a hidden sector $U(1)_H$. Such a $U(1)_H$ would still mix with the $U(1)_{EM}$. So long as $U(1)_{EM}$ is involved, an intriguing possibility is an explanation for the origin of the intergalactic magnetic fields [26]. While the magnetic field averaged over the domains is zero, a deviation from the average, proportional to square root of the number of domains may suffice to provide the requisite seed [22]. An explanation for the g_F value $\sim O(1)$ may be provided by the existence of a hidden gauge group of the form $SU(N)_H \otimes U(1)_H$. In this case the situation may be similar to the neutron, with the fermion F a strongly coupled neutral bound state with large anomalous magnetic moment of the $U(1)_H$.

A distinctive prediction of this scenario is that the Dark Energy dominated era must end. As the Universe expands, the density n_F decreases and when the interaction stipulated in eq. (10) becomes insignificant, spontaneous magnetism vanishes. The time scales are expected to be comparable to cosmic time as per the discussion about stability of the walls. The disappearance of the domain walls would release some entropy. This and other possible signatures of this scenario are model dependent and need further investigation.

If this scenario is correct then there is no fundamental Cosmological Constant, returning General Relativity to its status where Einstein left it. Why the Higgs mechanism of electroweak theory induces no vacuum energy remains an open problem [3].

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