

# Simulating a faint gamma-ray burst population

D. M. Coward\*

*School of Physics, University of Western Australia, M013, Crawly WA 6009, Australia*

17 March 2018

## ABSTRACT

There have now been three supernova-associated  $\gamma$ -ray bursts (GRBs) at redshift  $z < 0.17$ , namely 980425, 030329, and 031203, but the nearby and under-luminous GRBs 980425 and 031203 are distinctly different from the ‘classical’ or standard GRBs. It has been suggested that they could be classical GRBs observed away from their jet axes, or they might belong to a population of under-energetic GRBs. Recent radio observations of the afterglow of GRB 980425 suggest that different engines may be responsible for the observed diversity of cosmic explosions. Given this assumption, a crude constraint on a luminosity function for faint GRBs with a mean luminosity similar to that of GRB 980425 and an upper limit on the rate density of 980425-type events, we simulate the redshift distribution of under-luminous GRBs assuming BATSE and *Swift* sensitivities. A local rate density of about 0.6% of the local supernova Type Ib/c rate yields simulated probabilities for under-luminous events to occur at rates comparable to the BATSE GRB low-redshift distribution. In this scenario the probability of BATSE/HETE detecting at least one GRB at  $z < 0.05$  is 0.78 over 4.5 years, a result that is comparable with observation. *Swift* has the potential to detect 1–5 under-luminous GRBs during one year of observation.

**Key words:** cosmology: observations – gamma-rays: bursts – supernovae: general

## 1 INTRODUCTION

Discovery of GRB 980425, associated with the very nearby supernova (SN) 1998bw (at  $z = 0.0085$ , corresponding to about 40 Mpc), heralded a new era in understanding the origin of GRBs (Galama et al. 1998a,b). Recently, the detection of spectroscopic features in the light curve of GRB 030329, similar to those seen in SN 1998bw (Stanek et al. 2003; Hjorth et al. 2003), has strengthened the SN 1998bw/GRB 980425 association. These observations support the ‘collapsar’ model (Zhang, Woosley & MacFadyen 2003) in which a Wolf-Rayet progenitor, possibly in a binary system, undergoes core collapse, producing a compact object surrounded by an accretion disk, which injects energy into the system and thus acts as a ‘central engine’. The energy extracted from the system gives rise to a Type Ib/c SN explosion and drives collimated jets along the progenitor rotation axis, producing a prompt GRB and afterglow emission—see the review by Zhang & Mészáros (2004).

Discovery of the GRB-SN association was an important breakthrough, but GRB 980425 had an unusually low luminosity—it was under-luminous in  $\gamma$ -rays by three orders of magnitude compared to ‘classical’ GRBs. It was suggested that it could be a very rare event and not a member of the

classical GRB population. This explanation seems unlikely given the discovery of the under-luminous and nearby GRB 031203 ( $z = 0.105$ ), the first analogue of GRB 980425.

There are presently three GRBs (980425, 030329, and 031203) definitely associated with extremely energetic Type Ib/c SNe (Prochaska 2004; Malesani et al. 2004), all occurring at  $z < 0.17$ . GRB 030329 is classified as classical in the context of energy emission, but was relatively close at  $z = 0.17$ . However, it is difficult to reconcile the under-luminous GRBs 980425 and 031203 with the classical population. One simple explanation is that under-luminous bursts are GRBs observed away from the jet axis.

Guetta et al. (2004) argue that a unified picture can only be obtained by using a luminosity function (LF) that includes all luminosities down to that of GRB 980425, so that the probability of observing the three low- $z$  events is non-negligible. They show that for GRBs 980425, 030329, and 031203 to belong to the classical burst population, the LF must be a broken power-law. This is an attractive proposal in that GRBs 980425 and 031203 can be explained within the bounds of currently popular GRB progenitor models by extending the LF to accommodate GRB 980425. They calculate that if this is the case, no bright burst within  $z = 0.17$  should be observed by a HETE-like instrument within the next  $\sim 20$  yr.

\* E-mail: coward@physics.uwa.edu.au

## 2 ANOMALOUS GRBS

### 2.1 Evidence for intrinsically sub-energetic events

Soderberg et al. (2004) argue that if GRB 031203 was observed ‘off axis’, then the radio afterglow should brighten as the ejecta slows down, but they did not observe any re-brightening. They find that the afterglow is faint, indicating that the explosion was under-energetic. Similarly, there is no evidence of re-brightening for GRB 980425, despite radio calorimetry since 1998. Soderberg et al. (2004) conclude that GRBs 980425 and 031203 were intrinsically sub-energetic events.

If SN 1998bw was a rare and unusually sub-energetic SN distinct from local SNe and GRBs, Soderberg et al. (2004) claim the characteristics of SN 1998bw/GRB 980425 are not a result of the observer’s viewing angle but of the properties of its central engine. SN 1998bw was an engine-driven explosion (Li & Chevalier 1999), in which 99.5% of the kinetic energy ( $\sim 10^{50}$  ergs) was coupled to relativistic ejecta of Lorentz factor 2 (Kulkarni et al. 1998), while a mere 0.5% went into the ultra-relativistic flow. In contrast, ‘classical’ GRBs couple most of their energy into  $\gamma$ -rays. Berger et al. (2003b) claim that the observed diversity of cosmic explosions (SNe, x-ray flashes (XRF), and GRBs) could be explained with a standard energy source but with a varying fraction of that energy injected into relativistic ejecta. Different engines may be responsible for the observed diversity of cosmic explosions, implying that classical GRBs represent one class of event, one in which  $\gamma$ -rays channel most of the energy away from a central engine.

It is evident that SN 1998bw could be a member of a distinct class of SN explosions. But how rare is SN 1998bw in the context of Type Ib/c SNe and classical GRBs? Berger et al. (2003a) carried out a systematic program of radio observations of Type Ib/c SNe using the Very Large Array to place the first constraint on the rate density of SN 1998bw type events. Of the 33 SNe observed from late 1999 to the end of 2002, they conclude that the fraction of events similar to SN 1998bw is at most 3%. Furthermore they find, by comparison of the SN radio emission to that of GRB afterglows, that none of the observed SNe could have resulted from a classical GRB.

### 2.2 Evidence for an off-axis model for GRB 031203

The evidence that GRB 031203 was an intrinsically faint and nearly spherical explosion is not widely accepted. Ramirez-Ruiz et al. (2004) disagree with this interpretation and argue, from models fitted to the observed x-ray light curve and the radio afterglow, that GRB 031203 was a classical GRB viewed off axis. They find that most spherical models under-predict the x-ray flux at late times by at least two orders of magnitude, and prefer to interpret GRB 031203 as a highly collimated GRB viewed off-axis.

For the case of an observer located outside the jet aperture,  $\theta_{\text{obs}} > \theta_0$ , the prompt GRB emission and its early afterglow are considerably weaker than for on axis,  $\theta_{\text{obs}} < \theta_0$ . An observer at  $\theta_{\text{obs}} > \theta_0$  sees a rising afterglow light curve at early times, which approaches that seen by an on-axis observer at late times. The emission remains low until the cone of the beam intersects the observer’s line of sight.

In the off-axis jet scenario, with viewing angle  $\theta_{\text{obs}} \sim 2\theta_0$ , GRB 031203 can be modelled as a GRB viewed a few degrees outside of a conical jet. Ramirez-Ruiz et al. (2004) show that if GRB 031203 is modelled as an off-axis observation, its energy emission in  $\gamma$ -rays is about  $10^{53}$  ergs, consistent with classical GRBs.

The off-axis model of Ramirez-Ruiz et al. (2004) used to explain the weak afterglow of GRB 031203 casts some doubt on the claim that it is an intrinsically weak event. Furthermore, because GRBs 031203 and 980425 share a common deficit in  $\gamma$ -ray emission, it is possible that these ‘outliers’ are a result of off-axis observations. The possibility that GRBs may represent a class of cosmic explosions with a broad range of energies provides the basis for the following simulation.

### 2.3 A population of under-luminous GRBs

Given the potential of the *Swift* satellite, a multi-wavelength GRB observatory (<http://swift.gsfc.nasa.gov/>) launched on 2004 November 20, to localize hundreds of GRBs, observations of events similar to GRB 980425 will provide new insight into GRB progenitor populations. This is strong motivation to constrain the probability of detecting similar GRB-SNe assuming a simple, if highly uncertain, model. The first upper limits on the rate density of Type Ic SNe associated with GRBs are being made, based on the presence of jetted emissions (e.g. Berger et al. 2003a). Given such upper limits, one can at least provide further constraints on GRB progenitor populations using continued satellite observations. Furthermore, we provide an example to demonstrate how an under-luminous GRB population would manifest during the BATSE observation period and the era of *Swift*.

Firstly, we assume GRB 980425 is a ‘typical’ member of a class of relatively rare GRB (compared to classical GRBs). Based on the luminosity of GRB 980425, the mean luminosity of this population is about 3 orders of magnitude less than that of the classical population. Secondly, the local rate density must be less than 3% of the Type Ib/c SN rate. Another observational constraint is based on the observed rate of GRB 980425 type events out to  $z = 0.0085$  assuming a 4.5 yr (BATSE) observation period; the probability of occurrence of this very nearby GRB must be compatible with the BATSE GRB distribution. Finally, we assume the  $\gamma$ -ray emissions are isotropic (not beamed) based on the radio observations of the afterglows of GRBs 980425 and 031203 (Soderberg et al. 2004). With these constraints, we simulate the observed GRB distribution for BATSE and *Swift* sensitivities.

## 3 THE GRB LUMINOSITY FUNCTION

The GRB LF, together with the flux sensitivity threshold of an instrument, determine the fraction of all GRBs potentially detectable with that instrument:

$$\psi_{\text{GRB}}(z) = S_d \int_{L_{\text{lim}}(z)}^{\infty} p(L) dL, \quad (1)$$

where  $\psi_{\text{GRB}}(z)$  is the GRB rate scaling function,  $S_d$  is the fraction of sky that the detector scans, and  $p(L)$  is the

GRB LF with  $L$  the intrinsic luminosity in units of photons  $\text{s}^{-1}$ . With  $f_{\text{lim}}$  denoting the instrumental flux sensitivity threshold, in photons  $\text{s}^{-1} \text{m}^{-2}$ , the minimum detectable luminosity can be expressed as a function of redshift by  $L_{\text{lim}}(z) = 4\pi D_L^2(z) f_{\text{lim}}$ , with  $D_L(z)$  the luminosity distance.

Most models for the classical GRB LF are based on the luminosity–redshift relation (Schaefer et al. 2001). But the models are biased by the redshift sample and the sensitivity limit related to the redshift estimate. This limit has to be lowered at least by an order of magnitude to encompass the complete range of luminosities. Firmani et al. (2004) show that by jointly fitting to the observed differential peak-flux and redshift distributions, the best fit for the LF takes a form that evolves weakly with redshift. However, there is no consensus on the form of LF for classical GRBs; possibilities include a single power law, a double power law and a log-normal distribution.

For an under-luminous population of GRBs modelled on the single GRB 980425, the choice of LF is so uncertain that the form of the function is somewhat arbitrary. Nonetheless, for definiteness and for comparison, we use log-normal distributions for the classical GRBs and an under-luminous population, both with observation-based statistical moments that fit the data:

$$p(L) = \frac{e^{-\sigma^2/2}}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{[\ln(L/L_0)]^2}{2\sigma^2}\right\} \frac{1}{L_0}, \quad (2)$$

where  $\sigma$  and  $L_0$  are the width and average luminosity, respectively. We take  $\sigma = 2$  and  $L_0 = 2 \times 10^{56} \text{ s}^{-1}$  for the classical GRBs, with  $f_{\text{lim}} = 0.2$  and  $0.04$  photons  $\text{s}^{-1} \text{cm}^{-2}$  for BATSE (classical GRBs) and *Swift* respectively, and take about  $0.1$  for  $S_d$  (Guetta et al. 2004). Assuming that GRB 980425 is representative of an under-luminous population, we take  $L_0 = 2 \times 10^{53} \text{ s}^{-1}$ , three orders of magnitude less than for the classical GRBs.

#### 4 GRB RATES AND DETECTION PROBABILITY

One can express the differential GRB rate in the redshift shell  $z$  to  $z + dz$  as

$$dR = \psi(z) \frac{dV}{dz} \frac{r_0 e(z)}{1+z} dz, \quad (3)$$

where  $dV$  is the cosmology-dependent co-moving volume element and  $R(z)$  is the GRB event rate, as observed in our local frame, for sources out to redshift  $z$ . Source rate density evolution is accounted for by the dimensionless evolution factor  $e(z)$ , which is normalized to unity in the present-epoch universe ( $z = 0$ ), and  $r_0$  is the  $z = 0$  rate density. The  $(1+z)$  factor accounts for the time dilation of the observed rate by cosmic expansion.

We assume a ‘flat- $\Lambda$ ’ cosmology with  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$  for the present-epoch density parameters, and take  $H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1}$  for the Hubble parameter at  $z = 0$ . The star formation rate (SFR) model SF2 of Porciani & Madau (2001) is re-scaled to this cosmology and converted to a normalized evolution factor  $e(z)$ . This SFR model levels off to an essentially constant rate density, of order 10 times the  $z = 0$  value, at  $z > 2$ ; a star formation cutoff at  $z = 10$  is assumed.

For the classical GRBs we use  $r_0 = 0.9 \text{ yr}^{-1} \text{Gpc}^{-3}$ , obtained by scaling the all-sky Universal rate to  $692 \text{ yr}^{-1}$ , the value implied by the GUSBAD<sup>1</sup> catalogue. This is comparable to the value of  $1.1 \text{ yr}^{-1} \text{Gpc}^{-3}$  from Guetta et al. (2004), obtained using a different SFR and a broken power-law luminosity function.

As GRBs are independent of each other, their distribution is a Poisson process in time: the probability for at least one event to occur in the volume out to redshift  $z$  during observation time  $T$  at a mean rate  $R(z)$  is given by an exponential distribution:

$$p(n \geq 1; R(z), T) = 1 - e^{-R(z)T}. \quad (4)$$

This formula can be used to define a ‘probability event horizon’—as observation time increases, how often will rarer, more local events, be observed? See Coward & Burman (2005) for a description. Based on equation (4), the probability of at least one GRB occurring in  $z < 0.17$  during 4.5 yr is about 0.5, implying that the observed GRBs in this volume need not be considered anomalous. But the probability of a GRB occurring in  $z < 0.01$  over 4.5 yr is 0.00015, implying that GRB 980425 ( $z = 0.0085$ ) is either an extreme outlier or a member of a different GRB population. We model the GRB redshift distribution under the latter assumption, with the constraint that the probabilities and rates are consistent with the observed BATSE GRB distribution.

#### 5 SIMULATING AN UNDER-LUMINOUS GRB REDSHIFT DISTRIBUTION

A GRB distribution comprised of two populations, with different local rate densities and mean luminosities can be expressed as:

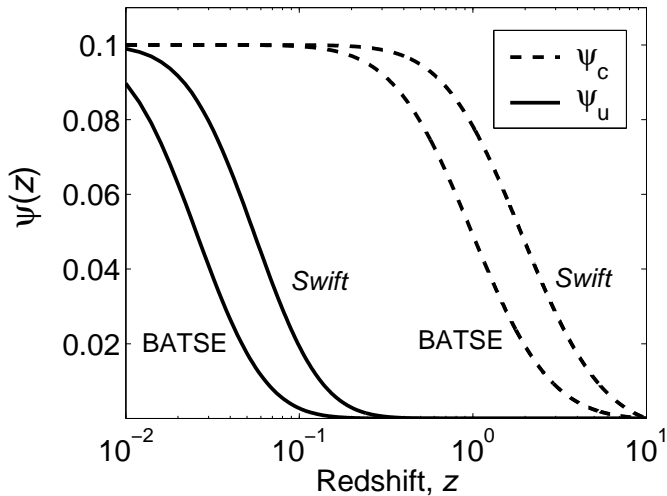
$$dR = \frac{dV}{dz} \frac{e(z)}{1+z} \left[ \psi_c(z) r_0^c + \psi_u(z) r_0^u \right] dz, \quad (5)$$

where  $\psi_c(z) r_0^c$  and  $\psi_u(z) r_0^u$  are the scaling functions and local rate densities of the classical and under-luminous GRBs respectively. It is assumed that both rates follow the SFR density so that  $e(z)$  is the same for both populations.

Figure 1 plots  $\psi_c(z)$  and  $\psi_u(z)$  for BATSE and *Swift* sensitivities. It is evident that at  $z > 0.1$ , the flux-limit of the detectors severely limits the potential detectability of the under-luminous GRB population (mean luminosity  $L_0 = 2 \times 10^{53} \text{ s}^{-1}$ ). For  $z < 0.1$  the scaling function is non-negligible even though events will be rare inside such a relatively small volume. The sensitivity of *Swift* implies that it could potentially detect most under-luminous events occurring inside a volume bounded by  $z = 0.01$ .

Clearly the observed redshift distribution of GRBs and expected rates for very energetic Type Ib/c SNe do impose constraints on the local rate density  $r_0^u$ . Importantly, GRBs 980425 and 031203 show no evidence for jets, implying that there is no geometric rate enhancement factor required to account for unseen bursts of similar type. As a first approximation to  $r_0^u$ , we take the classical rate  $r_0^c$  increased by the beaming rate enhancement factor for classical bursts, using a value of 250 from Frail et al. (2001). This gives  $r_0^u \approx 220 \text{ yr}^{-1} \text{Gpc}^{-3}$ , which is about 0.6%

<sup>1</sup> <http://www.astro.caltech.edu/~mxs/grb/GUSBAD/>



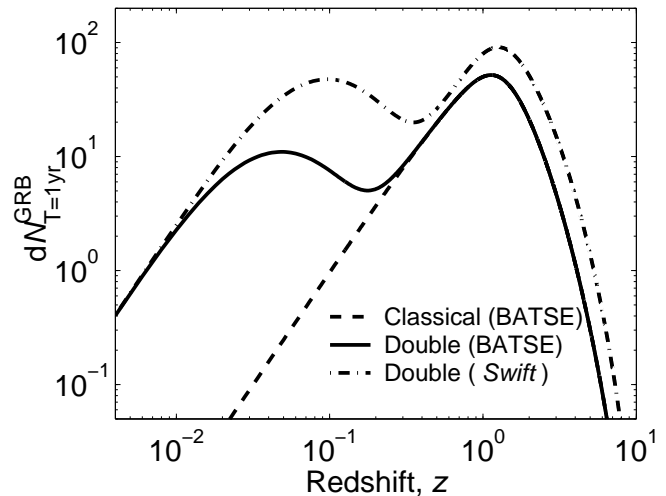
**Figure 1.** GRB scaling functions  $\psi_c$  for the classical population, and  $\psi_u$  for an under-luminous population, shown using BATSE and *Swift* sensitivities.  $\psi_u$  is calculated using the assumption that GRB 980425 is a typical event from an under-luminous population of mean luminosity  $L_0 = 2 \times 10^{53} \text{ s}^{-1}$ , three orders of magnitude less than the mean luminosity of the classical GRBs. A value of 0.1 for the scaling function corresponds to all GRBs in the field of view of the detector (0.1 str) being potentially detectable.

**Table 1.** The probability of observing at least one GRB inside volumes bounded by  $z = 0.0085$ , 0.05, and 0.17 during 4.5 yr of observation using a single GRB distribution (classical) with BATSE sensitivity and a double distribution (classical + under-luminous) with BATSE and *Swift* sensitivities, (labelled as Double (BATSE) and Double (*Swift*)) respectively.

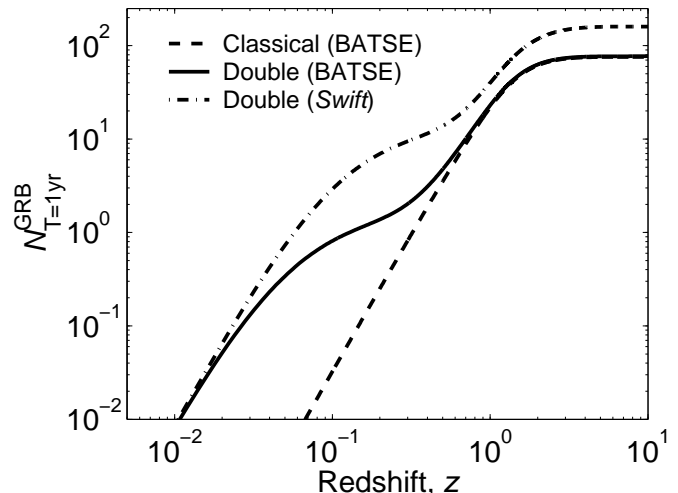
	$p(z < 0.0085)$	$p(z < 0.05)$	$p(z < 0.17)$
Classical (BATSE)	0.00015	0.02	0.5
Double (BATSE)	0.04	0.78	0.99
Double ( <i>Swift</i> )	0.04	0.96	0.99

of the local SN Type Ib/c rate,  $r_0^{\text{SN Ib/c}} \approx 3.7 \times 10^4 \text{ yr}^{-1} \text{ Gpc}^{-3}$  (Izzard, Ramirez-Ruiz & Tout 2004)—a result that supports the view that SN1998bw was a relatively rare and unusually energetic SN. We note that the quoted SN Type Ib/c rates may be underestimated because many core-collapse SN are lost to extinction in most surveys to date.

The mean of the luminosity function needs to be reduced by 3-4 orders of magnitude and the variance reduced from 2 to 1.5 to crudely fit the resulting probability distribution with the observed rates of GRBs at small redshift; that is, the probability of occurrence of the very nearby GRB 980425 ( $z = 0.0085$ ) must be compatible with present observations. For this condition to be satisfied we find that  $\sigma$  must be smaller than that used for modelling the classical population ( $\sigma = 2$ ), otherwise the rate of events at small  $z$  would be too large. If a broader luminosity distribution (larger variance) is employed, for example  $\sigma = 2.5$ , the mean luminosity must be reduced to 4 orders of magnitude less than the mean classical GRB luminosity, to yield observationally consistent probabilities. For definiteness and consistency, we assume  $\sigma = 1.5$  and  $L_0 = 2 \times 10^{53} \text{ s}^{-1}$  in all calculations



**Figure 2.** The differential number of GRBs as a function of redshift for an observation time of 1 yr using the three models described in Table 1. The BATSE classical and double distribution models predict similar numbers from  $z = 0.2 - 10$ .



**Figure 3.** As for Figure 2 but plotting the cumulative number of GRBs as a function of redshift for an observation time of 1 yr. A comparison of the double and classical models for BATSE shows that the under-luminous population causes a significant increase in numbers at  $z < 0.1$ , resulting in a higher probability of observing small- $z$  GRBs. *Swift* could potentially detect over 100 GRBs during one year with up to 5 in  $z < 0.1$

Table 1, using the same parameters as figure 1, shows the probabilities for detecting at least one GRB in volumes bounded by  $z = 0.0085$ , 0.05, and 0.17 during 4.5 yrs observation, assuming the sensitivities of both BATSE/HETE and *Swift*. Equation (3) is used to calculate the rates based on a classical distribution and equation (5) for a distribution comprised of both classical and under-luminous bursts. The double distribution model increases the detection probability of the very nearby GRB 980425 ( $z = 0.0085$ ) to a still small (0.04), but significant, level compared with the extremely small probability (0.00015) from the classical distribution alone.

Figures 2 and 3 plot the number distribution and cumulative number of GRBs observed over a 1 yr period for the classical and double distributions assuming BATSE and *Swift* sensitivities. For BATSE sensitivities it is evident that the under-luminous GRBs have no effect on the observed distribution at  $z > 0.2$ . They do contribute to the cumulative number at  $z < 0.2$ , a result that is compatible with the detection of GRBs 980425 and 031203. The cumulative number increases from about 1 at  $z = 0.1$ , for a BATSE sensitivity, to about 4 or 5 for a *Swift* sensitivity.

## 6 CONCLUSIONS

We have shown that a population comprised of classical and under-luminous GRBs is compatible with the currently observed GRB redshift distribution that includes the nearby and under-luminous GRBs 980425 and 031203. We find that a local rate density for an under-luminous GRB population of  $r_0^u \approx 220 \text{ yr}^{-1} \text{ Gpc}^{-3}$ , which is about 0.6% of the local SN Type Ib/c rate,  $r_0^{\text{SN Ib/c}} \approx 6.3 \times 10^4 \text{ yr}^{-1} \text{ Gpc}^{-3}$ , fits the observed low-redshift GRB distribution. Assuming that GRB 980425 is typical in luminosity for an under-luminous population, we make a first crude constraint on such a population by taking a mean luminosity of  $L_0 = 2 \times 10^{53} \text{ s}^{-1}$ —about 3–4 orders of magnitude less than the mean luminosity of the classical GRBs. These two constraints yield a probability of BATSE/HETE detecting at least one GRB at  $z < 0.05$  to be 0.78 over 4.5 years, a result that is compatible with the presently observed low-redshift GRB distribution.

GRBs 980425 and 031203 may only appear faint and anomalous because of the sensitivity limit of BATSE/HETE. If such an under-luminous population is present, the increased sensitivity of *Swift* should enable it to detect and localize 5 times more GRBs than BATSE at redshifts  $z < 0.1$ .

It seems reasonable that the observed broad distribution of observed GRB luminosities may represent related, but different classes of engine-driven emissions powered by rotating massive compact stellar remnants. The definite association of some nearby GRBs with Type Ib/c SNe—a SN type that exhibits considerable diversity—supports the idea that there could be a diverse class of inner engines driving at least a fraction of GRBs. These classes may even form a continuum that encompasses Type Ib/c SNe, XRF, faint GRBs and classical GRBs. Hence the distribution of sources in redshift in this scenario would consist of the sum of the individual populations with different inner engines and local rate densities. There is most likely overlap between the emission characteristics of the various sub-populations, so much so that they may form a continuum of luminosities ranging from XRF to hard GRBs.

If the anomalous GRBs are all shown to be off-axis observations of classical GRBs, then the evidence for different classes of cosmic explosions related to GRBs will become more tenuous. *Swift* should provide new information on the diversity of inner engines and on the progenitors that produce them, providing a wealth of data to help solve the cosmic riddle of identifying GRB progenitor populations.

## ACKNOWLEDGMENTS

I thank the referee for useful suggestions that have led to improving the clarity of this work. I also thank B. P. Schmidt for discussing the idea of multiple populations of GRBs and R. R. Burman and D. G. Blair for providing useful comments. D. M. Coward is supported by an Australian Research Council fellowship and grant DP0346344.

## REFERENCES

- Berger E., Kulkarni S.R., Frail D.A., Soderberg A.M., 2003a, *ApJ*, 599, 408  
 Berger E., et al., 2003b, *Nat.*, 426, 154  
 Bromm V., Loeb A., 2002, *ApJ*, 575, 111  
 Coward D.M., Burman R.R., 2005 (submitted)  
 Firmani C., Avila-Reese V., Ghisellini G., Tutukov A.V., 2004, *ApJ*, 611, 1033  
 Frail D.A. et al., 2001, *ApJ*, 562, L55  
 Galama et al., 1998, I. A. U. C., n 6895, 7 May 1998, p 1 pp.  
 Galama et al., *Nat.*, 1998, 395, 670  
 Guetta D., Perna R., Stella L., Vietri M., *ApJL*, 615, L73  
 Hjorth J. et al., 2003, *Nat.*, 423, 847  
 Izzard R. G, Ramirez-Ruiz E., Tout C A., 2004, *MNRAS*, 348, 1215  
 Kulkarni S.R. et al., 1998, *Nat.*, 395, 663  
 Li Z., Chevalier R.A., 1999, *ApJ*, 526, 716  
 Malesani D. et al., *ApJL*, 609, L5  
 Peebles P.J.E., 1993, *Principles of Physical Cosmology*, Princeton Univ. Press, Princeton, New Jersey, p. 312  
 Phinney E. S., 1991, *ApJ*, 380, L17  
 Porciani C., Madau P., 2001, *ApJ*, 548, 522  
 Prochaska J., et al., *ApJ*, 611, 200  
 Ramirez-Ruiz E. et al., 2004, preprint (astro-ph/0412145 v1)  
 Sazonov S.Yu., Lutovinov A.A., Sunyaev R.A., 2004, *Nat.*, 430 646  
 Schaefer B.E., Deng M., Band D.L., 2001, *ApJ*, 563, L123  
 Soderberg A.M. et al., 2004, *Nat.*, 430, 648  
 Stanek K.Z. et al., 2003, *ApJ*, 591, L17  
 Zhang W., Woosley S.E., MacFadyen A.I., 2003, *ApJ*, 586, 356  
 Zhang, B., Mészáros P., 2004, *Int. J. Mod. Phys.*, 19, 2385