

Absorption of dark matter by a supermassive black hole at the Galactic center: role of boundary conditions

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The evolution of the dark matter distribution at the Galactic center is analyzed, which is caused by the combination of gravitational scattering on Galactic bulge stars and absorption by a supermassive black hole at the center of the bulge. Attention is focused on the boundary condition on the black hole. It is shown that its form depends on the energy of dark matter particles. The modified flux of dark matter particles onto the black hole is calculated. Estimates of the amount of dark matter absorbed show that the fraction of dark matter in the total mass of the black hole may be significant. The density of dark matter at the central part of the bulge is calculated. It is shown that recently observed γ radiation from the Galactic center can be attributed to the annihilation of dark matter with this density.

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1 INTRODUCTION

Investigations of the distribution of dark matter in the nuclei of galaxies currently attract the interest of many researchers in view of both the effect of dark matter on the growth of supermassive black holes and the search for its possible annihilation radiation. In this work, we analyze the evolution of the dark matter distribution at the Galactic center under the effects of scattering by stars at the Galactic center and absorption by the central black hole. We assume that dark matter consists of particles that undergo only gravitational interaction and possibly slightly annihilate (so-called weakly interacting massive particles, WIMPs) [1].

The interaction of the central black hole with its environment, including dark matter at the Galactic center (bulge), was analyzed in [2, 3, 4, 5, 6] and elsewhere. An approach in which the distribution function of the dark matter is written in terms of invariants of motion seems to be most consistent. In the case of spherically symmetric, sufficiently slow evolution (compared to dynamical time), such variables are the orbital angular momentum m , its projection m_z onto the z axis, and the radial action (adiabatic invariant)

$$I = \frac{1}{\pi} \int_{r_-}^{r_+} v(r) dr.$$

We use a model of the dark matter halo formation in which its spatial density profile before the formation of the Galaxy and baryonic compression had the form $\rho(r) \sim r^{-12/7}$, and the initial distribution function in terms I, m, m_z has the form [7]

$$f(I, m, m_z) = f_0 I^{1/8} \delta(m^2 - l_0^2 I^2), \quad (1)$$

where $l_0 \sim 0.1$ is a small parameter. We emphasize the convenience of choosing radial action I rather than particle energy E as a basic variable when analyzing evolution in a slowly varying potential.

1.1 Kinetic equation. In the zeroth approximation, the distribution function of dark matter in the chosen variables does not change upon baryonic compression. Under the assumption that the black hole is formed at the coordinate origin, the increase in the black hole mass M_{bh} causes the absorption of particles whose angular momenta m are less than the critical value $m_g = 4 G M_{bh}/c$, but the amount of such particles turns out to be negligibly small [4]. However, the situation is significantly changed due to effects associated with the gravitational scattering of dark matter particles on bulge stars. These effects lead to the diffusion of particles in phase space [4, 6].

In the first approximation, the evolution of the distribution function is described by the kinetic equation

$$\frac{\partial f(\{I_i\}, t)}{\partial t} = \frac{\partial}{\partial I_k} \left[R_{kl} \frac{\partial f}{\partial I_l} \right] \quad (2)$$

where $\{I_i\} = \{I, m, m_z\}$, and R_{kl} are the corresponding diffusion coefficients. Owing to the spherical symmetry of the evolution, m_z does not enter into the equations. In addition, I can be approximated as $I(E, m) \approx J(E) - \beta m$ [2], where $\beta = 1$ for the Coulomb potential of the black hole, and $\beta \simeq 0.6$ for the isothermal potential of the bulge. Hereafter, we will use J instead of I , because we are interested in the region of low angular momenta. Since the parameter l_0 is small, I can also be changed to J in the initial distribution (1). It can be shown that

the cross terms R_{12} are small in the region of interest [8]. Finally, the kinetic equation takes the form

$$\frac{\partial f(J, m, t)}{\partial t} = \frac{\partial}{\partial J} \left(R_{11} \frac{\partial f}{\partial J} \right) + \frac{1}{m} \frac{\partial}{\partial m} \left(m R_{22} \frac{\partial f}{\partial m} \right). \quad (3)$$

The diffusion coefficients are calculated in [8]. One-dimensional diffusion along the m axis turns out to be the most significant. It leads to the particle flux into the region of low orbital angular momenta and to their absorption by the black hole for $m \leq m_g$. This process was analyzed in detail in [4, 8]. The effect considered there is the following. Let us write Eq. (3) in the one-dimensional form

$$\frac{\partial f(J, m, t)}{\partial t} = \frac{1}{m} \frac{\partial}{\partial m} \left(m R_{22} \frac{\partial f}{\partial m} \right). \quad (4)$$

The initial condition is taken in the form (1), and the boundary condition at the black hole corresponds to absorption: $f(J, m = m_g, t) = 0$. The diffusion coefficient R_{22} calculated for the isothermal bulge has the form

$$R_{22} = 0.46 G M_s L_c \sigma_0, \quad (5)$$

where σ_0 is the velocity dispersion of bulge stars, M_s is the mass of a star (for simplicity, we assume that all stars have the same mass equal to M_\odot), and $L_c \simeq 10$ is the Coulomb logarithm.

In this formulation, the solution of the diffusion equation yields the following expression for the dark matter flux onto the black hole

$$S(t) = 2(2\pi)^3 \int_0^\infty f_0 J^{1/8} S_J(t) \propto R_{22}^{9/16} t^{-7/16}. \quad (6)$$

Here

$$S_J(t) = \frac{0.18}{\ln \frac{l_0 J}{2m_g}} \cdot \frac{1}{t} \exp \left(-\frac{l_0^2 J^2}{5 R_{22} t} \right) \quad (7)$$

is the flux in one-dimensional diffusion equation (4) with the initial condition $f(t=0) = \delta(m^2 - (l_0 J)^2)$.

However, the absorbing boundary condition is valid only for $\Delta m^2 \ll m_g^2$, where

$$\Delta m = \sqrt{2T(J) R_{22}} \simeq \frac{\sqrt{2\pi R_{22} J}}{\sigma_0} \quad (8)$$

is the rms change in the orbital angular momentum per orbital period $T(J)$. It is easy to show that this condition is violated even at rather low energies of the particle. In this work, we obtain a more accurate expression for the boundary condition and investigate its effect on the absorption rate of the dark matter. We start with the presentation of the Milky Way bulge model.

1.2 Bulge model. We assume that the star distribution in the bulge is isothermal, i.e., spherically symmetric and isotropic, and has the power-law density profile

$$\rho_{s(out)}(r) = \rho_0 \left(\frac{r}{R_0} \right)^{-2}, \quad \rho_0 R_0^2 = \frac{\sigma_0^2}{2\pi G}, \quad (9)$$

where σ_0 is the star velocity dispersion independent of the distance to the center. In the inner part of the bulge – the black hole influence region – the density profile has lower exponent [9]:

$$\rho_{s(in)}(r) = \rho_0 \left(\frac{r}{R_0} \right)^{-3/2}, \quad (10)$$

where ρ_0 and R_0 are the same values as in (9), which assures the continuity of the star density at the boundary of the black hole influence region. We define the radius of the influence region such that the total mass of stars inside the region is equal to the mass of the black hole:

$$M_{in} = \int_0^{R_0} \rho_{s,in}(r') 4\pi r'^2 dr' = \frac{8\pi}{3} \rho_0 R_0^3 = M_{bh}, \quad (11)$$

$$\frac{G M_{bh}}{R_0} = \frac{4}{3} \sigma_0^2$$

In Milky Way, $\sigma_0 = 85 \div 90$ km/s, and the value of R_0 corresponding to the observed black hole mass $M_{bh} = 3 \cdot 10^6 M_\odot$, is $R_0 = 1.3$ pc. These values also give the observation-consistent normalization for the star density profile at the center [See Eq.(10)].

2 ABSORPTION OF PARTICLES BY THE BLACK HOLE AND BOUNDARY CONDITIONS

Thus, as was mentioned above, the absorbing-boundary approximation $f(m_g, J, t) = 0$ is valid only if $\Delta m \ll m_g$, i.e., change in the orbital angular momentum of the particle per period is small compared to the characteristic problem scale – the boundary orbital angular momentum. At the same time, as is easily seen from Eqs. (8, 7), this condition is violated for Milky Way at present, because the maximum of the flux comes from the values of J for which $\Delta m \gg m_g$.

To describe correctly the absorption of particles by the black hole, we consider two limiting cases (see figure): the random-walk approximation (absorption for $\Delta m \ll m_g$, and the pinhole approximation for $\Delta m \gg m_g$ (the names are taken from Lightman&Shapiro [10], where this process was considered in application to stars).

The absorption models are different in the random-walk and pinhole approximations. In the former case, the distribution function $f(m, J, t) \rightarrow 0$ for $m \rightarrow m_g$, and its derivative at the absorption boundary $m = m_g$

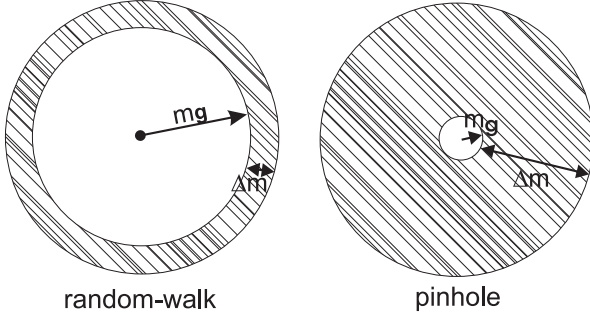


Figure 1: Two absorption regimes

determines the flux. In the latter case, $f(m) \rightarrow f_g$ where f_g is a nonzero boundary limit, and the absorption rate is determined as follows. There is a nonzero probability per orbital period that the angular momentum m of a particle increases by δm such that $m + \delta m < m_g$ and, hence, the particle is captured by the black hole. Flux onto the black hole is determined as the sum of the absorption probabilities for all particles per period. At the same time, it is equal to the diffusion flux from higher m values, where the diffusion approximation is valid ($\Delta m < m$).

2.1 Boundary condition. Different absorption regimes can be described by the following modification of the boundary condition:

$$\left(f - m_g \alpha \frac{\partial f}{\partial m} \right) \Big|_{m=m_g} = 0. \quad (12)$$

Values $\alpha \ll 1$ correspond to the random-walk approximation, and $\alpha \gg 1$ to the pinhole approximation.

Therefore, if we take the value of the distribution function at the boundary $f(m = m_g) = f_g$ and assume that the flux is continuous near the boundary and varies only slightly at low m , then we obtain the following expression for f near the boundary:

$$f(m) = f_g \left(1 + \frac{1}{\alpha} \ln \frac{m}{m_g} \right) \quad (13)$$

Our aim is to obtain expressions for α and for modified flux $S_J(t)$ of particles with radial action J on the black hole in the limit $\alpha \gg 1$.

For this purpose, we calculate the number of particles that diffuse into the region $m < m_g$ and are absorbed by the black hole during one orbital period. We assume that the probability distribution for obtaining a given increment δm of the particle angular momentum is Gaussian with the variance $\overline{\delta m^2} = \Delta m^2$ and mean value

$\overline{\delta m} \ll \Delta m$ [10]:

$$p(m, \delta m) = \frac{1}{\sqrt{2\pi} \Delta m} \exp\left(-\frac{\delta m^2}{2 \Delta m^2}\right) \times \frac{1 + \delta m/m}{\frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{m}{\sqrt{2} \Delta m}\right) \right] + \frac{\Delta m}{\sqrt{2\pi}} \exp\left(-\frac{m^2}{2 \Delta m^2}\right)} \quad (14)$$

The normalization is taken such that $\int_{-m}^{\infty} p(m, \delta m) d\delta m = 1$.

The total number of particles absorbed per period is given by the expression

$$Q = \int_{m_g}^{\infty} 2m dm f(m) \int_{-m}^{-(m-m_g)} p(\delta m) d\delta m = S_J T(J). \quad (15)$$

At the same time, it is equal to the flux from larger m values per period:

$$Q = S_J T(J) = m R \frac{\partial f}{\partial m} T = \frac{f_g}{\alpha} R T = \frac{f_g \Delta m^2}{2\alpha}. \quad (16)$$

In the limit $\Delta m \gg m_g$, we obtain

$$\alpha = 2.8 \left(\frac{\Delta m}{m_g} \right)^2, \quad (17)$$

which agrees with the result obtained in [10].

2.2 Dark matter absorption. To determine the expression for the particle flux onto the black hole $S_J(t)$, we use the same procedure as for the pinhole limit considered in [6]. Namely, knowing the general form of the expression $S_J(t) \propto \frac{1}{t} \exp(-\frac{m_0^2}{5 R t})$, we combine the solution $f(m, t)$ given by Eq. (13) and the Gaussian diffusion solution for the initial condition $f(t = 0) = \delta(m^2 - m_0^2)$. Thus, we obtain

$$S_J(t) = \frac{0.18}{\alpha + \ln \frac{m_0}{2 m_g}} \frac{1}{t} \exp\left(-\frac{l_0^2 J^2}{5 R t}\right) \quad (18)$$

This expression for flux differs from the one obtained in previous investigation (7) by an additional term α in the denominator. As was pointed out, at present $\alpha(J) \gg 1$ for the values of J from which flux is maximal and, therefore, the total flux is much lower than the value obtained without corrections. Indeed, the total flux is given by the expression

$$S(t) = 2(2\pi)^3 \int_0^{\infty} dJ f_0 J^{1/8} S_J(t) \approx 9.5 f_0 \left(\frac{5 R_{22} t}{l_0^2} \right)^{9/16} t^{-1} (\beta + 1)^{-8/9} \quad (19)$$

where the quantity

$$\beta = \frac{\sqrt{5} R_{22} t 2\pi R_{22}}{20 l_0 \sigma_0^2 m_g^2} \approx 7 \left(\frac{t}{10^{10}} \right)^{\frac{1}{2}} \left(\frac{M_{bh}}{3 \cdot 10^6 M_\odot} \right)^{-2} \quad (20)$$

reflects the correction due to the modification of boundary condition (12). If the correction is disregarded (by setting $\beta = 0$), the expression (19) yields a power-law increase of the black hole mass $M_{bh}(t) \propto t^{9/16}$ [6]. The estimate for the mass directly depends on the parameters f_0 and l_0 of the initial dark matter distribution. Taking $f_0 = 6 \cdot 10^9 \text{ g (cm}^2/\text{s)}^{-9/8}$ and $l_0 = 0.1$ for Milky Way according to [6], we estimate the mass as $M_{bh} \simeq 11 \cdot 10^6 M_\odot$ for the present time ($t = 10^{10} \text{ yr} = 3 \cdot 10^{17} \text{ s}$) and for negligibly low seed black hole mass (under the assumption that mass increases only due to the absorption of dark matter; i.e., it is a lower estimate for the mass).

The inclusion of the flux modification significantly changes the situation. As follows from Eq. (20), if the black hole mass is low, then $\beta \gg 1$ and hence the dark matter flux is also low. This is an evidence of the baryonic nature of a seed black hole. An upper estimate for the absorbed dark matter mass M_d can be obtained as the difference $M_d = M_0 - M_{bh}(0)$ between the final mass $M_{bh}(t_0 = 3 \cdot 10^{17} \text{ s}) = M_0 = 3 \cdot 10^6 M_\odot$, and the initial mass $M_{bh}(0)$ obtained by solving the equation $dM_{bh}/dt = S(t)$ backwards in time. For the values f_0 and l_0 adopted above, we obtain $M_d = 0.67 M_0 = 2 \cdot 10^6 M_\odot$. If the value f_0 is ten times lower, then $M_d = 0.11 M_0$. Another estimate can be obtained by the supposition that the growth of the black hole due to absorption of both dark and baryonic matter has a power-law form $M_{bh}(t) = M_0(t/t_0)^\gamma$. In this case, the mass of dark matter absorbed is given by the expression $M_d = \int_0^{t_0} S(t, M_{bh}(t)) dt$. For the growth law with the exponent $\gamma = 1/2$ (obtained in [5] analyzing the absorption of stars) $M_d = 0.36 M_0$; it is proportional to the normalization constant f_0 .

Thus, the correct formulation of the boundary condition considerably (approximately by an order of magnitude) reduces the estimate for the mass of dark matter absorbed by the black hole. The absorbed mass comprises up to $1/3 - 1/2$ of the current black hole mass for the chosen normalization of the dark matter density. We emphasize that a similar consideration for the absorption of stars shows that the modification of the boundary condition is insignificant [5]. This difference results from the fact that dark matter particles initially have low orbital angular momenta and sooner reach the absorption boundary. This means that the flux at a given time comes from higher J values than those for stars. Hence, the boundary condition for dark matter changes its form at large J values more considerably.

3 DENSITY PROFILE OF DARK MATTER AND ITS ANNIHILATION

We consider the detection of γ radiation from the Galactic center, which is probably due to annihilation of weakly interacting particles of dark matter (WIMPs), as a possible observational test of the current dark matter distribution. Photons of energies of several TeVs have been detected on the H.E.S.S. telescope, whose angular resolution of $3' - 5.8'$ corresponds to a spatial region of about 10 pc in size (the distance to the Galactic center is taken to be 8.5 kpc). As was shown in [11], the observed photon flux can be explained by the annihilation of supersymmetric particles with a mass of about 12 TeV whose density has a power-law profile and a mean value of about $\sim 10^3 M_\odot/\text{pc}^3$ inside the central 10 pc.

Knowing the distribution function of dark matter in phase space, it is easy to calculate its density profile as a function of the distance from the coordinate origin:

$$\rho(r) = \frac{2\pi}{r^2} \int_{J_{min}}^{\infty} dJ \frac{\partial E}{\partial J} \int_{m_g}^{m_{max}} 2m dm \frac{f(J, m, t)}{\sqrt{2(E - \Psi(r)) - \frac{m^2}{r^2}}} \quad (21)$$

Let us calculate the dark matter density in the central region 10 pc in size. For simplicity, we assume that the potential in this region is determined by the central star cluster with a density $\rho_s(r) = \sigma_0^2/2\pi G r^2$, where σ_0 is the star velocity dispersion. As follows from (13, 18),

$$f(J, m, t) = f_0 J^{1/8} \frac{S_J(t)}{R} \left(\alpha + \ln \frac{m}{m_g} \right) = f_0 J^{1/8} \frac{\alpha + \ln \frac{m}{m_g}}{\alpha + \ln \frac{l_0 J}{2m_g}} \frac{0.18}{R t} \exp \left(-\frac{l_0^2 J^2}{5 R t} \right) \quad (22)$$

For $r \sim 10 \text{ pc}$ $\alpha \sim 100$, therefore, the first fraction in Eq.(22) is close to unity. The quantity m_{max} in Eq.(21) is determined from the condition of zero denominator: $m_{max} = 2\sigma_0 r \sqrt{\ln[\sqrt{\pi} J/\sigma_0 r]} \simeq 2\sigma_0 r$ for $J > J_{min} = \sigma_0 r/\sqrt{\pi}$. However, approximation (22) is applicable only for low m values and, in particular, the total integral $\int_0^{m_{max}} f(J, m, t) 2m dm$ cannot be larger than $f_0 J^{1/8}$ according to the normalization condition. This gives the restriction $m \leq m_0 = \sqrt{5 R t}$, which becomes important for $r > r_* \simeq 2 \text{ pc}$. Thus, the internal integral in Eq.(21) is given by the expression

$$\int_{m_g}^{\min(m_{max}, m_0)} \frac{2m dm r}{\sqrt{m_{max}^2 - m^2}} \frac{0.18 f_0 J^{1/8}}{R t} \exp \left(-\frac{l_0^2 J^2}{5 R t} \right) = 2r \frac{0.18 f_0 J^{1/8}}{R t} \exp \left(-\frac{l_0^2 J^2}{5 R t} \right) \times$$

$$\times \begin{cases} m_{max} = 2\sigma_0 r & , \quad r < r_* \\ 2\sigma_0(r - \sqrt{r^2 - r_*^2}) & , \quad r > r_* \end{cases}$$

Since $\partial E/\partial J = 2\pi/T(J) = 2\sigma_0^2/J$, we have

$$\begin{aligned} \rho(r) &= 16\pi \sigma_0^3 \min \left[1, 1 - \sqrt{1 - \left(\frac{r_*}{r}\right)^2} \right] \times \\ &\frac{0.18}{Rt} \int_{J_{min}}^{\infty} dJ \frac{f_0 J^{1/8}}{J} \exp\left(-\frac{l_0^2 J^2}{5 Rt}\right) \simeq \\ &\simeq \frac{15 \sigma_0^3}{Rt} f_0 \left(\frac{5 Rt}{l_0^2}\right)^{1/16} \min \left[1, 1 - \sqrt{1 - \left(\frac{r_*}{r}\right)^2} \right] \simeq \\ &\simeq 10^3 \frac{M_\odot}{\text{pc}^3} \cdot \left(\frac{r}{10 \text{ pc}}\right)^{-2} \end{aligned}$$

Thus, the estimate for the density agrees in order of magnitude with observations. In any case, the dark matter density does not exceed the star density, which is estimated as $\rho_s(r) = 1.2 \cdot 10^6 (r/0.4 \text{ pc})^\beta M_\odot/\text{pc}^3$, where $\beta \simeq 1.5$ for $r < 0.4 \text{ pc}$ and $\beta \simeq 2$ for $r > 0.4 \text{ pc}$ [9]. For a more accurate determination of the dark matter density profile, it is necessary to take into account the diffusion along the J axis and a more exact expression for $f(J, m, t)$ in the limit $m \gg m_g$.

4 CONCLUSIONS

In this work, the diffusion of dark matter at the Galactic center has been considered. This diffusion is caused by scattering of dark matter particles on bulge stars and is considered in the $\{J, m, m_z\}$ phase space, where J is the modified radial action. Diffusion along the axis of the orbital angular momentum m plays the main role in the absorption of dark matter. The presence of the black hole determines the boundary condition at $m = m_g$ that has different forms for small and large values of J . The dark matter flux $S(t)$ calculated using the modified boundary condition is much smaller than the value obtained in previous works, where this modification was disregarded (i.e., the absorbing boundary approximation was applied). The amount of dark matter absorbed by the black hole may at present compose a significant fraction of the black hole mass. The dark matter density profile induced by diffusion within the central region 10 pc in size can be responsible for the observed γ radiation from the Galactic center, which arises upon the annihilation of dark matter particles.

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