Classifying the Future of Universes with Dark Energy

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Abstract. We classify the future of the universe for general cosmological models including matter and dark energy. If the equation of state of dark energy is less then -1, the age of the universe becomes finite. We compute the rest of the age of the universe for such universe models. The behaviour of the future growth of matter density perturbation is also studied. We find that the collapse of spherical overdensity region is greatly changed if the equation of state of dark energy is less than -1.

1. Introduction

The Universe is replete with dark energy whose nature is almost completely unknown, excepting that its "equation of state", $w = p/\rho$, is negative. Dark energy determines the future of the Universe. Recently, there is renewed interest in the future of the Universe partly because the state of the future of the universe would dramatically be changed in the presence of dark energy [1, 2]. Moreover, another motivation comes from the fact that the equation of state of dark energy is being constrained by cosmological observations [3] and the discovery of a new type of the future singularity with $w < -1$ dark energy (called phantom [4, 5]): the big rip [6]. This singularity is intriguing because a spacetime singularity appears even if the weak energy condition ($\rho + p \geq 0$) is violated. The big rip is the singularity where the universe expands so rapidly due to the repulsive rather than the attractive nature of gravity that both the scale factor and the Hubble parameter diverge there.

In this paper, including $w < -1$ dark energy, we classify the future of the universe and the conformal diagrams for cosmological models with dark matter and dark energy with general equation of state. We also calculate the remaining age of the Universe if the age is finite. We also investigate the future evolution of matter density perturbation in the universe with dark energy.

Throughout the paper, we limit ourselves to a constant w. However, the case with time varying w could be obtained by combining our results (however, see [7] for exceptional cases).

2. Diagram of the Universe

The Friedmann equation for the universe with (nonrelativistic) matter and dark energy is

$$\left(\frac{H}{H_0}\right)^2 = \left(\frac{\dot{a}}{H_0 a}\right)^2 = \Omega_M a^{-3} + \Omega_X a^{-3(1+w)} + (1 - \Omega_M - \Omega_X)a^{-2}, \qquad (1)$$

where H_0 is the present Hubble parameter and w is the equation of state of dark energy and Ω_M and Ω_X is present density parameter for matter and dark energy, respectively. The scale factor has been normalized such that $a = 1$ at present.

The Friedmann equation Eq.(1) can be read as the "energy equation":

$$\frac{\dot{a}^2}{H_0^2} + V(a) = E, \qquad (2)$$

where the potential energy term $V(a)$ is

$$V(a) = -\Omega_M a^{-1} - \Omega_X a^{-1-3w}, \qquad (3)$$

and the total energy $E(= \Omega_K = -K/H_0^2)$ is $E = 1 - \Omega_M - \Omega_X$. If there are roots of $E - V(a) = 0$ with $a > 0$, the universe can stop its expansion ($\dot{a} = 0$) and recollapse or bounce. Let us discuss the roots of $E - V(a) = 0$ in the following.

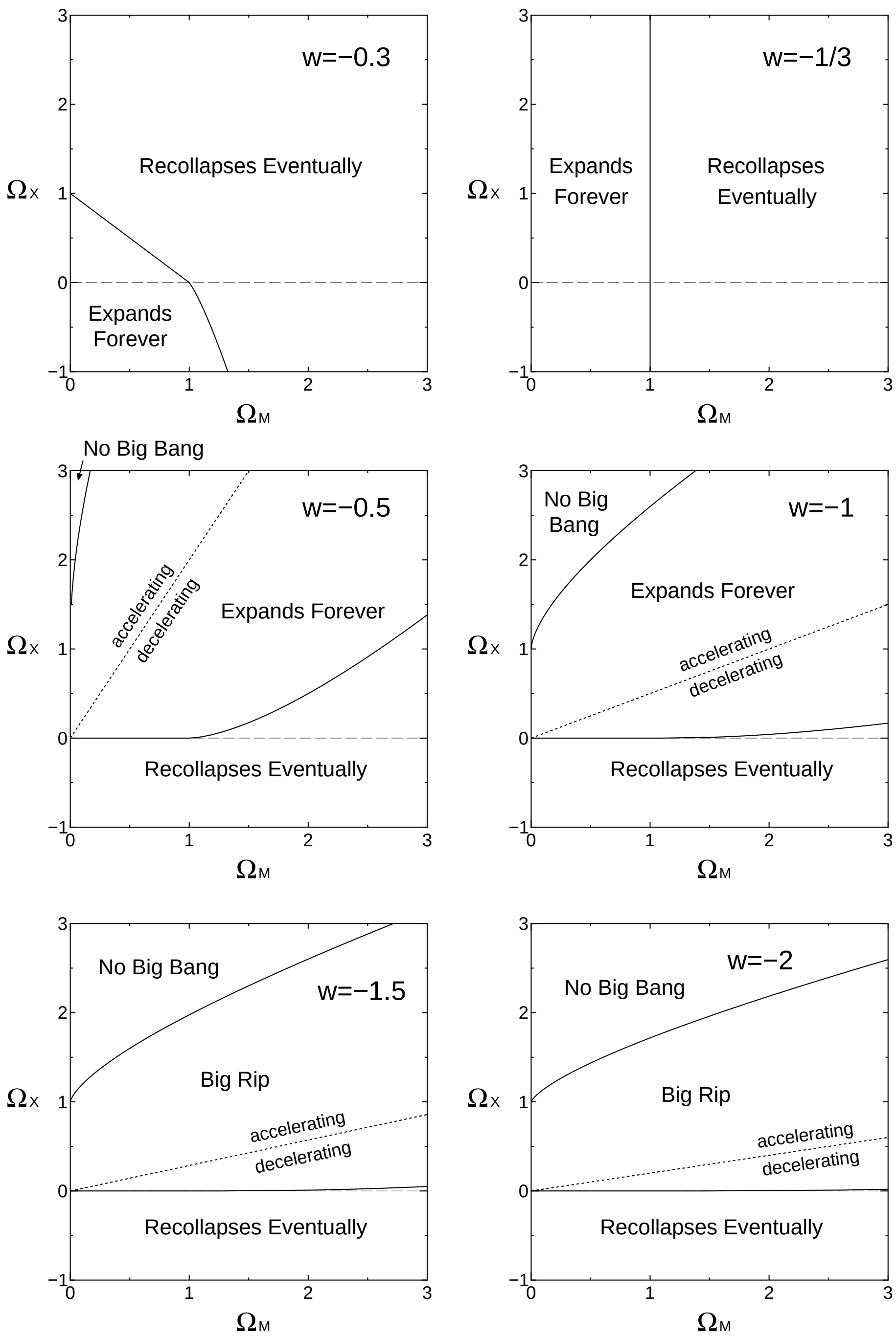

Figure 1. The "phase diagram" of the universe in $\Omega_M - \Omega_X$ plane for $w = -0.3, -1/3, -0.5, -1, -1.5, -2$. The regions, labeled "no big bang", "expands forever", "big rip", and "recollapses eventually" are divided by the solid lines. The horizontal dashed line is $\Omega_X = 0$. The region of "no big bang" represents the bounce cosmologies. In the region of "expands forever" for $w \geq -1$, the universe will expand forever. In that of "big rip" for $w < -1$, the universe will end in a big rip. The expansion of the universe is currently accelerating above the dotted line, while decelerating below it. In the region of "recollapses eventually", the universe will eventually stop its expansion and recollapse to a big crunch.

A: $\Omega_X > 0$ & $0 > w > -1/3$. In this case, the dark energy behaves like a matter with small negative pressure. If E $(= \Omega_K) < 0$, since $E - V(a) = 1 > 0$ at $a = 1$ and $E - V(a) \to E(< 0)$ for $a \to \infty$, there is a root of $E - V(a)$ at $a > 1$. Hence the universe will recollapse in the closed model $\Omega_K < 0$. Interestingly, if $E > 0$ there is no root and the universe will expand forever. The future of the Universe would be quite different from that with a cosmological constant: The Universe with negative dark energy density can expand forever without big crunch.

In top left panel of Fig.1, we show the "phase diagram" of the universe for $w = -0.3$. The two regions, labeled "recollapses eventually" and "expands forever", are divided by solid line. The horizontal dashed line is $\Omega_X = 0$.

B: $w = -1/3$. In this special case, the Friedmann equation Eq.(1) is reduced to $(H/H_0)^2 = \Omega_M a^{-3} + (1-\Omega_M)a^{-2}$ and is independent of Ω_X. Dark energy with $w = -1/3$ behaves like a curvature, so that the destiny of the universe depends only on the matter density. For $\Omega_M \leq 1$ the universe will expand forever, while for $\Omega_M > 1$ it will recollapse. In top right panel of Fig.1, we show the "phase diagram" for $w = -1/3$.

C: $\Omega_X > 0$ & $w < -1/3$. The universe can recollapse or bounce if $E < V_m = \max(V(a))$. Let a_m be the scale factor at the maximum of $V(a)$, where a_m is given by

$$a_m = \left(\frac{\Omega_M}{-(1+3w)\Omega_X} \right)^{-1/3w}, \tag{4}$$

and V_m is

$$V_m = -\frac{3w}{1+3w} \frac{\Omega_M}{a_m}. \tag{5}$$

Hence the critical condition $E - V_m = 0$ is rewritten as,

$$\left| \frac{1 - \Omega_M - \Omega_X}{3\ w\ \Omega_X} \right|^{-3w} = \left| \frac{\Omega_M}{(1+3w)\ \Omega_X} \right|^{-1-3w}. \tag{6}$$

This equation is the same as Eq.(5) in Moles (1991) if we set $w = -1$. The bounce would occur if $E < V_m$ and $a_m < 1$; the recollapse would occur if $E < V_m$ and $a_m > 1$ [8].

In Fig. 1, we display the "phase diagram" of the universe depending on the equation of state of dark energy ($w = -0.5, -1, -1.5, -2$). For $w < -1/3$, the three regions are divided by the two solid lines. The region of "no big bang" represents the bounce cosmologies. As w becomes negatively larger, the region of bounce

becomes larger. This can be understood from

$$\frac{\partial V_m}{\partial w} = \frac{V_m}{w} \left(\ln a_m - \ln(\Omega_X/\Omega_M) \right) \tag{7}$$

for fixed Ω_M and Ω_X. This implies that for $a_m < 1$ (or $\Omega_M < -(1+3w)\Omega_X$), V_m increases as w decreases when Ω_M and Ω_X fixed, so that the universe can bounce more easily. On the other hand, for $a_m > 1$, V_m decreases as w decreases when Ω_M and Ω_X fixed, so that the universe can recollapse more hardly.

If the equation of the state of dark energy is less than -1 [4, 5], then the universe expands so rapidly that the scale factor will diverge and the space will eventually be torn apart and the universe will consequently result in the "big rip" [6]: the singularity with the positively divergent Hubble parameter. In the bottom panels of Fig.1 ($w = -1.5, -2$), the region of "big rip" represents the cosmological models with big rip singularity. ‡

D: $\Omega_X < 0$ & $w \neq -1/3$. For $0 > w > -1/3$ the universe will recollapse if $E < V_m$. While for $w < -1/3$ there is a root at $a > 1$, since $E - V(a)$ is 1 at $a = 1$ and $\Omega_X a^{-1-3w}$ (< 0) at $a \gg 1$. Hence the universe will recollapse in $\Omega_X < 0$ & $w < -1/3$.

3. The Life of the Universe

For $w < -1$, the age of the Universe becomes finite even if the energy density of dark energy is positive. Then the immediate question would be, "how much time is left for the Universe?". So we calculate numerically the remaining age of the universe, $t_{\rm left}$, for general non-flat universe with $w < -1$:

$$t_{left} = \int_1^\infty \frac{da}{\dot{a}} = \int_1^\infty \frac{da}{aH} = \frac{1}{H_0}\int_0^1 \frac{dx}{x\sqrt{\Omega_M x^3 + \Omega_X x^{3(1+w)} + (1-\Omega_M-\Omega_X)x^2}}, \quad (8)$$

where we have introduced $x = 1/a$. The case of flat universe is calculated in [4, 9, 10].

Fig. 2 is $H_0 t_{\rm left}$ for general non-flat universe models: for $w = -1.5$, $H_0 t_{\rm left}$ is 1.0 from top in step of 0.2; for $w = -2.0$, $H_0 t_{\rm left}$ is 0.6 from top in step of 0.2; for $w = -3.0$, $H_0 t_{\rm left}$ is 0.3 from top in step of 0.1; for $w = -5.0$, $H_0 t_{\rm left}$ is 0.2 from top in step of 0.1.

Fig. 3 is for a flat model: $H_0 t_{\rm left}$ is $5.0, 3.0, 2.0, 1.0, 0.5, 0.3, 0.2$ from top to bottom. From Fig. 2, we find that the remaining age is primarily determined by Ω_X. Moreover, for negatively larger w dependence on Ω_X becomes weak and the age is essentially determined by w (see Fig. 2 for $w = -5.0$). This fact can easily be found from Eq.(8); since for $|w| \gg 1$ dark energy becomes dominant during most of the remaining age of the universe, the integral can be approximately estimated as

$$H_0 t_{left} \simeq \int_0^1 \frac{dx}{\sqrt{\Omega_X} x^{(5+3w)/2}} = \frac{2}{3|1+w|\sqrt{\Omega_X}}, \quad (9)$$

which explicitely shows that t_{left} is more sensitive to w than to Ω_X. It is found from numerical calculations that this approximates the integral Eq.(8) well (within 10%) for $0.5 \lesssim \Omega_X \lesssim 2.0$ if $w \lesssim -1.3$.

‡ The properties of the singularity and the existence of a stable fixed point are studied in [11].

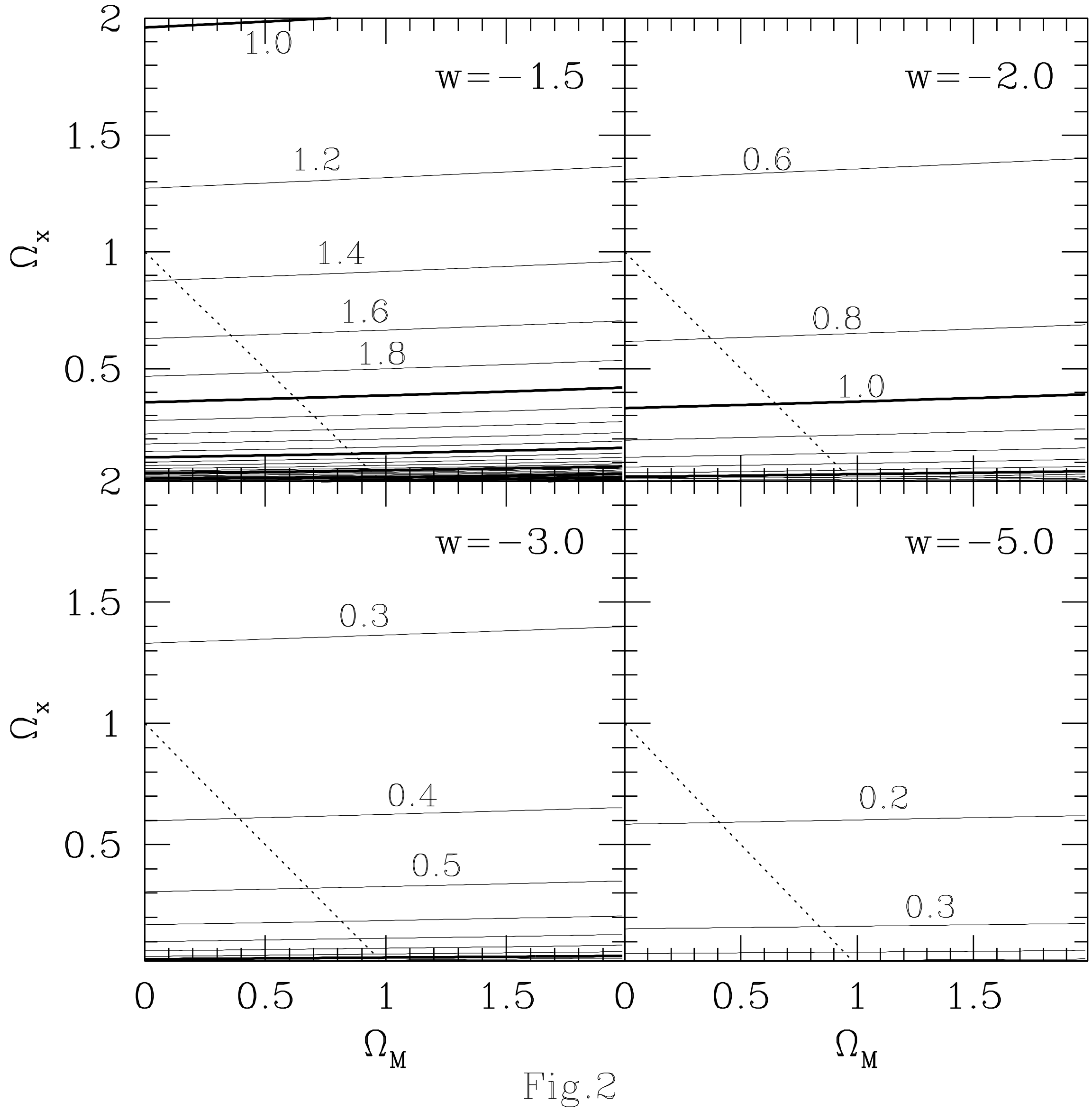

Figure 2. $H_0 t_{\rm left}$ for general universe model: for $w = -1.5$, $H_0 t_{\rm left}$ is 3.0 from top in step of 0.2; for $w = -2.0$, 0.6 from top in step of 0.2; for $w = -3.0$, $H_0 t_{\rm left}$ is 0.3 from top in step of 0.1; for $w = -5.0$, $H_0 t_{\rm left}$ is 0.2 from top in step of 0.1.

4. Conformal Diagrams

Similar to the fate of the universe, an interesting question would be "what is the entire structure of spacetime for the universe with dark energy?". We show the conformal diagrams of flat cosmological models with various dark energy. There are several works which have some overlap [12, 13, 17, 15, 16, 17, 18]. Our aim here is to collect and classify these results including matter and $w < -1$ case for comparison and for completeness.

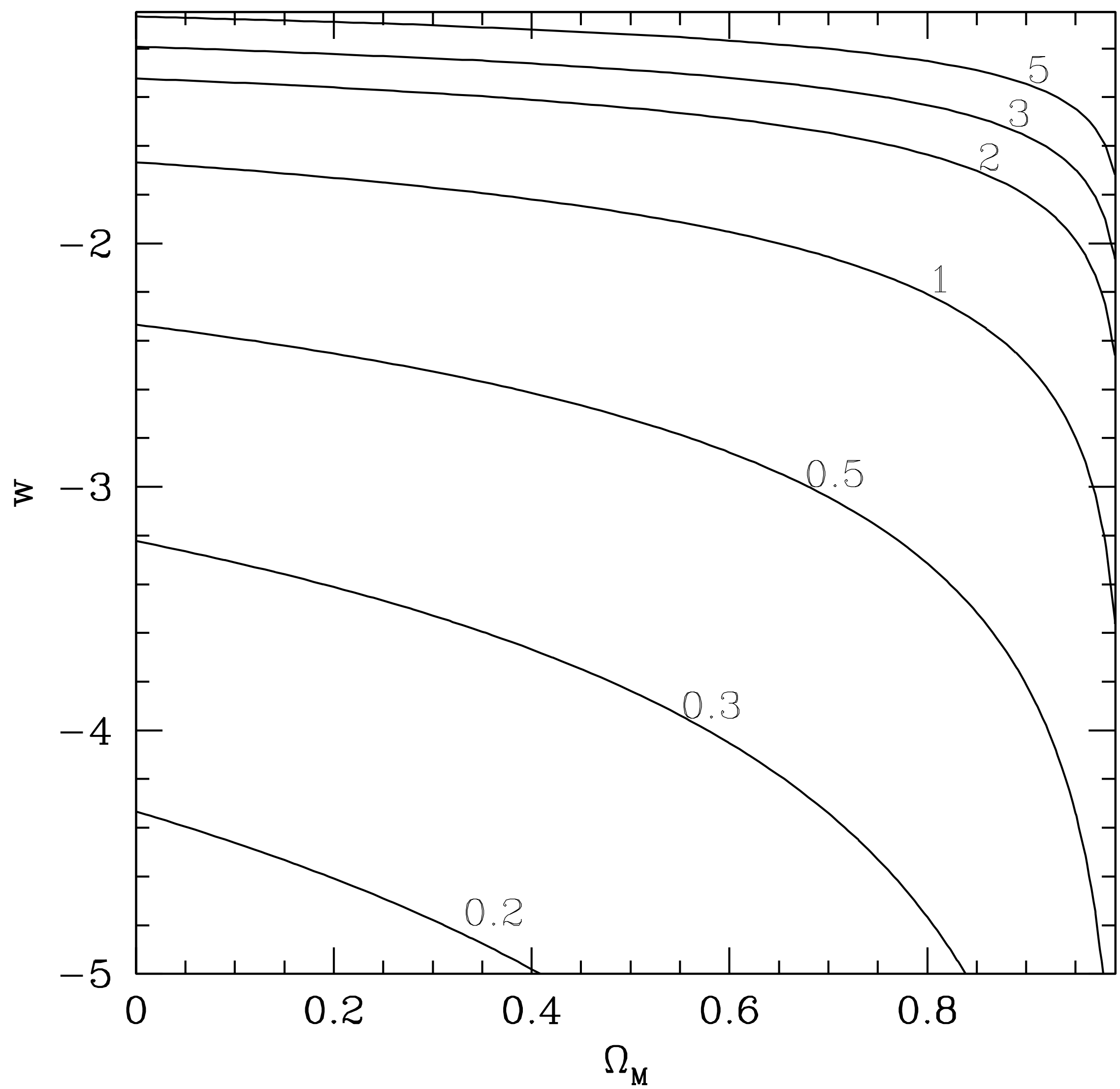

Fig.3

Figure 3. $H_0 t_{\rm left}$ for flat models. $H_0 t_{left} = 5.0, 3.0, 2.0, 1.0, 0.5, 0.3, 0.2$ from top to bottom.

4.1. Dark Energy without Matter

For simplicity, we first consider cosmological models without matter. The conformal diagrams of such cosmological models (for $w \geq -1$) are studied in [12, 14]. We include cosmological models with $w < -1$ dark energy as well.

Since the metric of flat models is conformal to Minkowski spacetime

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega^2) = a(\eta)^2(-d\eta^2 + dr^2 + r^2 d\Omega^2), \tag{10}$$

the conformal diagram is a subset of that of the Minkowski spacetime and the range of the conformal time η depends on w.

A: $w > -1/3$. Since $a(t) = t^{2/3(1+w)}$,

$$\eta = \int \frac{dt}{a} = \frac{3(1+w)}{1+3w} t^{(1+3w)/3(1+w)}. \tag{11}$$

The range of η corresponding to that of t ($0 \le t < \infty$) is, $0 \le \eta < \infty$, so that the conformal diagram is the upper half of the conformal diagram of Minkowski spacetime.

B: $w = -1/3$. Since $a = t$, $\eta = \ln t$. The range of η is $-\infty < \eta < \infty$, and the diagram is almost the same as that of the Minkowski spacetime excepting the presence of big-bang singularity at the past null infinity.

Since there is a past null singularity, the physical size of the past light cone d_H for an observer at $r = 0$ (particle horizon) is given by

$$d_H = a(t) \int_0^t \frac{dt'}{a(t')}, \tag{12}$$

and it is infinite for $w = -1/3$. Hence, there is no horizon problem in this case.

C: $-1 < w < -1/3$. Since $a(t) = t^{2/3(1+w)}$ and the exponent is greater than unity, $\eta \propto -t^{(1+3w)/3(1+w)}$. The range of η corresponding to that of t ($0 \le t < \infty$) is $-\infty < \eta \le 0$, so that the conformal diagram is the lower half of the conformal diagram of Minkowski spacetime with the big-bang singularity. Since there is a past null singularity, d_H is infinite in this case as well.

D: $w = -1$. Since $a = \exp(Ht)$ ($-\infty < t < \infty$), $\eta \propto -\exp(-Ht)$. Again, the conformal diagram is the lower half of the conformal diagram of Minkowski spacetime but without the big bang singularity.

E: $w < -1$. Since $a = (-t)^{2/3(1+w)}$, $\eta \propto -(-t)^{(1+3w)/3(1+w)}$. The range of η corresponding to that of t ($-\infty < t \le 0$) is $-\infty < \eta \le 0$, so that the conformal diagram is again the lower half of the conformal diagram of Minkowski spacetime but this time with the big rip singularity at the future spacelike infinity. Since the past null singularity is null, d_H is infinite in this case as well.

These results are shown in Fig. 4. For $w < -1/3$, since the conformal time is bounded above, there is a future cosmological event horizon [19, 14]. The light rays emitted beyond the horizon never reach an observer at $r = 0$. Hence, the asymptotic region of spacetime cannot be measured, and there is no S-matrix [14].§ The proper radius of the horizon R_c is given by

$$R_c = a(t) \int_t^\infty \frac{dt'}{a(t')} = -\frac{3(1+w)}{1+3w} t, \tag{13}$$

for $-1 < w < -1/3$. $R_c = H^{-1}$ for $w = -1$, and $R_c = (3(1+w)/(1+3w))(-t)$ for $w < -1$. (Note that the range of t is bounded above for $w < -1$.) The size of the

§ However, the situation is not improved much for $w > -1/3$. see [20].

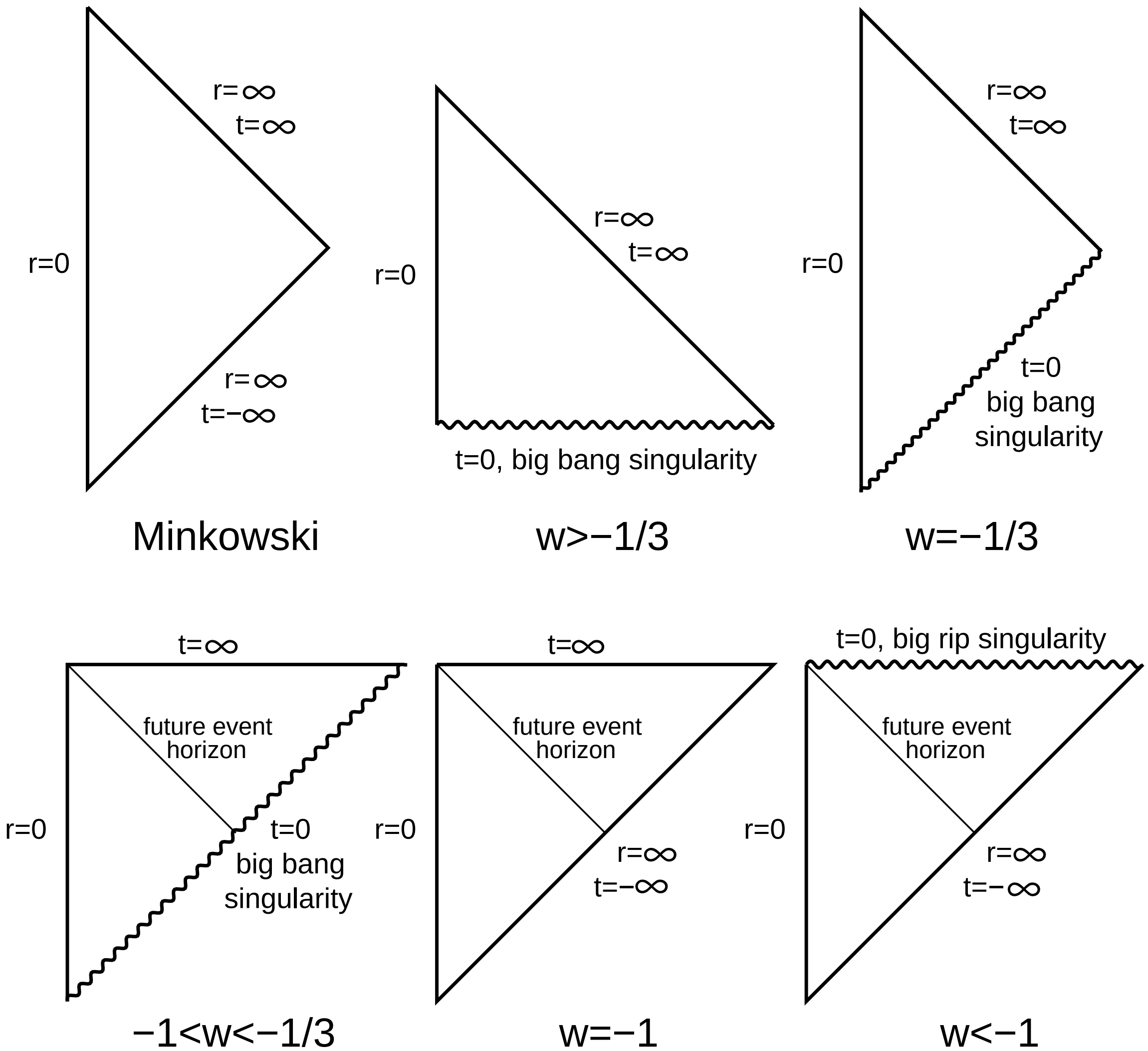

Figure 4. Conformal diagrams of flat cosmological models with dark energy but without matter for $w > -1/3, w = -1/3, -1 < w < -1/3, w = -1, w < -1$. The thin solid line is the future event horizon.

horizon grows for $-1 < w < -1/3$ and remains constant for $w = -1$ but *decreases* for $w < -1$ because phantom matter violates the dominant energy condition.

4.2. Including Matter

When matter is included, lower half parts of the diagrams are replaced with big bang singularity. We plot the corresponding diagrams in Fig. 5. Because matter was dominated in the past,] the size of the particle horizon becomes finite even for $w \leq -1/3$. However, the presence of the cosmological event horizon is not affected by including matter, since the universe would be dominated by dark energy in the future (see [15, 16, 17, 18] for the detailed discussion of the evolution of the horizon).

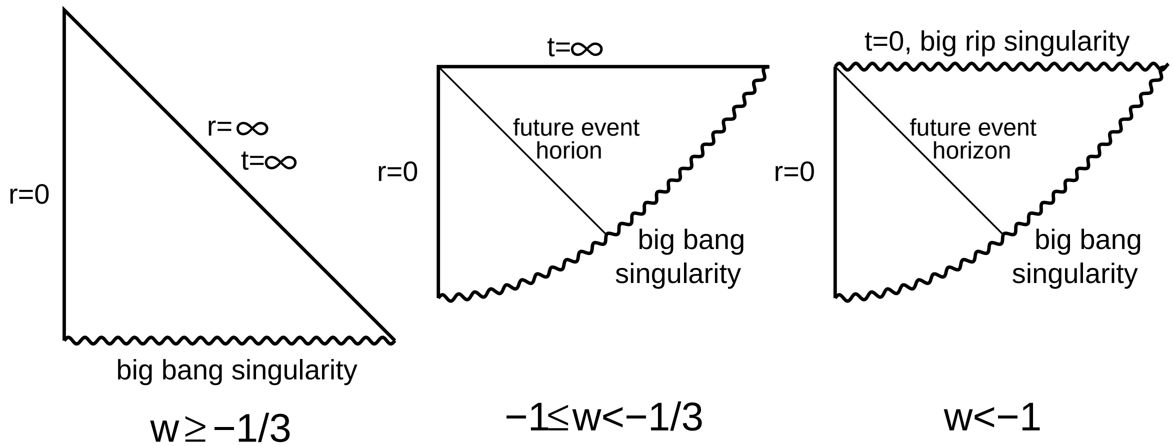

Figure 5. Conformal diagrams of flat cosmological models with matter and dark energy for $w \geq -1/3, -1 \leq w < -1/3, w < -1$. The thin solid line is the future event horizon.

5. Growth of Structures

In this section, we study the formation of structures in flat universe models with dark energy. First, we consider the evolution of linear matter perturbation, and then on the base of it, we consider the nonlinear evolution of perturbation using spherical collapse model.

5.1. Linear perturbation

The growth of linear matter density perturbation $\delta = \delta\rho_M/\rho_M$ in various flat cosmological models with dark energy is determined by the following equation [21]:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_M\delta = 0. \tag{14}$$

Eq.(14) can be rewritten in terms of $g = \delta/a$ as

$$(\Omega_M + \Omega_x a^{-3w})\frac{d^2 g}{d\ln a^2} + \left(\frac{5}{2}\Omega_M + \frac{5-3w}{2}\Omega_X a^{-3w}\right)\frac{dg}{d\ln a} + \frac{3}{2}(1-w)\Omega_X a^{-3w} g = 0. \tag{15}$$

The exact solution of the linear perturbation equation (15) was obtained by Silveira and Waga [22, 23] (see also [24] for $w = -1$) for a constant w. Denoting the solution as $D(a)/a$, it is given by

$$\frac{D(a)}{a} = {}_2F_1\left(-\frac{1}{3w}, \frac{w-1}{2w}, 1 - \frac{5}{6w}; -\frac{\Omega_X}{\Omega_M}a^{-3w}\right), \tag{16}$$

where ${}_2F_1$ is the Gauss's hypergeometric function. Here $D(a)$ is normalized so that $D(a) \rightarrow a$ at $a \rightarrow 0$.

We compute the evolution of density perturbation as a function of scale factor or cosmic time. The results are shown in Fig. 6 and Fig. 7. Matter density perturbation is normalized by its present value, $D_0 \equiv D(a = 1)$. We assume $\Omega_M = 1 - \Omega_X = 0.3$. The future growth of the density perturbation becomes more suppressed for negatively

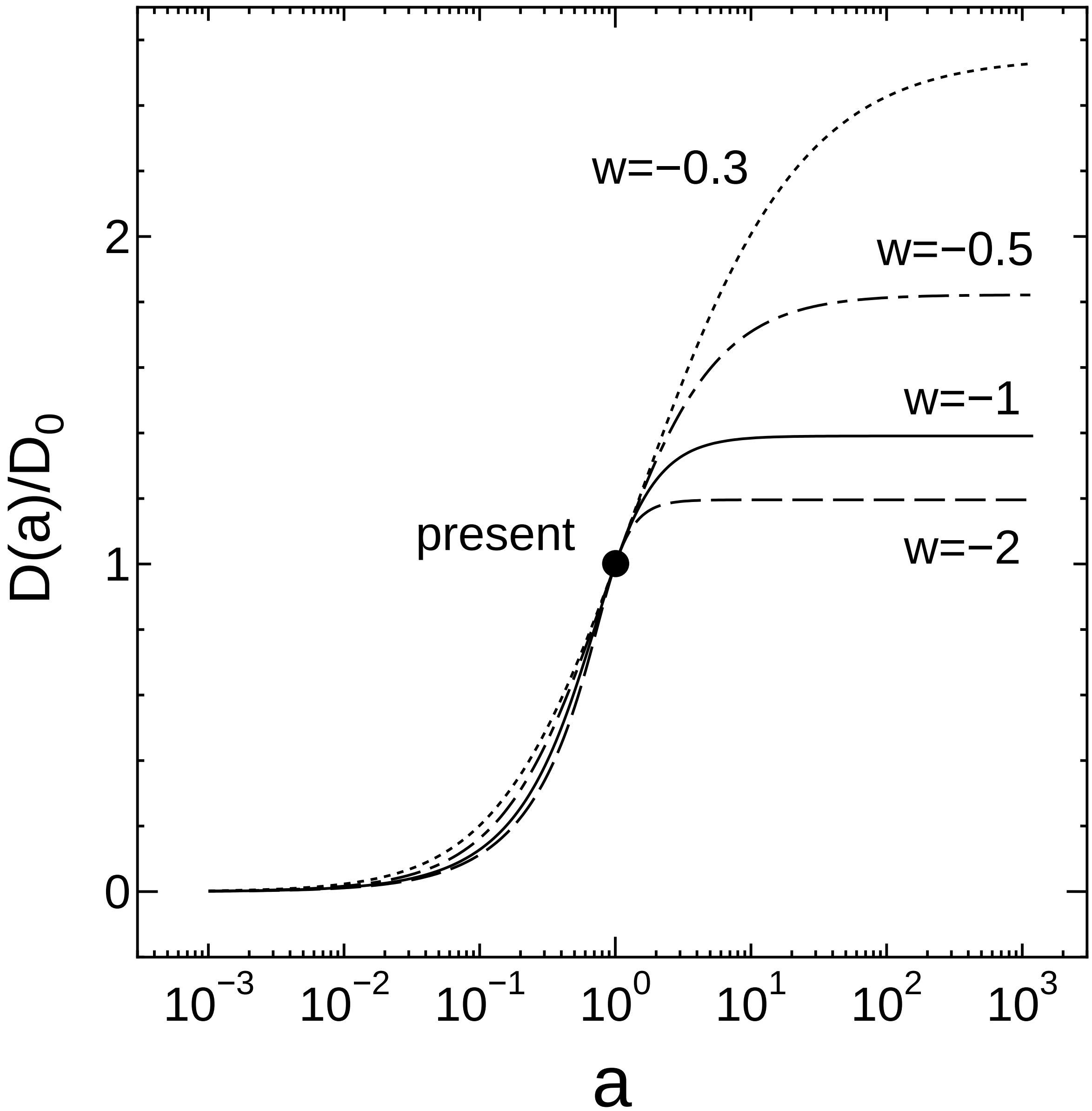

Figure 6. The linear growth rate as a function of scale factor for $w = -0.3$(dot), -0.5(dot-dash), -1(solid) and -2(dash) with $\Omega_M = 0.3$. $D(a)$ is normalized by its present value, $D_0 \equiv D(a = 1)$.

large w. We find from Eq.(16) that $D(a)$ asymptotically converges toward a constant value in the limit of $a \to \infty$ for $w < -1/3$ [25],

$$D(a) \to \frac{\Gamma(1-\frac{5}{6w})\Gamma(\frac{1}{2}-\frac{1}{6w})}{\Gamma(1-\frac{1}{2w})\Gamma(\frac{1}{2}-\frac{1}{2w})} \left(\frac{\Omega_X}{\Omega_M}\right)^{1/3w} \Bigg(1 + \frac{\Gamma(-\frac{1}{2}+\frac{1}{6w})\Gamma(1-\frac{1}{2w})\Gamma(\frac{1}{2}-\frac{1}{2w})}{\Gamma(\frac{1}{2}-\frac{1}{6w})\Gamma(\frac{1}{2}-\frac{1}{3w})\Gamma(-\frac{1}{3w})} \left(\frac{\Omega_X}{\Omega_M}\right)^{\frac{1}{6w}-\frac{1}{2}} a^{-\frac{1}{2}+\frac{3w}{2}}\Bigg), \quad (17)$$

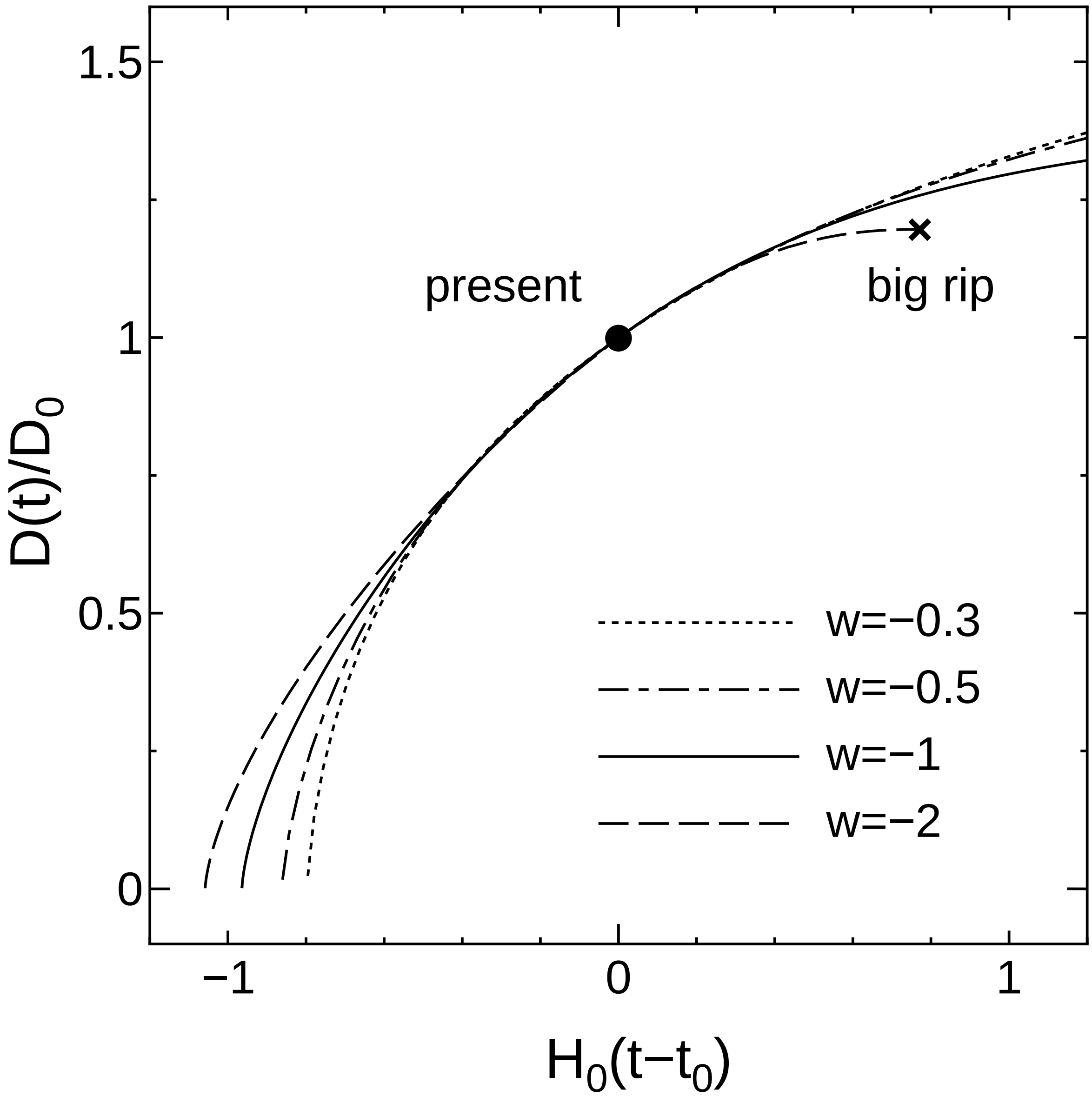

Figure 7. Same as Fig.6, but as a function of cosmic time. For $w = -2$, the dashed line ends in big rip singularity (cross).

while for $0 > w > -1/3$

$$D(a) \to \frac{\Gamma(1-\frac{5}{6w})\Gamma(\frac{1}{2}-\frac{1}{6w})}{\Gamma(1-\frac{1}{2w})\Gamma(\frac{1}{2}-\frac{1}{2w})}\left(\frac{\Omega_X}{\Omega_M}\right)^{1/3w}\left(1+\frac{w-1}{w(6w-5)}\left(\frac{\Omega_M}{\Omega_X}\right)a^{3w}\right). \quad (18)$$

This is because the time scale for expansion $\sim H^{-1} \sim (G\rho_X)^{-1/2}$ is much less than that for growth of density perturbation $\sim (G\rho_M)^{-1/2}$ in the future ($\rho_X \gg \rho_M$) [21]. The asymptotic constant in Eq.(17) and E.(18) is the same for both cases and is an increasing function of w. Eq.(17) and Fig.6 show that for negatively larger w, the growth of D is saturated earlier in the future. This is due to the rapid growth of ρ_X relative to ρ_M. The difference of D between various w may be suppressed when D is plotted as a function of cosmic time since time elapses more slowly for negatively large w.

5.2. Spherical collapse

In the previous section we studied the growth of linear density perturbation. In this section we consider spherical collapse model in order to investigate nonlinear growth of perturbation. It is already studied both analytically and numerically [26] for a cosmological constant and the fate of structures for $w < -1$ is briefly described in [6]. Our approach here will be more analytic than the former and more quantitative than the latter for general equation of state of dark energy.

We consider a evolution of spherical overdense region with uniform density ρ_{sp} and radius R [27]. The evolution of radius is described by

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\rho_{sp} - \frac{4\pi G}{3}(1+3w)\rho_X, \tag{19}$$

where $\rho_{sp} \propto R^{-3}$ and the mass is $M = (4\pi\rho_{sp}/3)R^3$. The first term represents the gravitational force which contracts the sphere, while the second term represents the repulsive force for $w < -1/3$ due to the dark energy and it prevents the contraction.

Defining dimensionless radius $y \equiv R/R_0$, where R_0 is the radius at present, the above equation (19) is rewritten as,

$$\left(\Omega_M + \Omega_X a^{-3w}\right)\frac{d^2}{d\ln a^2}\left(\frac{y}{a}\right) + \frac{1}{2}\left[\Omega_M + (1-3w)\,\Omega_X a^{-3w}\right]\frac{d}{d\ln a}\left(\frac{y}{a}\right) - \frac{1}{2}\Omega_M\frac{y}{a} + \frac{1}{2}\Delta_0\Omega_M\left(\frac{y}{a}\right)^{-2} = 0, \tag{20}$$

where $\Delta_0 \equiv \rho_{sp}(R_0)/\rho_{M,0}$ is the ratio of sphere to background density at present and it represents nonlinear density contrast. We also define linear density contrast δ_0 which is the density contrast at present if the density perturbation evolve by linear growth rate $D(a)$. Then, δ_0 is given by

$$\delta_0 = \lim_{a\to 0}\frac{\delta(a)}{D(a)}D_0 = \lim_{a\to 0}\frac{1}{D(a)}\left[\frac{\rho_{sp}(R(a))}{\rho_M(a)} - 1\right]D_0. \tag{21}$$

The analytical solution of Eq.(20) at $a \ll 1$ is obtained by

$$\frac{y}{a} = \Delta_0^{1/3}\left[1 - \frac{1}{3}\frac{\delta_0}{D_0}a + \mathcal{O}(a^2)\right]. \tag{22}$$

We use the above equation as the boundary condition to solve Eq.(20). Either Δ_0 or δ_0 can be determined by a condition of $y(a=1) = 1$, and hence there is one free parameter.

In order to relate the mass M of sphere to the density contrast δ_0, we assume that the overdense region is formed from the 1σ high-density peak of mass fluctuation. Then, the linear perturbation at present δ_0 is equal to the linear mass fluctuation δ_M for the mass M, $\delta_0 = \delta_M$. Here, δ_M is given by

$$\delta_M^2 = \frac{1}{2\pi^2}\int dk k^2 P(k) W^2(kr), \tag{23}$$

where $P(k)$ is the power spectrum and $W(kr)$ with $r = (2M/\Omega_M H_0^2)^{1/3}$ is the top-hat window function [28]. We normalise δ_M so that $\delta_M = \sigma_8 = 0.9$ at $r = 8h^{-1}$ Mpc. We assume $\Omega_M = 0.3$, the baryon density $\Omega_b = 0.04$ and the Hubble parameter $h = 0.7$.

Evaluating the dimensionless radius $y(a)$ in Eq.(20) for various values of parameters Δ_0, M and δ_0, we find that the fate of over-density sphere can be classified into three cases. We display these cases in Fig.8. The left panel is $\Delta_0 - w$ plane, while the right panel is $M - w$ plane (we also show δ_0).

(i) The region of "Monotonously Expands". For low density region, $\Delta_0 \lesssim 10$, the sphere monotonously expands forever. This is because the linear perturbation at far future, $\lim_{a\to\infty} \delta(a) = \delta_0/D_0 \lim_{a\to\infty} D(a)$, is a constant value from Eq.(17) and Eq.(18) and cannot reach the critical over density $\delta_c \sim 1.68$ [29] which is δ when the sphere collapses. In the far future, $a \gg 1$, the radius increases in proportional to the scale factor, $R \propto a$.

(ii) The region of "Collapses". For high density region, the sphere stops its expansion and collapses to $R = 0$.

(iii) The region of "Expands-Contracts-ReExpands". For intermediate mass density, $10 \lesssim \Delta_0 \lesssim 20$ with $w < -1$, the sphere stop its expansion and turns around. But the repulsive force due to the dark energy, which increases with time, prevents its contraction and the sphere re-expands forever.

In Fig.9, we show the radius R as a function of time with $w = -1.5$ for $M = 10^{14} M_\odot$ (dot-dash), $4 \times 10^{13} M_\odot$ (solid) and $2 \times 10^{13} M_\odot$ (dash), as an example of these three types. The radius at present is $R_0 = [3M/(4\pi\rho_{M,0}\Delta_0)]^{1/3}$. As shown in Fig.9, we note that the behavior of R is not symmetric about the turn-around time for $w \neq -1$.

The fraction of collapsed objects with mass greater than M at the scale factor a is given by [29]

$$F(M, a) = \frac{2}{\sqrt{2\pi}\delta_M(a)} \int_{\delta_c(a)}^{\infty} d\delta e^{-\delta^2/2\delta_M^2(a)}, \qquad (24)$$

where $\delta_M(a)$ is the mass fluctuation at a, $\delta_M(a) = \delta_M D(a)/D_0$. The critical density $\delta_c(a)$ is the linear perturbation when the sphere collapses at a. In Fig.10, we show $F(M, a)$ as a function of the scale factor with $M = 10^{11} M_\odot$ (dash), $M = 10^{13} M_\odot$ (solid), $M = 10^{14} M_\odot$ (dot), $M = 10^{15} M_\odot$ (dot-dash) and $M = 10^{16} M_\odot$ (dot-dot-dash). For larger w the objects with the larger mass $M \gtrsim 10^{15} M_\odot$ can form, while for smaller w the growth of density fluctuation is suppressed and the mass fraction becomes constant in the near future.

6. Summary

In this paper, we have classified the future of the universe with dark energy with various equation of state. Moreover we have investigated the future structure formation of the universe.

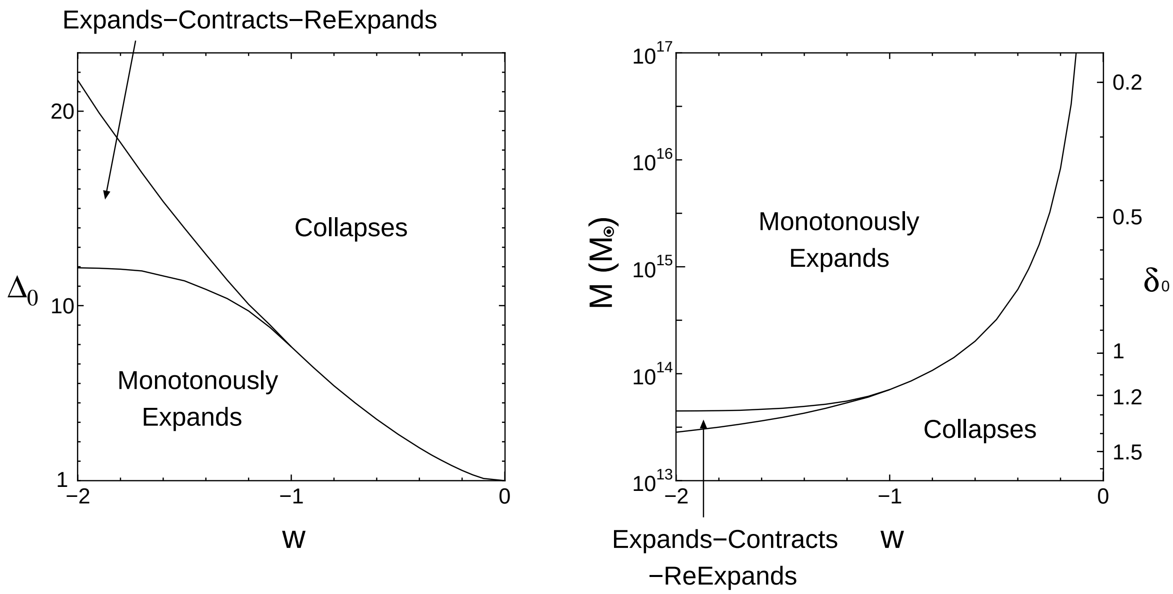

Figure 8. The fate of spherical overdense region. Δ_0 is the ratio of spherical density to background density at present, M is its mass and δ_0 is the linear perturbation at present. In the region of "Monotonously Expands", the sphere monotonously expands forever. In the region of "Collapses", the sphere stops its expansion and collapses to $R = 0$. In the region of "Expands-Contracts-ReExpands", the sphere stops its expansion and turns around. But the repulsive force due to the dark energy, which increases with time, prevents its contraction and the sphere re-expands forever.

We have found that the phase diagrams of the universe in $\Omega_M - \Omega_X$ plane are drastically different between $w \geq -1/3$ and $w < -1/3$. Infinitesimally small change of w from $w = -1/3$ to $w < -1/3$ moves regions of "expands forever" and "recollapses eventually" to completely different places. Moreover, for $w < -1/3$, there is a region of "no big bang", which does not exist for $w \geq -1/3$. On the other hand, phase diagrams of $-1/3 > w \geq -1$ or $w < -1$ are about same, except the "expands forever" region of the former is replaced with the "big rip" region of the latter.

This discontinuity at $w = -1/3$ can be seen in conformal diagrams too such that the event horizon appears for $w < -1/3$, but $w \geq -1/3$. The size of the event horizon grows, remains constant and decreases for $-1 < w < -1/3$, $w = -1$, and $w < -1$, respectively.

Concerning the structure formation, we have found the linear evolution asymptotically converges to a constant value for $w < 0$ and obtained the value in the limit of $a \to \infty$ as a function of w.

However, it is not always the case that these linear perturbations eventually turn into collapsed objects. We have classified the fate of the overdense regions into three categories, i.e., "monotonously expands", "collapses", and "expands-contracts-re expands". And we have shown each region in $w - \Delta_0$ plane, where Δ_0 is the ratio of density of the spherical region to background density at the present epoch. We have also found the largest structure in the future universe as a function of w.

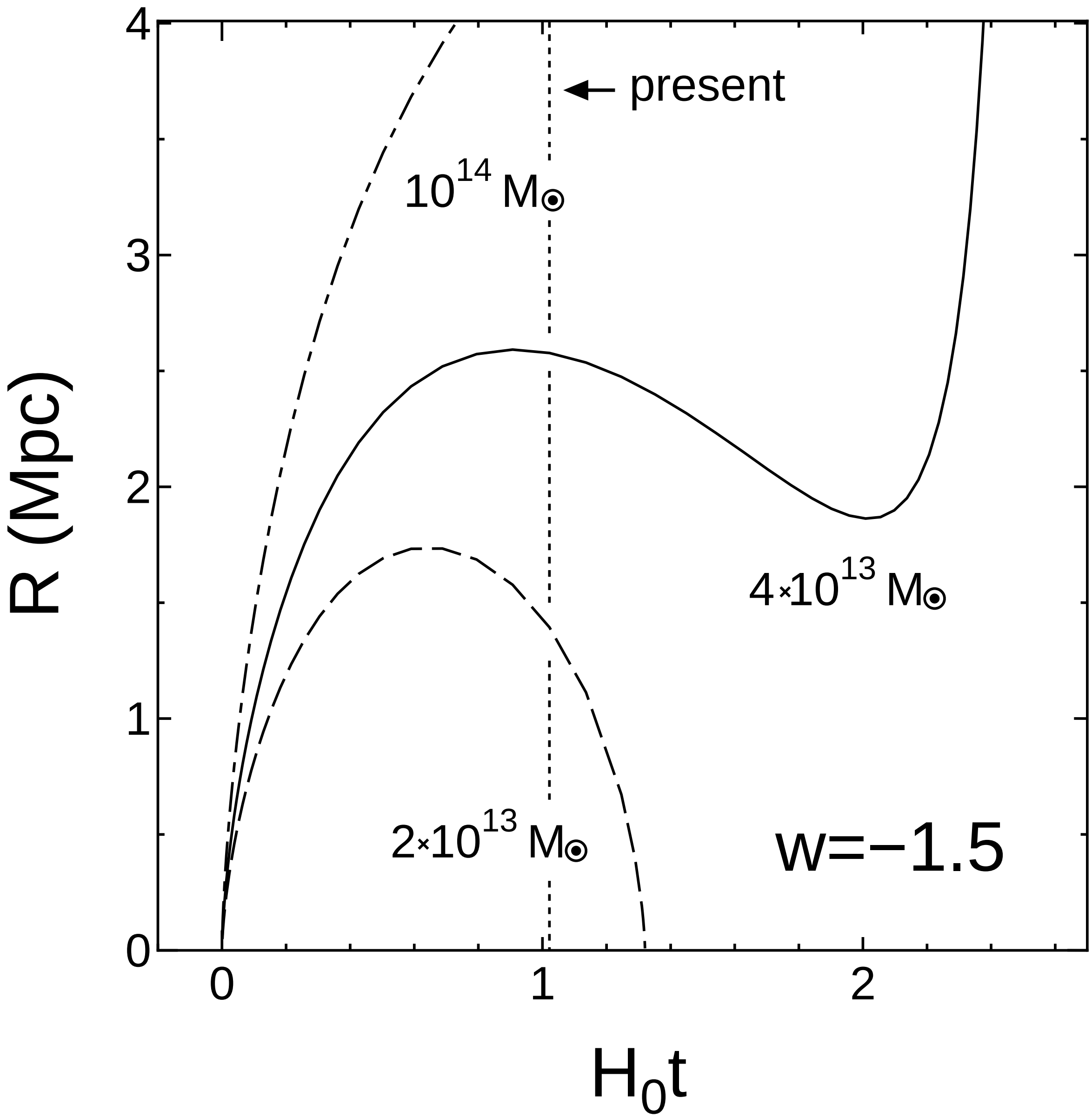

Figure 9. The radius R as a function of the cosmic time with $w = -1.5$ for $M = 10^{14}M_{\odot}$ (dot-dash), $4 \times 10^{13}M_{\odot}$ (solid) and $2 \times 10^{13}M_{\odot}$ (dash).

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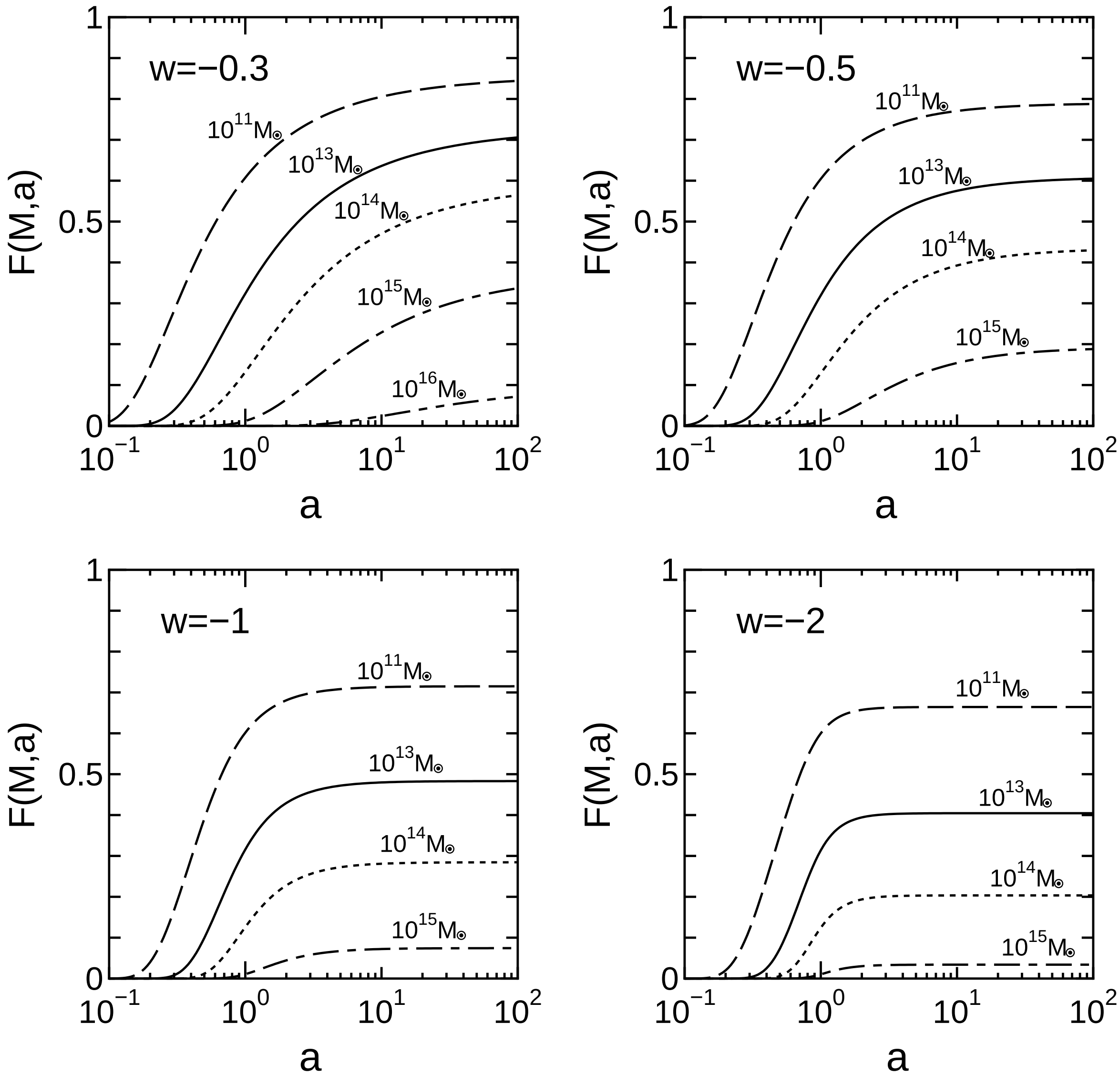

Figure 10. The fraction of collapsed objects with mass greater than M as a function of a.

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