

The end of locked inflation

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Abstract. We investigate the end of the inflationary period in the recently proposed scenario of locked inflation, and consider various constraints arising from density perturbations, loop corrections, parametric resonance and defect formation. We show that in a scenario where there is one long period of locked inflation, it is not possible to satisfy all of these constraints without having a period of saddle inflation afterwards, which would wipe out all observable signatures. On the other hand, if one does not insist on satisfying the loop correction constraint, saddle inflation can be avoided, but then inflation must have ended through parametric resonance.

1. Introduction

For a paradigm that has become so universally accepted as the one responsible for the generation of the primordial density fluctuations in the universe, as well as the observed homogeneity, isotropy and spatial flatness, it is remarkable that the Inflationary Universe Scenario appears to sit so uncomfortably in most current particle physics models. Following the initial work of Guth [1], where he demonstrated how old inflation could occur, it was soon realised that inflation could not end gracefully. The next variant, new inflation, where the false vacuum region of the potential was replaced by a slowly rolling scalar avoided this problem but at the expense of having to account for a significant degree of fine tuning of parameters in the underlying inflaton potential [2, 3, 4].

Over the past 20 years or so there have been many models of inflation, generally implementing the idea of slow roll inflation through the addition of extra fields. The best known of these, due to Linde, is known as hybrid inflation [5]. However, it is fair to say that in all these models there are parameters which have to be fine tuned to the correct values. Even the most recent ideas of inflation arising in brane worlds rely on slow roll to provide the required amount of inflation [6, 7, 8, 9] (For nice reviews of inflation models in particle physics and string theory see Refs. [10, 11]).

One of the problems facing models of inflation in the context of spontaneously broken supergravity is that the moduli fields which would naively be expected to be natural candidate inflaton fields generally fail to satisfy the required slow roll condition $\eta \sim m^2/H^2 \ll 1$, where m is the mass of the field and H the Hubble parameter during inflation, since the fields generally have protected masses $m \sim H$. This is the η -problem in inflation and has proven to be a real headache for those working in the field (see Ref. [9] for a recent example, and Refs. [12, 13, 14, 15, 16] for a recent proposal on how a new shift symmetry in the Kähler potential could alleviate the η -problem).

Dvali and Kachru [17, 18] recently revisited the idea of old inflation, asking whether there was a way we could make use of the fact that it did not rely on slow roll inflation, whilst avoiding the graceful exit problems it usually has associated with it.

They proposed extending the old inflation scenario, calling it *new old inflation* or *locked inflation* (see also Ref. [19]), and developed a model which did not require a slow-roll potential, thereby alleviating a number of the usual problems associated with inflation model building. It requires two coupled fields, hence is in the spirit of hybrid inflation, but it differs from the usual picture in that the universe arises from a single tunnelling event as the inflaton leaves the false vacuum. The subsequent dynamics arising from the oscillations of the inflaton field keeps a second field trapped in a false minimum. For suitable values of the parameters this then leads to a period of 50 e-foldings of inflation inside the bubble, allowing the bubble to grow large enough to contain our present horizon volume. In this model, reheating is accomplished when the inflaton driving the last stage of inflation rolls down to the true vacuum, with the accompanying adiabatic density perturbations arising from the moduli-dependent Yukawa couplings of the inflaton to matter fields. In particular the usual paradigm for calculating density perturbations from slow roll inflation no longer applies and a formalism allowing for more general scenarios has to be adopted [20, 21, 22, 23, 24, 25].

Following the work in Refs. [17, 18], Easther et al. investigated some of the cosmological applications of locked inflation [26]. They showed that an important constraint on the allowed range of parameters on the model arises from the fact that it is possible to have a secondary phase of *saddle inflation* following the initial period of locked inflation. This subsequent period of inflation leads to a strongly scale dependent spectrum and the possible overproduction of massive black holes in the early universe. Avoiding this outcome places strong constraints on the parameter space available to models of locked inflation.

In this paper, we extend the investigation of locked inflation. As in Ref. [26], we concentrate on the dynamics associated with the end of an extended period of new old inflation, but we introduce new constraints emerging from considerations of different physical properties associated with these models. In general we find that it is very difficult to satisfy the pure outcome associated with locked inflation, as there is invariably some extra features which emerge to spoil the locked inflation.

In particular, adopting the model proposed in [17], we consider the constraints which arise from density perturbations, loop corrections, parametric resonance and

defect formation at the end of inflation. Our conclusions are quite strong. We show that it is not possible to satisfy all of the above constraints without having a period of saddle inflation afterwards, which would wipe out all observable signatures arising from the initial period of locked inflation period. The one exception to this is that if we relax our insistence that we satisfy the loop correction constraint. Then it is possible to avoid a period of saddle inflation, but in that case we show that the locked inflation must have ended through parametric resonance, which may have important implications.

Following a discussion of the basics of the locked inflation model in Section 2, we begin to address the constraints imposed on the model in Section 3. These include recapping the density perturbation constraint arising from saddle inflation [26], and demonstrating how this bound becomes much tighter once we account for the fact that the period of inflation could end with the production of topological defects. Introducing the quantum loop corrections into the potential (expected to occur of course because supersymmetry has to be broken today), we show that we are very far from the slow roll regime. Turning our attention to the issue of parametric resonance we show how the evolution equations for the second waterfall field to which the inflaton is coupled, can be written as a modified Mathieu equation. The evolution of this system depends on the ratio of the mass scales of the inflaton (m_Φ) and waterfall field (m_ϕ). The loop correction constrains this parameter, $b \sim \frac{m_\phi^2}{m_\Phi^2} \leq 2$. We show how the system behaves for large and small values of b , corresponding to ignoring and satisfying the loop correction constraint respectively. In particular, we show that in both cases, inflation ends through parametric resonance. In Section 4, we investigate the consequences of the parametric resonance for the inhomogeneous modes, and show that in the case of small b , the non-linear effects associated with the resonance rapidly cut off the inflation, meaning that no inflation takes place. Finally we conclude in Section 5.

2. Locked inflation

The model discussed in Ref. [17], is based on the same potential as normal hybrid inflation [5],

$$V(\Phi, \phi) = \frac{1}{2}m_\Phi^2\Phi^2 + \frac{1}{2}\lambda\Phi^2|\phi|^2 + \frac{\alpha}{4}(|\phi|^2 - M_*^2)^2, \quad (1)$$

where Φ and ϕ are scalar fields, possibly with multiple components. At the origin, the “waterfall” field ϕ has a negative mass-squared $\mu^2 = -m_\phi^2 = -\alpha M_*^2$. The potential has symmetry-breaking minima at $|\phi| = M_*$ with $\Phi = 0$.

We assume that initially $|\Phi| > \Phi_C = \sqrt{\alpha/\lambda}M_*$, so that the expectation value of ϕ vanishes and the symmetry is restored. The usual slow-roll parameter η_Φ is given by

$$\eta_\Phi = M_{\text{Pl}}^2 \frac{V''}{V} = \frac{m_\Phi^2}{3H^2}, \quad (2)$$

where $M_{\text{Pl}} = (8\pi G)^{-1/2} \approx 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Slow roll inflation requires $\eta_\Phi \ll 1$, which poses a problem for attempts to build inflationary models from string theory or supergravity, because in those cases m_Φ is of order H . This is known

as the η -problem. More generally, one may ask how necessary slow roll is for inflation to occur. For these reasons, it is interesting to study this model with $m_\Phi \gtrsim H$.

In locked inflation, $m_\Phi > H$, but $|\Phi|$ is small enough so that the energy density is dominated by the constant contribution

$$V_0 = V(0, 0) = \frac{\alpha}{4} M_*^4. \quad (3)$$

This leads to an approximately exponential inflationary solution with the expansion rate

$$H^2 \approx \frac{V_0}{3M_{\text{Pl}}^2} = \frac{m_\phi^2 M_*^2}{12M_{\text{Pl}}^2}. \quad (4)$$

At the same time, Φ is oscillating with a decreasing amplitude

$$\Phi(t) \approx \Phi_0(t) \cos(m_\Phi^2 - 9H^2/4)^{1/2} t, \quad (5)$$

where $\Phi_0(t) = \Phi_0 e^{-3Ht/2}$.

During every oscillation, Φ passes through the instability region $|\Phi| < \Phi_C$, but if its velocity is fast enough, it will leave it before the instability has had an effect. In Ref. [17], the authors assumed that inflation goes on until the “effective mass”

$$m_{\text{eff}}^2 \approx \lambda \Phi_0(t)^2 - \alpha M_*^2, \quad (6)$$

becomes negative. This happens when $\Phi_0(t) \approx \Phi_C$, and implies that the number of e-foldings is

$$N = \frac{2}{3} \ln \frac{\Phi_0}{\Phi_C} \approx \frac{1}{3} \ln \frac{\lambda \Phi_0^2}{m_\phi^2}. \quad (7)$$

A different way to estimate when inflation ends was mentioned above Eq. (4) in Ref. [17]. The time Φ spends in the instability region is approximately

$$\Delta t = \frac{2\Phi_C}{m_\Phi \Phi_0(t)}. \quad (8)$$

This should be compared with the instability time scale $1/m_\phi$, which tells us that inflation ends when

$$\Phi_0(t) = \Phi_{C2} \approx \frac{m_\phi}{m_\Phi} \Phi_C \approx \frac{1}{\lambda^{1/2}} \frac{m_\phi^2}{m_\Phi}. \quad (9)$$

As we will discuss later in Section 3.4, this line of argument is the more appropriate one in certain cases.

For the actual parameters used in Ref. [17], the choice between Φ_C and Φ_{C2} makes no difference, because $m_\Phi \approx m_\phi$. The authors chose the “natural” values $M_* \approx M_{\text{Pl}}$ and $\lambda \approx 1$. They expressed the other parameters α and m_ϕ in terms of a mass scale M by writing $\alpha \approx M^4/M_{\text{Pl}}^4$ and $m_\phi^2 \approx M^4/M_{\text{Pl}}^2$, from which the approximate equality between the two masses follows. The initial amplitude of the Φ field was chosen to be $\Phi_0 \approx M_{\text{Pl}}$.

In this paper, we generally assume that $\lambda \sim 1$ and $\Phi_0 \sim M_{\text{Pl}}$. Because these parameter essentially only enter the calculations logarithmically through Eq. (7), their precise values are not very important. For discussing the values of the remaining

parameters, it turns out to be most useful to use the quantities η_Φ defined in Eq. (2), η_ϕ defined in Ref. [26],

$$\eta_\phi = \frac{4M_{\text{Pl}}^2}{M_*^2} = \frac{m_\phi^2}{3H^2}, \quad (10)$$

and the number of e-foldings N calculated with Eq. (7), which we can also write as

$$N \approx \frac{1}{3} \ln \frac{M_{\text{Pl}}^2}{m_\phi^2} \approx -\frac{1}{3} \ln \alpha + \frac{1}{3} \ln \frac{\eta_\phi}{4}, \quad (11)$$

and where the second term is generally negligible. It can be seen that to have the $N \gtrsim 50$ e-foldings required to solve the flatness and horizon problems, one needs an incredibly weak coupling $\alpha \sim 10^{-60}$ and a very low mass scale $m_\Phi \sim m_\phi \sim 10^{-12}$ GeV, but it is believed that supersymmetry protects these from radiative corrections [17].

3. Constraints on locked inflation

3.1. Density perturbations arising from the waterfall field

Because the slow-roll conditions are not satisfied during inflation, density perturbations cannot be created by the same mechanism as in ordinary inflationary models. In Ref. [17], the authors proposed an alternative mechanism based on the modulus field that controls the decay rate of ϕ . However, it was later pointed out in Ref. [26], that perturbations may be generated by the usual slow-roll mechanism if there is a period of “saddle inflation” after the locked inflation. If the period of saddle inflation is too short, the spectrum of these perturbations is inconsistent with observations and may lead to production of massive black holes. Therefore only parameters for which saddle inflation last for more than 50 e-foldings or does not occur at all are possible. In the former case, saddle inflation would wipe out all possible observable signatures of locked inflation, but the latter case of no saddle inflation is more interesting.

More specifically, the authors of Ref. [26] calculated the amount of saddle inflation in terms of the parameter η_ϕ defined in Eq. (10). This is the usual slow-roll parameter in the ϕ direction (up to the sign), but even if it is greater than unity, some amount of saddle inflation is possible. Defining a function

$$f(\eta_\phi) = \frac{3}{2} \left(\sqrt{1 + \frac{4}{3}\eta_\phi} - 1 \right) \approx \sqrt{3\eta_\phi}, \quad (12)$$

they showed that the number of e-foldings due to saddle inflation is

$$N_{\text{saddle}} \approx \frac{1}{f(\eta_\phi)} \ln \frac{\sqrt{2}}{f(\eta)} \frac{M_{\text{Pl}}}{H} \approx \frac{1}{\sqrt{3\eta_\phi}} \ln \sqrt{2} \frac{M_{\text{Pl}}}{m_\phi}. \quad (13)$$

In terms of the number of e-foldings N due to locked inflation, this is

$$N_{\text{saddle}} \approx \sqrt{\frac{3}{4\eta_\phi}} N. \quad (14)$$

In order to avoid saddle inflation, we need $N_{\text{saddle}} \lesssim 1$, and that gives the constraint

$$\eta_\phi \gtrsim \frac{3}{4} N^2 (\approx 2000 \text{ for } N = 50). \quad (15)$$

3.2. Defect formation

Because the end of inflation in this model involves a symmetry-breaking phase transition, topological defects are produced if the model allows them. This is the case if ϕ has fewer than four real components.

For real ϕ the defects would be domain walls, which are known to have disastrous cosmological consequences. They would soon dominate the energy density of the universe, and cause it to collapse in a short time.

If ϕ is complex, the defects are global strings. They have a logarithmic long-range interaction, which is difficult to study numerically, and their effects are therefore not as well understood as those of gauged strings. Nevertheless, it is believed that the general consequences would be similar. In particular, for large enough string tension they would produce observable temperature fluctuations in the cosmic microwave background. There is therefore an observational upper limit for the tension μ , i.e., the energy per unit length, of a string of [27]

$$G\mu \lesssim 10^{-6}. \quad (16)$$

For global strings, μ is logarithmically diverging,

$$G\mu \approx \frac{M_*^2}{4M_{\text{Pl}}^2} \log \frac{r}{r_0} = \frac{1}{\eta_\phi} \log \frac{r}{r_0}. \quad (17)$$

Thus, even if we ignore the logarithmic divergence, we have the constraint $\eta_\phi \gtrsim 10^6$, which is even stronger than that in Eq. (15).

A very similar constraint applies if ϕ is an $\text{SO}(3)$ triplet, because in that case global monopoles are formed. Unlike gauged 't Hooft-Polyakov monopoles, they have a linearly increasing interaction potential, and therefore they behave essentially as endpoints of cosmic strings, and have similar cosmological consequences.

Making the field ϕ charged under a gauge group would not relax these constraints, because the constraint from the cosmic strings is essentially the same in that case. In fact, formation of gauged monopoles would only make things more difficult, because it would lead to the well-known monopole problem.

Thus, we conclude that defect formation imposes a constraint $\eta_\phi \gtrsim 10^6$, unless ϕ has at least four components, in which case no defects exist. In Ref. [17], it was suggested that ϕ could be an $\text{SU}(2)$ doublet field to avoid this problem.

3.3. Loop corrections from the waterfall field

Because supersymmetry is broken, it cannot fully protect the potential from radiative corrections. According to Ref. [18], the one-loop correction from the ϕ field is

$$\Delta V_1 \approx \frac{m_\phi^2}{64\pi^2} \Phi^2 \ln \frac{\Phi^2}{Q^2}, \quad (18)$$

where Q is some mass scale. A quadratic tree-level mass term can be absorbed in the definition of Q . The potential in Eq. (18) has a local minimum at some point $\Phi > \Phi_C$, unless

$$1 + \ln \frac{\Phi_C^2}{Q^2} > 0. \quad (19)$$

The field would then become trapped in this minimum and inflation would never end. We must therefore satisfy Eq. (19), but this means that the effective mass defined as the curvature of the effective potential satisfies

$$m_\Phi^{\text{eff}}(\Phi)^2 > \frac{m_\phi^2}{32\pi^2} \left(2 + \ln \frac{\Phi^2}{\Phi_C^2} \right). \quad (20)$$

Using Eq. (7) one can express the effective mass at the time when inflation started in terms of the number of e-foldings,

$$m_\Phi^2(\Phi_0) \gtrsim \frac{m_\phi^2}{32\pi^2} (2 + 3N) \approx \frac{N}{100} m_\phi^2. \quad (21)$$

In terms of the slow-roll parameter η_Φ defined in Eq. (2), this can be written as

$$\eta_\Phi \gtrsim \frac{N}{100} \eta_\phi \left(\approx \frac{\eta_\phi}{2} \text{ for } N = 50 \right). \quad (22)$$

Using Eq. (15), one finds that $\eta_\Phi \gtrsim 1000$, which means that we are very far from the slow-roll regime.

3.4. Parametric resonance

Let us now consider the equation of motion for the waterfall field ϕ ,

$$\ddot{\phi} + 3H\dot{\phi} + [\lambda\Phi(t)^2 - m_\phi^2] \phi = 0. \quad (23)$$

Defining a new time variable $\tau = m_\Phi t$ and rescaling the field as $\chi = \exp(3Ht/2)\phi$, we can write this as

$$\chi'' + [2q(\tau)(1 - \cos 2\tau) - b] \chi = 0, \quad (24)$$

where

$$b = \frac{m_\phi^2}{m_\Phi^2} + \frac{9}{4}h^2, \quad q(\tau) = q_0 e^{-3h\tau}, \quad q_0 = \frac{\lambda\Phi_0^2}{4m_\Phi^2}, \quad h = \frac{H}{m_\Phi} = \frac{1}{\sqrt{3}\eta_\Phi}. \quad (25)$$

We can also write

$$b = \frac{\eta_\phi}{\eta_\Phi} \left(1 + \frac{3}{4\eta_\phi} \right), \quad (26)$$

which, together with the constraint (15) implies that $b \approx \eta_\phi/\eta_\Phi$. The loop constraint (22) can then be written as $b \lesssim 100/N$, and the saddle inflation constraint (15) as

$$h = \sqrt{\frac{b}{3\eta_\phi}} \lesssim \frac{2\sqrt{b}}{3N} \left(\approx 0.015\sqrt{b} \right). \quad (27)$$

The rescaled equation (24) is nothing but the Mathieu equation [28] with time-dependent parameters $q = q(\tau)$ and $a = 2q(\tau) - b$. Because h is small, $q(\tau)$ is varying slowly, and we can assume that at any given time, the evolution is well approximated by the ordinary Mathieu equation with those parameters.

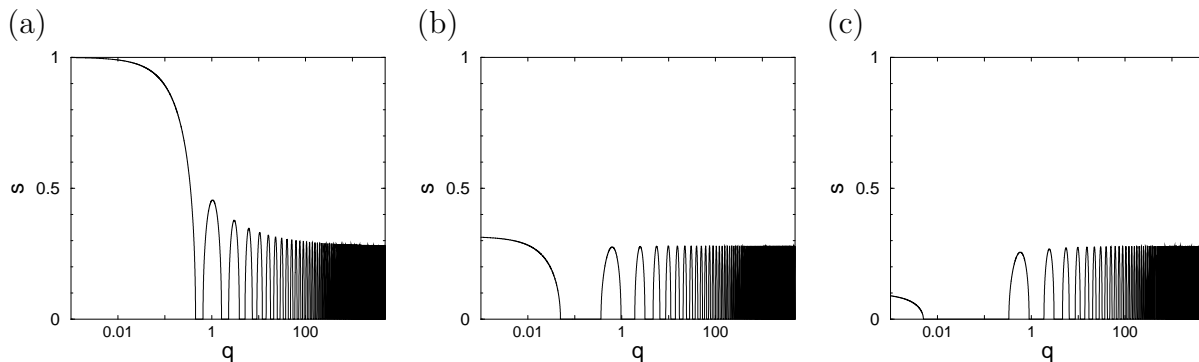


Figure 1. Floquet exponents for (a) $b = 1$, (b) $b = 0.1$ and (c) $b = 0.01$.

3.4.1. Small b We will first assume that b is of order one or smaller. This is required by the loop correction constraint (21), which implies that $b \lesssim 2$ for $N = 50$ e-foldings.

The Mathieu equation has stable and unstable regions in its parameter space. Floquet's theorem [28] shows that the solutions are of the form

$$\chi(\tau) = e^{s\tau} f(\tau), \quad (28)$$

where f is periodic, $f(\tau + \pi) = f(\tau)$, and the Floquet exponent s can be complex. We calculated the exponent numerically, by solving the equation in the interval $0 \leq \tau \leq \pi$, and have plotted the results for selected values of b in Fig. 1.

When $q \lesssim b/4$, Eq. (24) is dominated by the constant term b , and indeed, the plot shows that at those values, the system undergoes a normal tachyonic instability. For higher q , there are instability bands due to parametric resonance, and within each of them, the maximum value of the exponent is around 0.3. The mean value of the exponent, averaged over a range of q , is approximately $\bar{s} \approx 0.11$ for any $b \lesssim 1$. We can therefore expect that the solution of the full equation (24) behaves as $\chi(\tau) \sim \exp(\bar{s}\tau)$, implying that

$$\phi(t) \sim e^{(\bar{s}-3h/2)\tau}. \quad (29)$$

To check this assumption, we solved Eq. (24) numerically for two sets of parameters. As can be seen from Fig. 2, the results agree well with Eq. (29).

According to Eq. (29), ϕ is growing exponentially unless

$$h \gtrsim 0.07 \text{ (or equivalently } \eta_{\Phi} \lesssim 70 \text{)}. \quad (30)$$

This constraint was mentioned on page 7 of Ref. [17]. If we combine it with Eq. (27), we find $b \gtrsim N^2/100$, which is only compatible with the original assumption of small b if locked inflation lasts less than 10 e-foldings. Parametric resonance is therefore inevitable in a long period of locked inflation, if we want to satisfy both the saddle inflation and loop correction constraints. The consequences of this will be discussed in section 4.1.

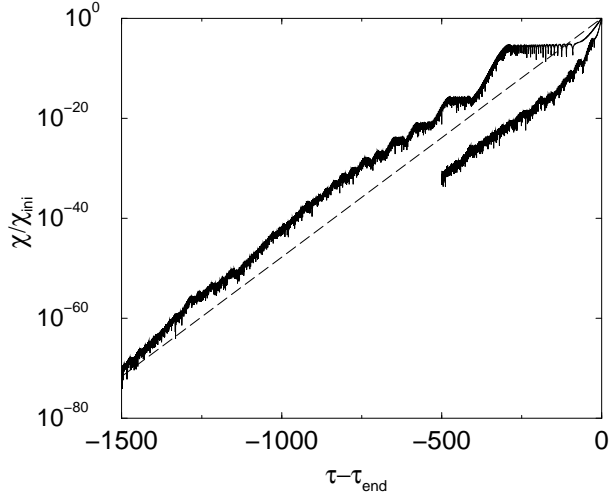


Figure 2. Numerical solutions of Eq. (24) for $(b = 0.1, h = 0.003)$ (top curve) and $(b = 1, h = 0.01)$ (bottom curve). The dashed line is the function $\exp[\bar{s}(\tau - \tau_{\text{end}})]$ with $\bar{s} = 0.11$. In this plot, τ_{end} is the time when the evolution becomes tachyonic. In other words, $q(\tau) = (b/4) \exp[-3h(\tau - \tau_{\text{end}})]$.

3.4.2. Large b One may also take the more phenomenological view that it is not necessary to satisfy the loop correction constraint, because the underlying fundamental theory is unknown. Locked inflation then loses its main advantage over the usual slow-roll scenario, but it still remains an interesting alternative. On the other hand, one can imagine a scenario with several shorter periods of locked inflation [17]. In both cases b can have larger values, the behaviour of the Floquet exponent changes, and Eq. (30) is not valid.

In Fig. 3, we show the Floquet exponent for larger b . It can be seen from this curve that the exponent becomes large well before the tachyonic range. Empirically, we find that the maximum value of the exponent is well described by

$$s_{\text{max}} \approx 0.3 \frac{b}{\sqrt{q}} \quad (31)$$

for a wide range of $q \gtrsim b$. This means that the resonance becomes stronger well before the tachyonic range. In fact, the criterion presented in Eq. (9) for the end of inflation can be written as $q_{\text{end}} \approx b^2/4$, which is fully consistent with Eq. (31), indicating that inflation ends when $s_{\text{max}} \approx 0.6$. However, this does not take into account the expansion of the universe.

Instead, we have to estimate whether the Hubble rate is high enough to suppress this resonance. Because the resonance bands are broader for large b , the mean exponent \bar{s} is greater than one half of s_{max} . By integrating the numerical solution for s over τ , we find that the mean value is approximately

$$\bar{s}(\tau) \approx 0.25 \frac{b}{\sqrt{q(\tau)}}. \quad (32)$$

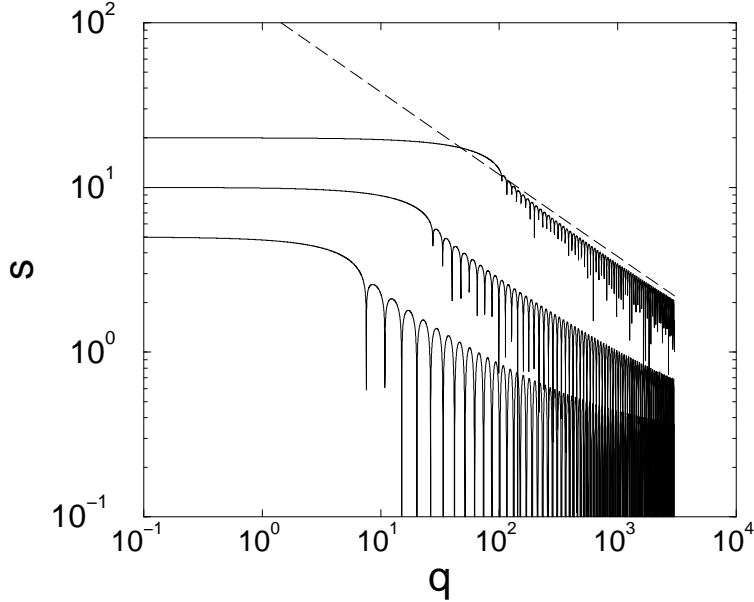


Figure 3. Floquet exponents for $b = 400$, $b = 100$ and $b = 25$ (from top to bottom). The dashed line shows the curve $s = 0.3b/\sqrt{q}$ for the case $b = 400$.

Again, we solved Eq. (24) numerically to check if it is justified to use the mean exponent. Fig. 4 shows a reasonably good agreement with the numerical data. The resonance is strong enough to overcome the Hubble suppression when $\bar{s} \approx 3h/2$. If we assume that inflation ends then, the number of e-foldings is

$$N' \approx \frac{1}{3} \ln \frac{36h^2 q_0}{b^2} \approx \frac{1}{3} \ln \frac{\lambda \Phi_0^2}{\alpha M_{\text{Pl}}^2} \approx N - \frac{1}{3} \ln \frac{\eta_\phi}{4}, \quad (33)$$

where N is the number calculated from the tachyonic instability in Eq. (7). This logarithmic correction is typically small compared with N , so the resonance does not reduce the amount of inflation significantly. However, $N' < N$ for all realistic values of η_ϕ , and therefore we can conclude that inflation always ends resonantly.

4. Inhomogeneities under the influence of parametric resonance

In Section 3.4, we showed that that inflation always ends resonantly. For small b , this happens because it is impossible to have a high enough Hubble rate to suppress the resonant growth of ϕ without violating Eq. (15). If we do not insist on the loop correction constraint, b can have larger values, but in that case the Floquet exponent becomes so high at the late stages that the resonance cannot be suppressed by any Hubble rate.

Let us now examine the consequences of this parametric resonance for inhomogeneous modes. A mode with comoving momentum k satisfies the equation of motion

$$\chi_k'' + [2q(\tau)(1 - \cos 2\tau) - b(k, \tau)] \chi_k = 0, \quad (34)$$

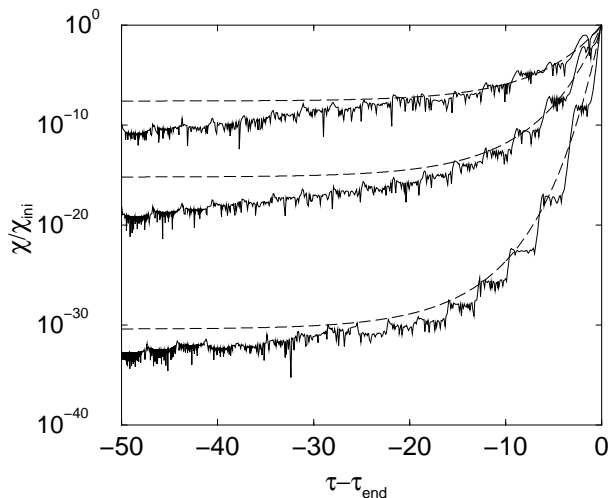


Figure 4. Numerical solutions of Eq. (24) for $b = 25$, $b = 100$ and $b = 400$ (from top to bottom) with $h = 0.1$. The dashed lines show the growth predicted by the mean Floquet exponent \bar{s} is Eq. (32). In all three cases, the late-time behaviour agrees with the prediction.

where $b(k, \tau) = b - \hat{k}^2 \exp(-2h\tau)$ and $\hat{k} = k/m_\Phi$.

4.1. Small b

Let us first discuss the case with small $b \lesssim 1$. As long as $q(\tau) \gtrsim -b(k, \tau)$, the mode behaves in the same way as the homogeneous mode, growing exponentially as $\chi_k \sim \exp(\bar{s}\tau)$.

Because $q(\tau)$ is generally much larger than b , we can ignore the constant in $b(k, \tau)$ and state that at any given time, modes with

$$\hat{k}^2 \lesssim q_0 e^{-h\tau} \quad (35)$$

resonate. Because the right-hand side of this equation is decreasing with time, fewer and fewer comoving modes are resonating.

At time τ , all modes with $k^2 \lesssim k_{\text{max}}^2(\tau)$, where

$$k_{\text{max}}^2(\tau) \approx q_0 m_\Phi^2 \exp(-h\tau) \approx \frac{\lambda \Phi_0^2}{4} \exp(-Ht), \quad (36)$$

have been continuously amplified since the start of inflation. The variance of those modes is the vacuum value multiplied by a k -independent amplification factor,

$$\langle \chi_k^2 \rangle(\tau) \approx e^{2\bar{s}\tau} \langle \chi_k^2 \rangle(0). \quad (37)$$

Let us now consider the equation of motion for Φ ,

$$\ddot{\Phi} + 3H\dot{\Phi} + (m_\Phi^2 + \lambda\langle\phi^2\rangle)\Phi = 0. \quad (38)$$

This equation becomes non-linear when the expectation value $\langle\phi^2\rangle = \exp(-3Ht)\langle\chi^2\rangle$ exceeds $\langle\phi^2\rangle_{\text{nl}} \approx m_\Phi^2/\lambda$. It is difficult to solve the non-linear equation, so we will

instead simply assume that inflation ends when the non-linearity appears. It would be interesting to carry out a more detailed study of the non-linear dynamics to check that assumption, but that is beyond the scope of this paper.

Assuming that all the modes with $k \lesssim k_{\max}$ are amplified by the same amount, we can calculate the expectation value after N e-foldings,

$$\langle \phi^2 \rangle \approx \frac{k_{\max}^2}{8\pi^2} [e^{(2\bar{s}/h-1)N} - 1] \approx \frac{\lambda\Phi_0^2}{32\pi^2} e^{-N} [e^{(2\bar{s}/h-1)N} - 1], \quad (39)$$

Because the constant prefactor in $\langle \phi^2 \rangle$ is much greater than $\langle \phi^2 \rangle_{\text{nl}}$, the evolution becomes non-linear during the first e-folding, and no inflation will take place. This rules out locked inflation with small b .

4.2. Large b

In the case of a larger $b \gg 1$, the Hubble rate can be higher and suppress the resonance. However, as Fig. 4 shows, the amplification becomes extremely fast at the late stages. We assume that Eq. (32) is valid for time-dependent b in the form

$$\bar{s}(k, \tau) \approx 0.25 \frac{b(k, \tau)}{\sqrt{q(\tau)}}. \quad (40)$$

At the time when the resonance starts, i.e., $h\tau = N'$, inhomogeneous modes start to resonate if $\hat{k}^2 \exp(-2N') \ll b$. This includes all superhorizon modes, for which $\hat{k} \exp(-N') \lesssim h$.

If the picture of amplification with the mean exponent \bar{s} is valid, all these modes grow very rapidly and inflation ends instantaneously. Therefore, we do not expect any unwanted effects such as density fluctuations on superhorizon scales or exceedingly strong fluctuations on small scales. However, a fully non-linear study is needed to understand the details of the resonant stage.

5. Conclusions

The prospect of avoiding the fine tuning associated with slow roll inflation is very appealing. In their recent work, Dvali and Kachru have provided a mechanism to do that by introducing a model which effectively uses both the physics of old and new inflation [17, 18]. The motivation behind the proposal is perhaps reinforced when we take on board the recent suggestions concerning the string landscape [29, 30, 31, 32, 33, 34, 35, 36], which invokes the possibility that there are a discrete set of closely spaced metastable vacua in string theory.

Following on from their work, Easter et al. [26] pointed out a number of constraints which the model would have to satisfy if its distinctive signatures were not to be completely obliterated by a second extended period of saddle inflation. In particular they showed that the effective slow roll parameter associated with the second waterfall field, η_ϕ has to be huge, of order 1000, in order to prevent this from occurring.

In this paper we have also investigated the dynamics of locked inflation in some detail, concentrating on some of the non-linear features associated with the rich dynamics of the coupled scalar fields present. We have confirmed a number of the conclusions found in both [17] and [26] and have found some new even stronger constraints. In particular we have shown that the whole parameter space available for locked inflation is ruled out if we want to have a single, long period of inflation that would solve the flatness and horizon problems. This follows from the following three observations:

- (i) Avoiding saddle inflation requires $\eta_\phi \gtrsim 2000$
- (ii) Taking account of the loop correction requires $\eta_\Phi \gtrsim \eta_\phi/2$
- (iii) Parametric resonance stops inflation during the first e-folding unless $\eta_\Phi \lesssim 70$

In addition we have shown the possible production of cosmic strings at the end of the period of inflation places an even tighter constraint on η_ϕ , namely $\eta_\phi \gtrsim 10^6$. However, this particular constraint is avoided if the waterfall field has more than three real components.

There are two possible ways of making the scenario viable. If one is willing to have several shorter periods of inflation [17], the constraints become weaker. To illustrate this, we have calculated the same constraints for an arbitrary number of e-foldings. Alternatively, a model with only one inflationary period is possible if one does not insist on the loop correction constraint. In either case, our results show that parametric resonance is inevitable. It is therefore important to understand the consequences of this resonance to properly judge the viability of locked inflation.

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Bibliography

- [1] A. H. Guth, “The Inflationary Universe: A Possible Solution To The Horizon And Flatness Problems,” *Phys. Rev. D* **23**, 347 (1981).
- [2] A. D. Linde, “A New Inflationary Universe Scenario: A Possible Solution Of The Horizon, Flatness, Homogeneity, Isotropy And Primordial Monopole Problems,” *Phys. Lett. B* **108**, 389 (1982).
- [3] A. Albrecht and P. J. Steinhardt, “Cosmology For Grand Unified Theories With Radiatively Induced Symmetry Breaking,” *Phys. Rev. Lett.* **48**, 1220 (1982).
- [4] S. W. Hawking and I. G. Moss, “Supercooled Phase Transitions In The Very Early Universe,” *Phys. Lett. B* **110**, 35 (1982).
- [5] A. D. Linde, “Hybrid inflation,” *Phys. Rev. D* **49**, 748 (1994) [arXiv:astro-ph/9307002].
- [6] G. R. Dvali and S. H. H. Tye, “Brane inflation,” *Phys. Lett. B* **450**, 72 (1999) [arXiv:hep-ph/9812483].
- [7] P. J. Steinhardt and N. Turok, “Cosmic evolution in a cyclic universe,” *Phys. Rev. D* **65**, 126003 (2002) [arXiv:hep-th/0111098].

- [8] N. Jones, H. Stoica and S. H. H. Tye, “Brane interaction as the origin of inflation,” JHEP **0207**, 051 (2002) [arXiv:hep-th/0203163].
- [9] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and S. P. Trivedi, “Towards inflation in string theory,” JCAP **0310**, 013 (2003) [arXiv:hep-th/0308055].
- [10] D. H. Lyth and A. Riotto, “Particle physics models of inflation and the cosmological density perturbation,” Phys. Rept. **314**, 1 (1999) [arXiv:hep-ph/9807278].
- [11] F. Quevedo, “Lectures on string / brane cosmology,” Class. Quant. Grav. **19**, 5721 (2002) [arXiv:hep-th/0210292].
- [12] M. Kawasaki, M. Yamaguchi and T. Yanagida, “Natural chaotic inflation in supergravity,” Phys. Rev. Lett. **85**, 3572 (2000) [arXiv:hep-ph/0004243].
- [13] J. P. Hsu, R. Kallosh and S. Prokushkin, “On brane inflation with volume stabilization,” JCAP **0312**, 009 (2003) [arXiv:hep-th/0311077].
- [14] F. Koyama, Y. Tachikawa and T. Watari, “Supergravity analysis of hybrid inflation model from D3-D7 system,” Phys. Rev. D **69**, 106001 (2004) [arXiv:hep-th/0311191].
- [15] H. Firouzjahi and S. H. H. Tye, “Closer towards inflation in string theory,” Phys. Lett. B **584**, 147 (2004) [arXiv:hep-th/0312020].
- [16] J. P. Hsu and R. Kallosh, “Volume stabilization and the origin of the inflaton shift symmetry in string theory,” JHEP **0404**, 042 (2004) [arXiv:hep-th/0402047].
- [17] G. Dvali and S. Kachru, “New old inflation,” [arXiv:hep-th/0309095].
- [18] G. Dvali and S. Kachru, “Large scale power and running spectral index in new old inflation,” [arXiv:hep-ph/0310244].
- [19] K. Dimopoulos and M. Axenides, arXiv:hep-ph/0310194.
- [20] D. H. Lyth and D. Wands, “Generating the curvature perturbation without an inflaton,” Phys. Lett. B **524**, 5 (2002) [arXiv:hep-ph/0110002].
- [21] K. Enqvist and M. S. Sloth, “Adiabatic CMB perturbations in pre big bang string cosmology,” Nucl. Phys. B **626**, 395 (2002) [arXiv:hep-ph/0109214].
- [22] G. Dvali, A. Gruzinov and M. Zaldarriaga, “A new mechanism for generating density perturbations from inflation,” Phys. Rev. D **69**, 023505 (2004) [arXiv:astro-ph/0303591].
- [23] G. Dvali, A. Gruzinov and M. Zaldarriaga, “Cosmological Perturbations From Inhomogeneous Reheating, Freeze-Out, and Mass Domination,” Phys. Rev. D **69**, 083505 (2004) [arXiv:astro-ph/0305548].
- [24] M. Zaldarriaga, “Non-Gaussianities in models with a varying inflaton decay rate,” Phys. Rev. D **69**, 043508 (2004) [arXiv:astro-ph/0306006].
- [25] L. Kofman, “Probing string theory with modulated cosmological fluctuations,” arXiv:astro-ph/0303614.
- [26] R. Easther, J. Khoury and K. Schalm, “Tuning locked inflation: Supergravity versus phenomenology,” JCAP **0406** (2004) 006 [arXiv:hep-th/0402218].
- [27] E. Jeong and G. F. Smoot, “Search for cosmic strings in CMB anisotropies,” arXiv:astro-ph/0406432.
- [28] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, (Dover, New York, 1974).
- [29] R. Bousso and J. Polchinski, “Quantization of four-form fluxes and dynamical neutralization of the cosmological constant,” JHEP **0006**, 006 (2000) [arXiv:hep-th/0004134].
- [30] J. L. Feng, J. March-Russell, S. Sethi and F. Wilczek, “Saltatory relaxation of the cosmological constant,” Nucl. Phys. B **602**, 307 (2001) [arXiv:hep-th/0005276].
- [31] A. Maloney, E. Silverstein and A. Strominger, “De Sitter space in noncritical string theory,” arXiv:hep-th/0205316.
- [32] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240].
- [33] L. Susskind, “The anthropic landscape of string theory,” arXiv:hep-th/0302219.
- [34] S. Ashok and M. R. Douglas, “Counting flux vacua,” JHEP **0401**, 060 (2004) [arXiv:hep-th/0307049].

- [35] B. S. Acharya, “A moduli fixing mechanism in M theory,” arXiv:hep-th/0212294.
- [36] B. S. Acharya, “Compactification with flux and Yukawa hierarchies,” arXiv:hep-th/0303234.