

Magnetic flux captured by an accretion disk

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ABSTRACT

We suggest a possible mechanism of efficient transport of the large-scale external magnetic flux inward through a turbulent accretion disk. The outward drift by turbulent diffusion which limits the effectiveness of this process is reduced if the external flux passes through the disk in concentrated patches. Angular momentum loss by a magnetocentrifugal wind from these patches of strong field adds to the inward drift. We propose that magnetic flux accumulating in this way at the center of the disk provides the ‘second parameter’ determining X-ray spectral states of black hole binaries, and the presence of relativistic outflows.

Subject headings: black hole physics — MHD — accretion, accretion disks — magnetic fields — galaxies: active

1. Introduction

Magnetic fields in thin accretion disks can be of two conceptually different kinds. On the one hand, there are the chaotic, small-scale (compared with the disk radius) magnetic fields generated by the dynamo action and field-line stretching in the turbulent shear flow (Shakura & Sunyaev 1973; Stone et al. 1995; Brandenburg et al. 1996; Machida et al. 2000; Sano et al. 2004). These are the fields now believed to produce the effective viscous stress responsible for the accretion flow in most types of accretion disks (Balbus & Hawley 1991). On the other hand, one might think of fields captured from the environment. If such ‘trapped’ large scale fields can be dragged in with the accretion flow, they would be amplified to large strengths on their way to the center of the disk, because of the decreasing disk area (e.g., Bisnovatyi-Kogan & Ruzmaikin 1976).

Though both kinds of fields can be useful for interpreting a range of phenomena in accretion disks, there is an important conceptual difference between them. Locally generated turbulent fields can appear and disappear in situ. Like hydrodynamic shear turbulence, their average strength, length scales and time scales would reflect the local conditions in the disk. The statistical properties of these fields will therefore be predictable functions of the disk conditions, in particular the local accretion rate.

This is not the case for trapped fields. The net vertical magnetic flux through the disk surface is conserved, not changeable by local processes in the disk (as a consequence of $\text{div } \mathbf{B} = 0$). It can change only by advection into or diffusion out of the disk through its outer edge. If magnetic flux can be trapped at all, it therefore constitutes an *additional parameter*, or degree of freedom characterizing the disk, and one would expect this property to show up in observations.

2. Trapping ordered fields in disks

2.1. Indications of strong ordered fields

The X-ray emission from neutron star and black hole binaries can be classified into ‘soft’ and ‘hard’ states, understood as originating from a standard optically thick, cool accretion disk and an optically thin Comptonizing ‘cloud’, respectively. The hard states are typically associated with lower luminosities. The correlation between accretion rate (as inferred from total X-ray luminosity) and the spectral state of the source is not very direct, however. The so-called very high state in black hole transients, for example, is a hard state at high luminosity (prototypically seen in Nova Muscae, Ebisawa et al. 1994). In the persistent black hole binary GRS 1915 (Belloni et al. 2000) spectral state changes take place as loops of varying size in hardness-intensity or color-color diagrams. In the persistent neutron star accreters, there is generally a close relation between intensity and hardness of the X-rays, taken as indicative of an underlying dependence of these parameters on the accretion rate. This relation, however is not stable: day-to-day shifts are seen in these relations (e.g., Homan et al. 2002; Di Salvo et al. 2003; Schnerr et al. 2003).

These variations in the relation between spectrum and the (inferred) accretion rate are hard to understand in a conventional accretion disk scenario (with or without magnetic coronas), since this scenario is too deterministic. Apart from statistical fluctuations due to turbulence (e.g. Gilfanov et al. 1999), the local state of the accretion flow, in the inner region where the X-ray luminosity is produced, is governed by just one parameter: the instantaneous accretion rate in this region.

These observations are much easier to understand if there is a “second parameter” influencing the accretion flow. There are few plausible candidates for such a parameter. Trapped magnetic flux is one, and, as we shall argue here, it has ideal properties. For

example, it can plausibly vary on time scales of days or longer, corresponding to the time for accretion from the outer edge of the disk. At the same time shorter time scales are also possible, corresponding to internal instabilities of the trapped field further inside the disk.

A second, again indirect, indication for magnetic field trapping is the presence of relativistic jets, seen from both black hole and neutron star accreters (e.g. Cir X-1). They are present in many objects, but usually not all the time (a noticeable exception being the steady jet in SS433). The mechanism of choice for producing these is the magneto-centrifugal mechanism (e.g. Bisnovatyi-Kogan & Ruzmaikin 1976; Blandford & Payne 1981; for a tutorial see Spruit 1996) for which a strong open magnetic field has to be present above the disk. The field due to magnetic disk turbulence would produce a magnetized corona, but the small length scale of the turbulence (of the order of the disk thickness) will strongly limit the height to which the field extends above the disk, thereby also limiting the possible acceleration of an outflow. A large scale, ordered field such as that provided by trapped flux would certainly work better. Indeed, almost all calculations of magnetic outflows have in fact assumed such configurations.

A third consideration is the collimation of jets. Even magnetic jet acceleration requires an additional agent to focus the outflow, especially if the opening angle is to be as small as a few degrees. The toroidal field associated with a rotating magnetic outflow has some collimating effect, but only for modest degrees of collimation. For the high degree of collimation often seen, internal (kink) instabilities are particularly effective at destroying the toroidal field component that does the collimating, and an additional mechanism is needed (Eichler 1993, Spruit et al. 1997). An external dense medium, such as the pre-SN envelope in the collapsar model for GRB can have the right properties. In many cases, however, there is no visible evidence of such a collimating environment, especially in the case of the microquasars. The only plausible alternative left then is collimation by an

ordered poloidal magnetic field anchored in the disk (‘poloidal collimation’). As shown in Spruit et al. (1997), close to perfect collimation is possible for a range of parameters of such a poloidal field, as long as the size of the disk is large enough compared with the source region of the jet. This condition is satisfied for X-ray binaries and AGN disks, but not for the disks in cataclysmic variables, where the ratio of outer to inner disk radius is of the order of a factor 100 or less. In fact, no jets are known from CVs, though there is ample evidence for uncollimated outflows from these systems.

2.2. The difficulty of trapping flux in a disk

The virtues of trapped magnetic flux as described above are somewhat offset by a theoretical obstacle. The small scale processes that give rise to the exchange of angular momentum allowing mass to accrete through a disk (most likely magnetic turbulence, Shakura & Sunyaev 1973; Hawley et al. 1996) may also act like an effective magnetic diffusivity. As magnetic field lines are dragged in with the accretion flow, they diffuse out by the same process that causes the accretion. As noted first by van Ballegoijen (1989; see also Heyvaerts et al. 1996; Agapitou & Papaloizou 1996; Livio et al. 1999) this limits the effectiveness of field line ‘dragging’ to much lower values than one might at first sight expect. This is because the dragging causes a kink in the field line direction at the disk surface. As a result, the length scale relevant for the diffusion of the field is the disk thickness H rather than the radial scale r of the field. The outward diffusion speed is set by reconnection of the radial field component across the disk, not the radial diffusion of the vertical component. If the turbulent diffusivity for the magnetic field is similar to the turbulent viscosity (i.e., the effective magnetic Prandtl number of order unity), this limits the angle of the field lines with respect to the vertical to a value of order H/r , and dragging is inefficient.

A way around this obstacle is to argue that the magnetic Prandtl number could be significantly smaller than unity. The small scale motions associated with the exchange of angular momentum act in the $r - \phi$ plane [in cylindrical coordinates (r, ϕ, z)], whereas the diffusion of the radial field component across the disk is effected by exchanges in the (r, z) plane. A suitable form of anisotropy of the turbulence might lower the effective magnetic Prandtl number. Numerical simulations of magnetic disk turbulence in which the relevant components of the diffusivity were monitored with passively advected vector fields, however, have not shown any significant anisotropy (Brandenburg and Spruit 1998, unpublished). The diffusive properties of magnetic turbulence in disks appear to have received little attention in numerical simulations so far; we believe a closer look at this problem would be valuable.

We note as an aside, that, while the kink in the field at the disk surface causes a radial outward force (by the curvature of the field lines), this force is not what prevents the field from being dragged in with the accretion flow. The limiting effect is entirely kinematic, due to diffusion of the magnetic field in the assumed turbulence. A displacement by the radial force causes the angular momentum to deviate from its surroundings; as a result radial forces are almost instantaneously balanced by a small deviation from Keplerian rotation. A small radial drift comes in only as a result of drag associated with the difference in orbital velocity with respect to the surroundings (see section 3.6 below).

These considerations assume that the magnetic field is weak enough that the forces it exerts can be neglected compared to gravity. This looks like a perfectly reasonable assumption, since the main idea of creating a strong magnetic field in the inner regions is to start with a weak field in the outer regions of the disk. However, this is correct only if the field that is being dragged in is in a diffuse form. If it is intermittent and concentrated into small patches of high field strength, the forces it exerts can not be neglected, and the

situation is different.

While the accretion flow alone is thus not a very good way of trapping magnetic flux we find that other, quite effective mechanisms exist.

3. A better flux trap

In this section we describe a possible mechanism for transport of large-scale vertical magnetic flux inward across a turbulent accretion disk. A key element in our proposal is the realization that, in order to understand how magnetic flux moves around in such a disk, one has to look beyond the concept of ‘turbulent magnetic diffusivity’. Of particular importance is the phenomenon of turbulent diamagnetism, i.e., the property of turbulence to expel magnetic field from regions of strong fluid motions and to concentrate it in small patches at the boundaries of turbulent cells. As a result, the magnetic field in a turbulent medium is generally expected to be strongly intermittent (e.g., Vishniac 1995; Schekochihin et al. 2004). As we shall see later in this section, the spatial intermittency of the magnetic field should play a very important role in the overall transport of magnetic flux.¹ In fact, in our model we will take this notion to the limit by assuming that the plasma in the disk is segregated into two coexisting phases: most of the disk area is covered by a usual turbulent medium, whereas there is also a second phase comprised of small patches of very strong magnetic field that suppresses turbulence. We start with a brief review of these effects.

¹This effect has also been found to be important in the interstellar medium turbulence (e.g., Lazarian & Vishniac 1996).

3.1. Magnetic fields in turbulence

The interaction of an externally imposed magnetic field with a turbulent flow has been a subject of investigation since the 1960's, especially in connection with observations of the Sun, where it is responsible for the very patchy pattern that is so characteristic of magnetic fields at the its surface. These studies, as well as numerical magnetohydrodynamic (MHD) simulations, have led to a rather detailed theoretical picture for the advection of magnetic fields by a turbulent medium.

3.1.1. Flux expulsion

An image of the solar surface shows the magnetic field to be very fragmented: on all length scales the magnetic field is concentrated into flux bundles of strong field, separated by convective cells with little magnetic flux. In the magnetic regions, the field is strong enough to suppress convection. In a sunspot this is evident through the low amplitudes of its internal velocity field and its reduced temperature. On the other hand, the convective flow is observed to quickly concentrate any newly emerging magnetic flux into the vertices between convective cells (esp. with the high resolution observations now available, e.g., Lites et al. 2004; Berger et al. 2004). This ‘flux expulsion’ effect of convective turbulence has been understood theoretically since the early days of astrophysical MHD (Zeldovich 1956; Parker 1963).

Numerical simulations (Weiss 1966; Proctor & Weiss 1982) show that the expulsion process takes place in two stages. A pattern of overturning cells concentrates the flux of an initially weak magnetic field into its vertices, on a time scale of the order of one overturning time of the cells. This is initially a kinematic effect: the flux is just advected passively by the flow. The increasing strength of the field thus wound up then reacts back on the

flow, suppressing the flow speeds in the regions of concentrated flux. In the interior of the cells, the mixture of fields of opposite directions caused by the overturning motion decays by Ohmic diffusion on a longer time scale. At the end of the process, a nearly complete separation has taken place between the turbulent, field-free cells, and magnetic flux patches with suppressed flow.

An important aspect of the flux expulsion process is that it has to happen only once: when a flux bundle is formed by the process described, it tends to remain in concentrated form by its continued interaction with the surrounding fluid flow. It can fragment, and flux can be redistributed between neighboring bundles (processes observed abundantly on the Sun), but its field strength remains high compared with the that in the surrounding flow.

3.1.2. Analogy with superconductors

The separation between flow and magnetic flux has an interesting analogy with superconductivity. While magnetic flux is excluded from a superconductor, the material (a type I superconductor, say) is normally conducting again above a critical field strength. If boundary conditions are arranged such that a given fixed amount of magnetic flux passes through the material, cooling below the critical temperature causes a separation into superconducting regions without magnetic flux and normally conducting regions into which the magnetic flux is concentrated. The precise nature of the non-superconducting regions depends on the type of superconductor (1 vs 2). In the case of magnetic flux in a turbulent flow, the turbulence plays the role of superconductivity, expelling the flux. The suppression of turbulence by a strong magnetic field is analogous to the lifting of superconductivity by a magnetic field.

3.2. Behavior of concentrated fields

We now assume that a large portion of the net vertical flux threading a disk gets concentrated into patches of strong field (see Fig. 1). This has two important effects: first, it reduces reconnection across the disk, and second, the magnetic patches can lose angular momentum effectively. The first effect reduces the outward diffusion, the second causes the field to drift inward.

As is the case with magnetic fields in convective turbulence, we assume now that the field in an accretion disk locally becomes strong enough to suppress the turbulence generated by the magneto-rotational instability (MRI). Some evidence for this behavior appears to have been seen already in numerical simulations of magnetic disk turbulence (Machida et al. 2000).

The concentrated fields will have especially noticeable effects near the surface of the disk. Because of the magnetic pressure, the gas density inside the patch is reduced. The resulting buoyancy causes the field to remain nearly vertical at the surface (again as seen on the Sun).

If only the dynamic interaction with the surrounding flow is important, the strength of the field becomes of the order of equipartition with the turbulent kinetic energy of the flow: just high enough for the field to avoid most of the tangling and overturning effect of the flow.

The field strength can increase further by two effects. Near the surface, radiative cooling can reduce the internal pressure. To the extent that heat transport in the disk is carried by turbulence, its reduction inside the magnetic patch implies a lower heat flux, causing the surface to be cooler inside it. By vertical hydrostatic balance, this reduces the internal pressure and hence increases the field strength. This effect will probably be limited

mostly to regions where the disk is convective, as in the case of the Sun, where the reduction of convective heat transport causes sunspots to be cool. The effect could be strong in the outer regions of the disk most likely to be convective, hence of particular importance for the trapping of flux from an external field.

Secondly, mass loss through a wind will occur on just those field lines that connect from the disk surface to the external field. The resulting ‘emptying’ of the field lines also lowers the internal pressure in the field, and hence increases the field strength.

3.3. Reduced diffusion

As in the case of the Sun and the numerical simulations mentioned, the flux stays in concentrated form once patches are formed by interaction with the turbulence. The outward diffusion of trapped fields by turbulent reconnection across the disk that is seen in kinematic (advection + diffusion) calculations is then reduced once the magnetic field locally becomes strong enough to avoid being tangled. The remaining diffusion takes place not by physical displacement by the fluid motions in the magnetic turbulence, but through reconnection in the atmosphere of the disk (see § 3.6.1).

3.4. Enhanced angular momentum loss in patches

In addition to the reduced diffusion of concentrated fields, it is quite likely that the patches will lose angular momentum rather effectively, leading to an additional inward drift.

The field of a patch, vertical near the surface, fans out above the disk into a more horizontal field (the ‘canopy’ in solar physics terminology). At the (radially) outward-facing side of a patch, this horizontal field has the right direction and strength to effectively

accelerate the plasma by the magneto-centrifugal mechanism (e.g., Bisnovaty-Kogan & Ruzmaikin 1976; Blandford & Payne 1982). Whereas the mean field would be insufficiently strong to cause much of a centrifugal flow, the concentrated patches will almost certainly be effective.

Angular momentum loss by a magnetic wind involves two steps. First, conditions at the disk surface have to be right to launch a mass flux on a field line. This requires a minimum field strength and a minimum outward inclination away from the vertical (Blandford & Payne 1982; Ogilvie & Livio 2001). Secondly, in order to carry a wind, a field line also has to be open, i.e., it has to extend to a sufficient distance from the disk instead of connecting back to some other location on the disk surface.

The first condition (launching) limits the angular momentum loss of a patch to its radially outward side. On the inward side the field lines have the wrong inclination. The second condition means that the angular momentum loss will take place just on those field lines that connect the external field to the disk. The foot points of the external field will change continually by reconnection against the turbulent disk field, but at any point in time conditions for launching will be satisfied on a significant fraction of them. The regions contributing to angular momentum loss are therefore not identical to the strong-field patches themselves. In order not to complicate the nomenclature in the following, however, we will use the term ‘patch’ interchangeably for an area of concentrated field and for the parts of them where angular momentum loss takes place.

Thus, we see a large number of small cool islands of strong magnetic field drifting through the stormy sea of the turbulent disk (Fig. 1). Even though the total covering fraction of these magnetized patches is small, they can carry a large fraction of the vertical magnetic flux. Because the magnetic torque is not linear but quadratic in the field strength, the angular momentum is removed from these patches much faster than from the rest of the

disk. As a result, the patches accrete faster than the bulk of the disk matter. But what’s important here is the fact that it is precisely these same patches that carry a large portion of the overall net vertical flux; therefore, the rapid inward motion of the patches leads to a very efficient transport of the flux. Note that it is not necessary for the patches to maintain their identity for more than an orbital time scale in order for the processes described to be effective.

Suppose that the disk is threaded by some large-scale vertical field with average flux density (i.e., the average field strength) B_0 . We assume that this average field is relatively weak so that it cannot suppress the MRI in the disk. This means that

$$\beta_0 \equiv \frac{8\pi P}{B_0^2} \gg 1, \quad (1)$$

where P is the disk gas pressure (in this paper we assume that gas pressure dominates over the radiation pressure inside the disk). Because of this, strong turbulence develops almost everywhere in the disk. This turbulence, in turn, leads to the concentration of the large scale magnetic flux into a large number of small sparsely distributed patches. The field in these patches increases until it is strong enough to locally suppress the MRI. Thus we can estimate the magnetic field strength $B_p \ll B_0$ inside the patches as

$$B_p \simeq \sqrt{\frac{8\pi P}{\beta_{\text{MRI}}}}, \quad (2)$$

where $\beta_{\text{MRI}} = O(1)$ is the minimal plasma-beta required for MRI to be unstable. In our simple model we assume that the vertical magnetic field B_p is uniform inside each patch and is the same in all patches. Under the assumption that most of the overall vertical magnetic flux threads the disk by going through the patches, we can express the surface covering fraction f of the patches as

$$f = \frac{B_0}{B_p} \simeq \sqrt{\frac{\beta_{\text{MRI}}}{\beta_0}} \ll 1. \quad (3)$$

Because of the strong magnetic field in the patches, the gas pressure inside them is lower than the ambient gas pressure. Indeed, from the horizontal pressure balance between a patch and the surrounding disk it follows that the patch gas pressure P_p is

$$P_p = P + P_{\text{turb}} - \frac{B_p^2}{8\pi} = P(1 + \alpha - \beta_{\text{MRI}}^{-1}), \quad (4)$$

where we have parametrized the turbulent pressure (both dynamic and magnetic) in the disk as $P_{\text{turb}} = \alpha P$. Thus we see that P_p may differ significantly from the outside pressure.

3.5. Inward drift

We can now address the question of the radial transport of vertical magnetic flux by the inward radial drift of magnetized patches. A strong magnetic field is very efficient in removing angular momentum from the patches by magnetic braking and/or through a thermally or magneto-centrifugally driven wind. The magnetic torque exerted on a patch of a surface area a^2 is

$$\tau_{\text{patch}} = \left| \frac{1}{4\pi} 2ra^2 B_p B_{\phi,p} \right| \frac{\gamma}{2\pi} ra^2 B_p^2, \quad (5)$$

where we have parametrized the strength of the toroidal magnetic field $B_{\phi,p}$ just above (and below) the surface of the patch as

$$|B_{\phi,p}| = \gamma B_p. \quad (6)$$

The value of γ depends on the mass flux in the wind. We discuss this dependence quantitatively in Appendix 1 using some well-known properties of cold magnetocentrifugal wind solutions. From equation A3 it is seen that γ can be larger or smaller than unity. Values larger than unity correspond to cases of large mass flux, for which the Alfvén surface is close to the base of the outflow, and the azimuthal field component becomes substantial already close to the base (Spruit 1996, Anderson et al. 2004). The angular momentum

flux from the patch will be dominated by those field lines (the outward facing ones) where mass-launching conditions are favorable and γ is large.

The magnetic torque (5) leads to an inward drift of the patch with a radial velocity $-v_{\text{dr}}$ which can be estimated by equating τ_{patch} and the angular momentum lost by the patch per unit time:

$$\tau_{\text{patch}} = - \frac{dJ_{\text{patch}}}{dt} v_{\text{dr}} \frac{d}{dr} J_{\text{patch}}(r). \quad (7)$$

The angular momentum of the patch is

$$J_{\text{patch}}(r) = M_{\text{patch}} v_K r M_{\text{patch}} \sqrt{GM r}, \quad (8)$$

where $M_{\text{patch}} = a^2 \Sigma = \text{const}$ is the patch mass. We thus get

$$\tau_{\text{patch}} = \frac{a^2 \Sigma}{2} v_K v_{\text{dr}}, \quad (9)$$

and by using (5), we find

$$v_{\text{dr}} = \frac{\gamma}{\pi} \frac{r B_p^2}{\Sigma v_K} \frac{8\gamma}{\beta_{\text{MRI}}} \frac{r P}{\Sigma v_K}. \quad (10)$$

Upon writing the disk surface density as $\Sigma = 2\rho H$ introducing the isothermal sound speed $c_s \equiv \sqrt{P/\rho}$, we can rewrite v_{dr} as

$$v_{\text{dr}} = \frac{4\gamma}{\beta_{\text{MRI}}} \frac{r}{H} \frac{c_s^2}{v_K}. \quad (11)$$

Then, using the estimate $H/r \simeq c_s/v_K$, which follows from vertical hydrostatic balance in the disk, we find

$$v_{\text{dr}} \simeq \frac{4\gamma}{\beta_{\text{MRI}}} c_s \gg v_{\text{accr}} \simeq \alpha \frac{H}{r} c_s, \quad (12)$$

that is, the magnetized patch drifts inward with a speed of the order of the speed of sound.

As a side remark, note that because the patches occupy only a minute fraction of the disk surface, their contribution to the overall angular momentum transport is negligible compared with that due to the MRI-induced turbulence in the bulk of the accretion disk. In

fact, using equation (5), the total magnetic torque per unit surface area that is attributed to patches is

$$\tau = f \frac{\gamma}{2\pi} r B_p^2 = \frac{4\gamma}{\beta_{\text{MRI}}} f r P, \quad (13)$$

whereas the torque per unit area due to the action of turbulence is

$$\tau_{\text{turb}} = \alpha r P \gg \tau, \quad (14)$$

because in this paper we assume the following ordering:

$$f \ll \alpha \ll \gamma = O(1). \quad (15)$$

3.6. Outward drift

The inward drift just computed is partly offset by an outward drift due to two processes: reconnection in the disk atmosphere and a drift associated with the radial magnetic force.

3.6.1. Reconnection in the disk corona

The magnetic field in the disk corona is constantly changing due to the MRI turbulence inside the disk. Because of the high Alfvén speed in the corona, the external open field lines may rapidly reconnect to this changing field. This causes an effective diffusion of the external field through the disk. If $L \simeq H$ is the radial length scale of the MRI turbulence, and $\epsilon\Omega$ is the rate at which the coronal field changes due to the turbulence in the disk, reconnection to this changing field causes diffusion with a coefficient that can be estimated as

$$D = \frac{1}{3} L^2 \epsilon \Omega \simeq \frac{\epsilon}{3} \Omega H^2. \quad (16)$$

We can estimate ϵ from the results of MRI simulations. These produce a field in which the azimuthal component dominates, $B_\phi/B_r \sim 10 - 20$ (e.g. Brandenburg et al. 1995).

This azimuthal field is created by stretching of a radial component in the Keplerian shear $\sigma = \frac{3}{2}\Omega$, which takes a time $B_\phi/(\sigma B_r)$. Thus, the rate at which the radial field component changes must be of the order $\sigma B_r/B_\phi$, hence $\epsilon = \frac{3}{2}B_r/B_\phi \simeq 0.1$, or $D \simeq 0.03H^2\Omega$. This is of the same order as the disk viscosity $\nu = \alpha c_s^2/\Omega$,

$$D/\nu = \frac{\epsilon}{3\alpha} \simeq \frac{0.03}{\alpha}. \quad (17)$$

The effect of this diffusivity by reconnection in the disk corona is the same as magnetic diffusion across the disk, and if it were the only effect present it would limit the angle at which an external field can be dragged in to $\simeq H/r \ll 1$, as before. Equivalently it would cause an outward drift at a speed of the order $v = \epsilon c_s/3$.

3.6.2. Viscous drift

If $B_r = kB_p$ is the radial component of the field of a patch (or rather, the part of it on which the magneto-centrifugal mass loss takes place) just above the disk surface, the outward force on a patch of area a^2 is

$$F_r = 2a^2kB_p^2/(4\pi). \quad (18)$$

To lowest order, this force is balanced by a small deviation from Keplerian rotation, so that the patch moves relative to its surroundings with velocity

$$\delta v_\phi = kv_A^2/2c_s. \quad (19)$$

This difference implies a frictional force on the patch in the azimuthal direction, which causes a gradual change in angular momentum, resulting in an outward drift. The friction can be computed either as a turbulent drag or a viscous drag based on the assumed turbulence in the disk. We assume here a viscous drag, turbulent drag would give qualitatively similar results. The drag force on a magnetic patch of area a^2 can then be

written as $C\rho\nu a\delta v_\phi$, where $\nu = \alpha c_s^2/\Omega$ is the disk viscosity and C is a constant numerical factor.

3.6.3. Net drift speed

The rate of angular momentum change by the combined effects of magnetic angular momentum loss and drag as estimated above is then

$$\dot{J}_{\text{patch}} = \frac{Ck\alpha}{2}raH \frac{B_p^2}{4\pi} - \gamma ra^2 \frac{B_p^2}{2\pi}. \quad (20)$$

This corresponds to a radial drift speed

$$v_d = \frac{\dot{J}_{\text{patch}}}{dJ_{\text{patch}}/dr} \frac{2v_A^2}{c_s} \left(\frac{C\alpha kH}{4a} - \gamma \right), \quad (21)$$

where the patch angular momentum J_{patch} is given by equation (8). To compute the net inward drift, the effect of reconnection in the atmosphere must be added. If the tangent of the angle between the external poloidal field and the vertical is $B_r/B_z = k$, then the effective reconnection of B_r across the disk causes the vertical field component to drift at a rate kD/H , with the diffusion coefficient D given by equation (17). Adding this to the above gives the net drift speed

$$v_d = \frac{2v_A^2}{c_s} (C\alpha kH/4a - \gamma) + \frac{k\epsilon}{3} c_s. \quad (22)$$

For a stationary state $v_d = 0$,

$$k = \frac{B_r}{B_z} = \frac{6v_A^2}{\epsilon c_s^2} (\gamma - C\alpha kH/4a). \quad (23)$$

For inward drift, $B_r/B_z > 0$, we must have

$$\gamma > \alpha kH/a, \quad (24)$$

that is, the angular momentum loss in the patches (measured by γ) must be large enough to overcome their outward viscous drift. If this is satisfied, the magnetic field is dragged in to the angle given by (23), i.e., $B_r/B_z \simeq (6\gamma/\epsilon) v_A^2/c_s^2$.

The significance of this result is that the angle B_r/B_z can be of *order unity*, compared with the turbulent diffusion model of external fields in a disk where it is only of order H/r . The requirement (24) still has to be satisfied, of course, and whether this is realistic will need further investigation.

To illustrate what this implies for the degree of inward concentration of the field that can be achieved, note that a potential field matching to a disk with a vertical magnetic field varying as $B_z \sim r^{-\mu}$ has a constant angle at the disk surface (Sakurai 1987):

$$B_r/B_z = \frac{\Gamma(\frac{\mu+1}{2})\Gamma(1 - \frac{\mu-1}{2})}{\Gamma(\frac{\mu}{2})\Gamma(1 - \frac{\mu}{2})}. \quad (25)$$

This is an increasing function of μ , and an angle of 45° for example, corresponds to $\mu = 1$. In an X-ray binary disk with outer radius $R_o = 10^{11}$ cm and inner edge $r_i = 10^7$ cm, for example, this would yield field strength ratios $B_i/B_o \simeq 10^4$ for $\mu = 1$.

3.7. Global Evolution of the Large-Scale Magnetic Field

Here we determine how the radial drift of individual patches of strong magnetic field contributes to the evolution of the disk's poloidal flux distribution on the global scale. In order to do this, we now consider all the local quantities discussed in the preceding section (e.g., B_p , f , B_0 , v_{dr}) to be functions of two variables — time t and cylindrical radius r . Then, the continuity equation for $\Psi(r, t) = \int_0^r B_0(r, t) 2\pi r dr$ — the vertical magnetic flux threading the disk inside the radius r — can be written as

$$\frac{\partial \Psi}{\partial t} = 2\pi r B_0(r, t) v_{\text{dr}}(r). \quad (26)$$

Correspondingly, the equation of evolution for the large-scale vertical magnetic field $B_0(r, t)$ is

$$\frac{\partial B_0}{\partial t} = \frac{1}{r} \partial_r [r B_0(r, t) v_{\text{dr}}(r)]. \quad (27)$$

As we look at the time evolution of the magnetic flux we assume that it takes place on timescales much shorter than the overall accretion timescale, so that the parameters describing the properties of the disk, such as M and \dot{M} , etc., can be taken as constant in time.

Since v_{dr} is independent of B_0 , equation (27) is linear in B_0 and so can be easily analyzed. For v_{dr} given by expression (12), we get:

$$\frac{\partial B_0}{\partial t} = \frac{4\gamma}{\beta_{\text{MRI}}} \frac{1}{r} \partial_r [r c_s(r) B_0(r, t)]. \quad (28)$$

where we have assumed γ and β_{MRI} to be constant.

For example, for a standard Shakura–Sunyaev disk model in the gas-pressure dominated regime with Thomson opacity (e.g., Frank et al. 2002) $H(r) \sim r^{21/20}$, i.e., the disk vertical scale height H scales almost linearly with radius at large distances ($r \gg r_g$). Thus we shall regard the ratio

$$\frac{H}{r} \simeq \frac{c_s}{v_k} \ll 1 \quad (29)$$

as a small constant parameter in our model. Then we can write

$$\frac{\partial B_0}{\partial t} = \frac{4\gamma}{\beta_{\text{MRI}}} \frac{H}{r} \frac{\sqrt{GM}}{r} \partial_r [\sqrt{r} B_0(r, t)]. \quad (30)$$

So far we assumed that the large-scale transport of the vertical magnetic flux is determined solely by an uninhibited inward drift of magnetic patches in accordance with formula (27). This corresponds to the assumption that there is an effective sink for the flux somewhere at small radii. Although this is not realistic, this situation is a fair approximation for what happens at large distances and at early times.

In particular, if we consider some intermediate range of radii in the middle of the disk (i.e., sufficiently far away from both the inner and outer disk edges), it follows from equation (30) that the steady state distribution of the net vertical field is:

$$B_0(r) \sim 1/\sqrt{r}. \quad (31)$$

At the same time, the magnetic field B_p inside the patches just scales as the square root of the gas pressure. In the case of a radially self-similar disk, we have [e.g., eqn (7.31) in Krolik 1999]:

$$\int p(z) dz = HP \sim r^{-3/2}, \quad (32)$$

and since we here take $H \sim r$, we get $P(r) \sim r^{-5/2}$, and hence $B_p \sim r^{-5/4}$. Therefore, the covering fraction of our patches scales with radius as

$$f \sim B_0(r)/B_p(r) \sim r^{3/4}, \quad (33)$$

i.e. f increases outward.

4. The central flux bundle

The inward drift of the patches is halted by the central object, so that a central bundle (called a Magnetically Arrested Disk by Narayan et al. 2003) of accumulating flux develops around it (see Fig. 2). In a steady state, mass continues to accrete through this bundle, but, due to the high field strength, the processes that mediate the accretion flow are now different from those in a normal accretion disk. For example, Narayan et al. (2003) proposed that accretion takes place in the form of blobs and streams (see their Fig. 1).

4.1. Accretion through the central bundle

The strong field suppresses MRI turbulence so that the effective viscosity is reduced. The resulting pile-up of mass outside of the bundle eventually becomes unstable to interchange instabilities at the bundle’s outer edge. In this way mass enters the bundle while magnetic flux from the bundle mixes outward into the disk. These instabilities have been studied by analytical means (in the WKB approximation) by Spruit et al. (1995) and by Lubow & Spruit (1995). The onset of small-scale modes typical of interchanges (as in Rayleigh-Taylor) takes place only at rather large field strengths, due to a stabilizing effect of the Keplerian shear.

Numerical simulations of a strong ordered vertical field in a disk (Stehle 1996; Stehle & Spruit 2001) show the behavior typical of interchange instability, but only at low shear rates (less than Keplerian). In Keplerian shear, on the other hand, a large-scale, a global (azimuthal order $m = 1$) mode sets in first, and dominates the nonlinear development of instability (see also Caunt & Tagger 2001). The simulations show that the net effect of this global instability, however, is similar to that of an interchange: the strong field spreads outward while mass accretes inward. This scenario agrees well with the recent 3D MHD simulations by Igumenshchev et al. (2003) who have observed radiatively-inefficient accretion in the form of blobs and streams produced by interchanges.

The field strength at which this instability becomes effective is most usefully expressed in terms of the degree of support against gravity provided by the magnetic stress $B_r B_z$. The simulations indicate that the instability becomes effective when the radial acceleration due to the magnetic force is of the order of a few per cent of the gravitational acceleration. In a cool disk, this corresponds to a field strength that is large compared with the gas pressure in an otherwise identical non-magnetic disk. Thus, as long as the field line inclination B_r/B_z is of order unity, there is a range in field strengths, between the value at which MRI

turbulence is suppressed and the value where dynamical instability of the bundle itself sets in, where no known instability operates (Stehle & Spruit 2001). In this range, no accretion can take place. Instead, mass would build up outside a region with such field strengths until the central bundle is compressed enough for instability to set in. At the edge of the bundle, where the field is more horizontal, B_r/B_z is larger and instability is expected already at lower values of the vertical component B_z (see illustration in Fig. 3).

In a steady state the flux bundle stays in a state somewhat above marginal stability. The instability settles at such an amplitude that it allows mass to accrete at the incoming rate, across the magnetic field and onto the black hole.

4.1.1. *A feedback effect*

The central flux bundle has an ‘attractive’ effect on nearby flux patches that have not yet merged with it, due to a slightly counterintuitive effect. The field of the central bundle fans out above the disk, so that its field lines skim the disk surface. This strong radial field affects nearby patches by pushing their field lines over in the radial direction as well (see Fig. 3). This makes the conditions for magneto-centrifugal mass loss from the patches more favorable. The consequent angular momentum loss increases the inward drift speed of the patches, as if they were attracted towards the central bundle.

4.2. Transition between central bundle and accretion disk

The radial acceleration by the magnetic field of the central bundle, on a thin disk of surface density Σ embedded in it is

$$g_m = B_r B_z / (2\pi \Sigma). \quad (34)$$

If at the onset of internal instabilities in the bundle this acceleration is a fraction ϵ of the acceleration of gravity, as described above, the magnetic field B_z at that point is larger than equipartition with the gas pressure by a factor of order $\sqrt{(\epsilon/k)(r/H)}$, where k is the ratio B_r/B_z at the disk surface:

$$B_z \approx B_{\text{eq}} \sqrt{(\epsilon/k)(r/H)}. \quad (35)$$

We can use this to estimate how the transition from a field consisting of inward moving patches to the field of the central bundle takes place. Since the filling factor of the strong-field patches is low outside the central bundle, the inclination k of the magnetic field in the bundle is high near its edge (Fig. 3), and B_z is correspondingly low [see eq. (35)]. The vertical field strength in the bundle thus decreases smoothly outward from a high value at the center to a value near B_{eq} at its edge. Near this point it matches the value of the magnetic patches approaching the bundle, which have field strength of order equipartition with the gas pressure (section 3.4). From equation (35), the inclination of the field at this point is of the order $k \simeq \epsilon r/H$. This is sketched qualitatively in Figure 3.

4.3. The size of the central bundle

The flux in the central bundle increases with time, as long as the sign of the external flux captured by the disk stays constant. When the sign of the accreted flux changes, all the processes described above remain unchanged, except that the flux in the central bundle starts to decrease. Thus, the amount of flux in the bundle reflects the history of the field trapped by the disk from its environment. Its variations with time will reflect conditions near the outer edge of the disk. These are likely to be very long compared with accretion time scales near the hole.

5. Discussion and conclusions

The capture of external magnetic flux by an accretion disk and its subsequent compression in the inner regions of the disk is an attractive possibility to explain both the missing ‘second parameter’ determining the X-ray spectrum and jet activity. In this paper, we have argued that this process can actually be more relevant than it is usually assumed, provided that the external field can be concentrated into patches of field comparable in strength to the magnetorotational turbulence in the disk.

Because the external field lines are open, they can carry a magnetically driven wind. The angular momentum loss through this wind is transmitted to the disk by the footpoints of these external field lines. As a result, these are the first parts on the disk to move inward, carrying the external field lines with them. At the same time, the patchiness of these field lines at the disk surface guarantees that conditions favorable for launching a wind exist on a significant fraction of them.

We find that the inward drift speed of the external field lines can reach a significant fraction of the sound speed, i.e., a factor $r/(\alpha H)$ faster than the viscous accretion velocity.

As a result of this inward drift of strongly magnetized patches, the external field lines accumulate at the center of the disk into a flux bundle. The poloidal flux of this bundle grows as long as the polarity of the external flux trapped by the disk does not change. As discussed in the Introduction, there is now extensive observational evidence that the accretion flow in X-ray binaries does not depend solely on the instantaneous mass accretion rate in the inner disk; instead, a ‘second parameter’ must also be involved. We suggest that the size of the central magnetic flux bundle is to be identified with this second parameter.

For accretion to proceed, there must be processes allowing mass to drift across the central flux bundle. These will be different from those acting in a normal accretion disk.

Since the central flux bundle is also the region where most of the gravitational energy is released from the accreting mass, the strong variability observed in the so-called hard states of black hole and neutron star accreters may also be a consequence of the processes specific to mass transfer across a central bundle of magnetic flux.

The arguments given here for the processes involved in the capture and inward drift of an external field are admittedly qualitative, but they may be tested to some extent by numerical simulations. Because of the large Alfvén speeds encountered in the disk atmosphere, and the need to resolve small scales in the disk as well as larger scales in the captured flux, these will be somewhat challenging, however, unless a way is found to use complementary approximations for the disk and its atmosphere. A useful first step in this direction has been made by Stehle & Spruit (2001), where the existence of instabilities that allow for mass to accrete across the central flux bundle was demonstrated.

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A. Angular momentum loss as a function of the outflow mass flux

In the cold magnetocentrifugal wind model, the angular momentum loss per unit of poloidal magnetic flux is

$$j = \dot{m}\Omega\varpi_A^2, \tag{A1}$$

where the mass flux per unit of poloidal magnetic flux is $\dot{m} = \rho v_p/B_p$ (which is constant along a field line), v_p and B_p are the poloidal velocity and field strength, and ϖ_A the

distance of the Alfvén surface from the rotation axis. The solution for a cold wind in a radial poloidal field yields (e.g., Spruit 1996):

$$j = \frac{B_0}{4\pi} \varpi_0 f(\mu), \quad (\text{A2})$$

where B_0, ϖ_0 are the poloidal field strength and distance from the axis at the base of the flow, and the dimensionless function f is

$$f(\mu) = \frac{3}{2} \mu (1 + \mu^{-2/3}), \quad (\text{A3})$$

where μ is a dimensionless mass flux:

$$\mu = 4\pi \dot{m} \Omega \varpi_0 / B_0. \quad (\text{A4})$$

At the base of the flow, where $v_p = v_{p0}$, $\rho = \rho_0$, this can be written as

$$\mu = \rho_0 v_{p0} 4\pi \Omega \varpi_0 / B_0^2 = v_{p0} \Omega \varpi_0 / v_{A0}^2, \quad (\text{A5})$$

where v_{A0} is the poloidal Alfvén speed at the base.

Per unit of surface area, the angular momentum flux is $\dot{J} = B_p j$, which can be written as

$$\dot{J} = \frac{B_0^2}{4\pi} \varpi_0 f(\mu). \quad (\text{A6})$$

While this result has been derived for the specific geometry of a radial poloidal field (the ‘split monopole’), numerical solutions show that it also holds well for more general geometries (Anderson et al. 2004).

Comparing this with the angular momentum flux in equation (5) (and noting that in this equation the sum has been taken over the two surfaces of the disk), we find that

$$\gamma = f(\mu). \quad (\text{A7})$$

Thus, $\gamma \gg 1$ if $\mu \gg 1$ and $\gamma \ll 1$ if $\mu \ll 1$.

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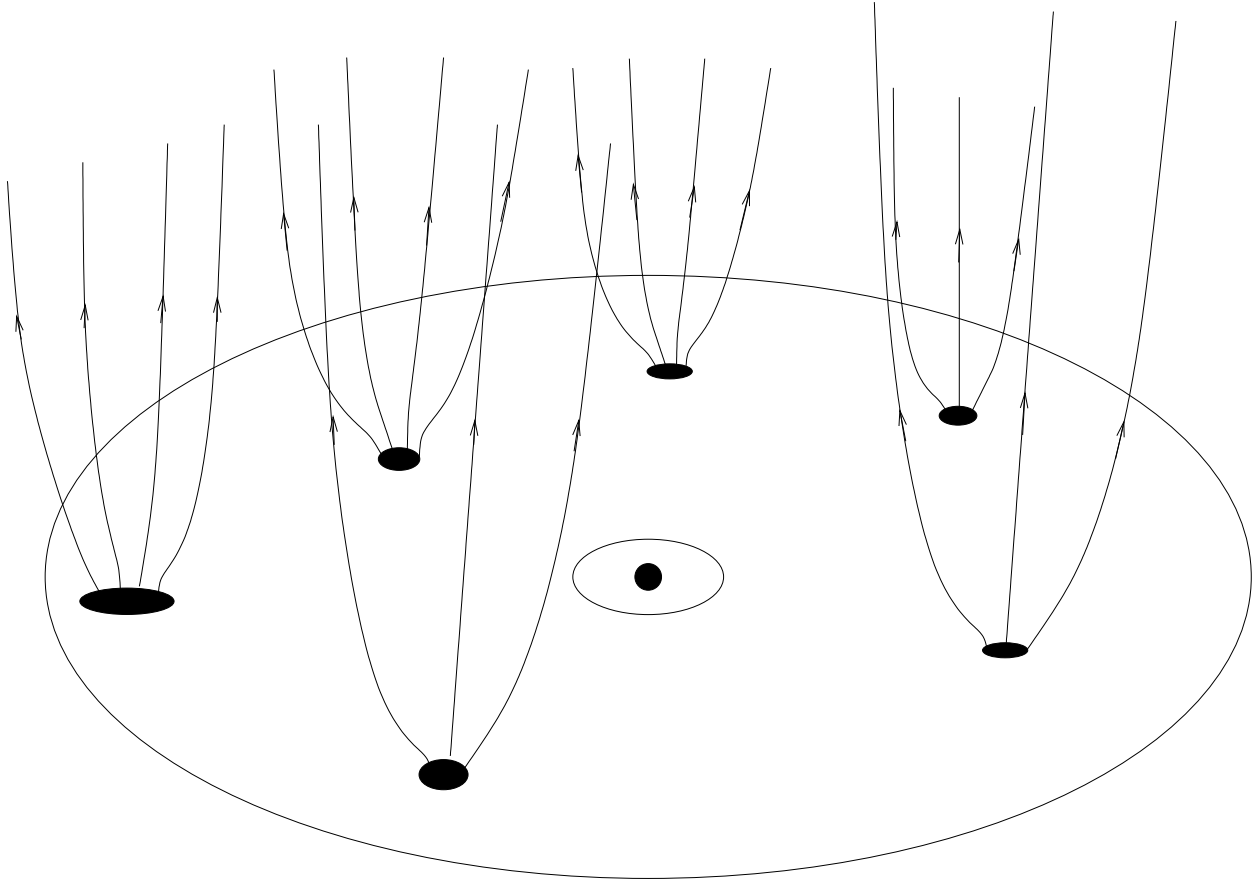


Fig. 1.— Magnetohydrodynamical turbulence in the accretion disk leads to the concentration of the weak external large-scale magnetic field threading the disk into small patches of strong field (shown by black ovals). These magnetized patches rapidly drift inward because of the enhanced angular momentum loss caused by magneto-centrifugally driven wind.

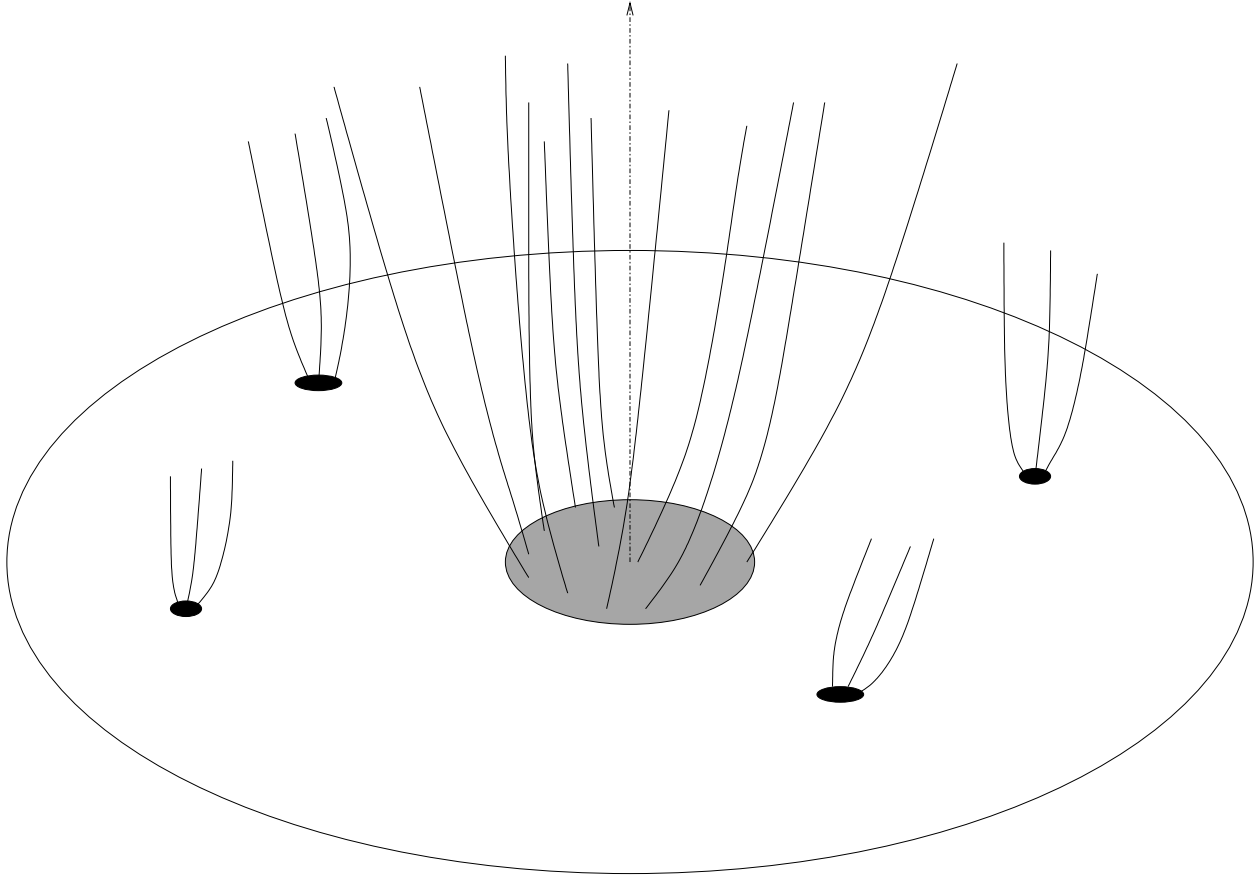


Fig. 2.— A large central magnetic flux bundle (gray) develops as a result of continuing accumulation of many small strongly-magnetized patches (black) drifting inward through the disk.

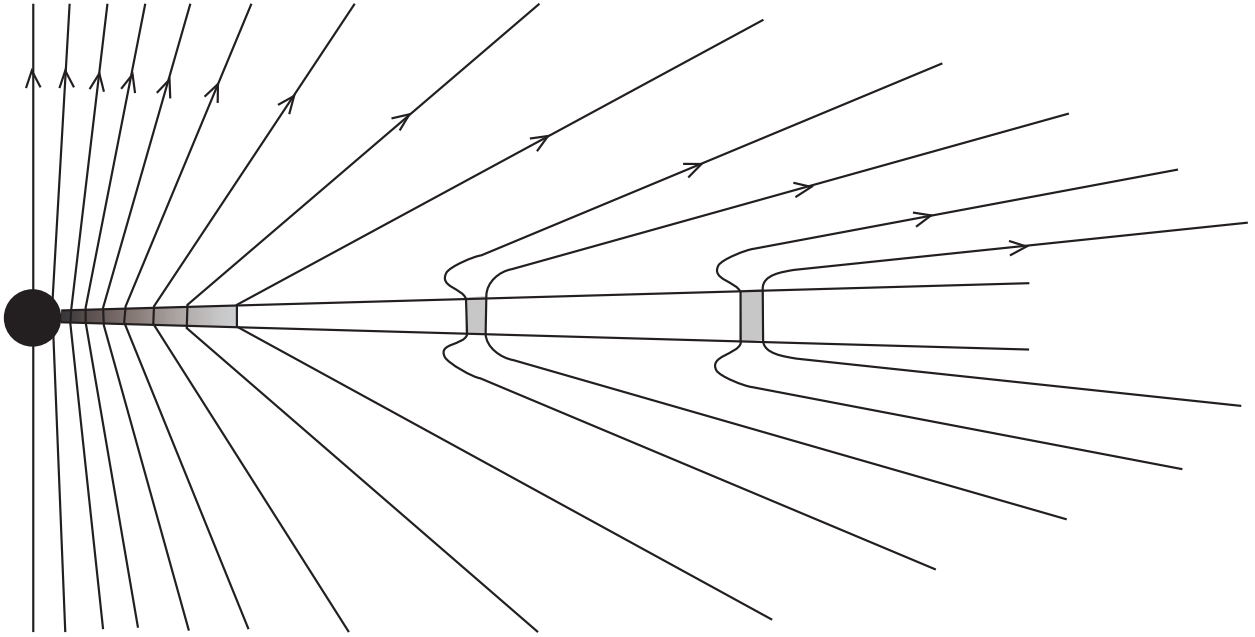


Fig. 3.— Accretion disk with magnetic patches approaching a central flux bundle (schematic). The figure shows the open field lines (connecting to infinity). Not shown is the turbulent small scale field in the disk itself. Shading indicates the strength of the open field lines at the points where they pass through the disk.