Dynamics of perfect fluid Unified Dark Energy models

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ABSTRACT

In this paper we show that a *one-to-one* correspondence exists between any dark energy model and an equivalent (from a cosmological point of view, in the absence of perturbations) quartessence model in which dark matter and dark energy are described by a single perfect fluid. We further show that if the density fluctuations are small, the evolution of the sound speed squared, c_s^2, is fully coupled to the evolution of the scale factor and that the transition from the dark matter to the dark energy dominated epoch is faster (slower) than in a standard ΛCDM model if $c_s^2 > 0$ ($c_s^2 < 0$). In particular, we show that the mapping of the simplest quintessence scenario with constant $w_Q \equiv p_Q/\rho_Q$ into a unified dark energy model requires $c_s^2 < 0$ at the present time (if $w_Q > -1$) contrasting to the Chaplygin gas scenario where one has $c_s^2 > 0$. However, we show that non-linear effects severely complicate the analysis, in particular rendering linear results invalid even on large cosmological scales. Although a detailed analysis of non-linear effects requires solving the full Einstein field equations, some general properties can be understood in simple terms. In particular, we find that in the context of Chaplygin gas models the transition from the dark matter to the dark energy dominated era may be anticipated with respect to linear expectations leading to a background evolution similar to that of standard ΛCDM models. On the other hand, in models with $c_s^2 > 0$ the expected transition from the decelerating to the accelerating phase may never happen.

Key words: cosmology: theory — cosmic microwave background

1 INTRODUCTION

A considerable effort has been devoted, in the past few years, to shape Unified Dark Energy (aka Quartessence) into a coherent phenomenological model. At the moment we are still lacking a solid theoretical motivation in favour of this unified scenario. Yet, the potential outcome of such models is enough to keep a significant amount of papers cyclicly emerging. The drive behind UDE is simple enough to be put into a few words: can dark matter and dark energy share a common unifying nature? Could they be different aspects of the same thing? Either way, an answer to these questions would tell us something of a fundamental nature.

Evidence for the existence of a dark energy component has been steadily piling over the years. It is clear from observations that most of the matter in the Universe is in a dark non-baryonic form (see, for instance, Tonry et al. 2003; Riess et al. 2004; Tegmark 2004; Spergel et al. 2006). However, there is at present no direct detection of non-baryonic dark matter or dark energy, their existence merely inferred from their cosmological implications through gravitational effects. Hence, one should not discard the possibility of a single component simultaneously accounting for both dark matter and dark energy.

Historically, the idea of UDE has sprung from the cosmological properties of the Chaplygin gas (Kamenshchik, Moschella & Pasquier 2001; Bilic, Tupper & Viollier 2002; Bento, Bertolami & Sen 2002), an exotic fluid with an equation of state $p = -A\rho^{-\alpha}$ (where A and α are positive constants). This special fluid has a dual behaviour: it mimics matter ($p \approx 0$) early in the history of the Universe and a cosmological constant much later (with a smooth transition in between) which is highly suggestive of a unified description of dark matter and dark energy.

Although we could assume that the Chaplygin gas is nearly homogeneous in both the radiation and matter eras much like a standard variable w-quintessence component (see for example Multamaki, Manera & Gaztanaga 2003), this would not explore the full potential of the Chaplygin gas as a UDE candidate. Hence, in the absence of any real clues regarding the microscopic properties of quartessence, we will make the simplest possible assumption: *that it can be approximated by a perfect fluid* whose properties are fully specified once the equation of state is known. However, we

shall refrain from making any further assumptions regarding the form of its equation of state (other than those which are required by observations that is) and leave the discussion as general as possible (see, however, Bento, Bertolami & Sen 2004 for a different approach).

An appealing feature of perfect fluid UDE models is the existence of formally equivalent models with a single scalar field, ϕ, described by the action

$$S = \int d^4x \sqrt{-g} p(X) \qquad (1)$$

where $p = p(X)$, $\rho = 2X dp/dX - p$ and $u_\mu = \nabla_\mu \phi / \sqrt{2X}$ are respectively the pressure, the energy density and the four velocity of the perfect fluid and $X = \nabla_\mu \phi \nabla^\mu \phi / 2$. If $p = X^{(w+1)/(2w)}$ the action describes a perfect fluid with equation of state $p = w\rho$ with constant $w \neq 0, -1$ (for $w = 1$ one obtains the usual massless scalar field action) while if

$$p(X) = -A^{\frac{1}{1+\alpha}} \left(1 - (2X)^{\frac{1+\alpha}{2\alpha}}\right)^{\frac{\alpha}{1+\alpha}} \qquad (2)$$

one gets a *generalized* Born-Infeld action describing the dynamics of the *generalized* Chaplygin gas. Here we are taking $\nabla_\mu \phi$ to be timelike. In all these models a single real scalar field accounts for both dark matter and dark energy.

The predictions of UDE models based on the Chaplygin gas have been tested using observational data including high-z supernovae (Avelino et al. 2003a; Bean & Doré 2003; Beça et al. 2003; Colistete & Fabris 2005; Bento et al. 2004; Zhu 2004; Bertolami et al. 2005), lensing (Makler, Quinet de Oliveira & Waga 2003; Silva & Bertolami 2003; Dev, Jain & Alcaniz 2004), high precision CMB (Carturan & Finelli 2003; Bean & Doré 2003; Amendola et al. 2003; Bento, Bertolami & Sen 2003) and Large Scale Structure (Fabris, Goncalves & De Souza 2002a; Fabris, Goncalves & De Souza 2002b; Bean & Doré 2003; Beça et al. 2003; Sandvik et al. 2004). As we'll later see in more detail, the Chaplygin gas attains very high sound speeds at recent times which has a significant negative impact on small scale structure formation. This was first studied by Sandvik et al. (2004) which concluded that in order to obtain the mass power spectra we observe today, the parameter α in the Chaplygin gas model had to be extremely fine tuned around zero ($\alpha = 0$ being the ΛCDM limit). However, it has been shown by Beça et al. (2003) that this problem could be alleviated (but not solved) by adding baryons to the mixture.

Yet, there is a caveat to most of these results: linear theory has been taken as a good approximation when comparing with cosmological observations. Ordinarily, this is a valid assumption to make; for example in standard ΛCDM models the non-linear collapse is not expected to have a large impact on the evolution of the average Universe. However, as discussed previously by Avelino et al. (2004), the small scale structure of the Chaplygin gas may influence the background evolution of the Universe, rendering linear theory invalid even on large cosmological scales. Hence, in order to confront perfect fluid UDE models with observations, linear theory may not enough; we may have to solve the full Einstein field equations which is obviously an enormous task. Nevertheless, in this paper we will show that the overall effect of the non-linear terms on the background evolution of the Universe may be understood using simple considerations.

2 LOCAL VS. GLOBAL EQUATIONS OF STATE

The dynamics of a homogeneous and isotropic Friedmann-Robertson-Walker Universe is partially described by

$$\frac{\dot{a}^2 + k}{a^2} = \frac{8\pi G}{3} \rho, \qquad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \qquad (4)$$

where the dot represents a cosmic time derivative, a is the scale factor, ρ the density, p the pressure and k is a constant (since we are mainly interested in the case of a flat Universe, we take $k = 0$ this point onwards). To fully specify the dynamics, one must supply an extra relation between pressure and density, in other words, an equation of state $p = p(\rho) = w(\rho)\rho$.

Since the cosmological principle is only approximately valid, so is the description given above. It applies only to the *average* Universe. In (3) and (4), p and ρ refer to *average* global measures of pressure and density obtained by smoothing over large enough regions, as opposed to local measures. Thus, it would probably be better to write them as $\langle p \rangle$ and $\langle \rho \rangle$. Note, however, that the extra prescription $p = w\rho$ is a relation between the local pressure and density and not necessarily between the smoothed versions. This is a subtle and important point.

Consider the perturbative decomposition of pressure and density

$$p = \langle p \rangle + \delta p + \cdots, \qquad (5)$$
$$\rho = \langle \rho \rangle + \delta \rho + \cdots.$$

The common procedure is to assume that $\langle p \rangle = w(\langle \rho \rangle)\langle \rho \rangle$. However, a priori, there is no reason to suppose that the relation between the average pressure and the average density is the same as the local one (for a related discussion see Ellis & Buchert (2005). The Chaplygin gas is an example of this. While locally it behaves as $p = -A\rho^{-\alpha}$, globally it does not (unless $\alpha = 0$):

$$\langle p \rangle = -A\langle \rho^{-\alpha} \rangle \neq -A < \rho >^{-\alpha}, \qquad (6)$$

except if perturbations are linear ($\delta = \delta\rho/\langle p \rangle \ll 1$) in which case we would have

$$\langle p \rangle = -A\langle \rho^{-\alpha} \rangle = -A\langle \rho \rangle^{-\alpha}\langle 1 - \alpha\delta \rangle \qquad (7)$$
$$= -A\langle \rho \rangle^{-\alpha}.$$

This discrepancy between local and global equations of state depends on what is happening to the fluid on small scales meaning that non-linearities are deeply involved. We further discuss this in Section 7. The important point is that this discrepancy complicates matters substantially. For instance, in the case of the Chaplygin gas, most researchers have naively used the local equation of state to relate $\langle p \rangle$ and $\langle \rho \rangle$ which in general is clearly not a valid assumption.

3 THE SIMPLEST UDE MODEL

The simplest unified dark energy model one can possibly conceive is that of a perfect fluid with a constant negative pressure: $p = -A$ with constant $A > 0$. We can also state this by saying that $w = -A\rho^{-1}$. Incidentally, this is the

case of the Chaplygin gas with $\alpha = 0$. From (3) and (4) it is straightforward to show that

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \,. \tag{8}$$

This has a simple solution if p is constant. In fact, we have that $\rho + p \propto a^{-3}$ and (3) can thus be rewritten as

$$\mathrm{H}^2 = \mathrm{H}_0^2(\Omega_m^{0*}a^{-3} + \Omega_\Lambda^{0*}) \,, \tag{9}$$

where $\mathrm{H} = \dot{a}/a$ is the Hubble parameter and $\Omega_\Lambda^{0*} = 1 - \Omega_m^{0*} = 8\pi Gp/3\mathrm{H}_0^2$ is the equivalent Λ energy fraction today. It is also easy to show that $w_0 \equiv (p/\rho)_0 = -\Omega_\Lambda^{0*}$ today. Here the index '0' indicates that the variables are to be evaluated at the present time, t_0.

The most important feature of this model is that it is totally equivalent to ΛCDM to all orders (Avelino, Beça, de Carvalho & Martins 2003). This fact translates the three year WMAP constraint on $\Omega_m^0 = 0.238^{+0.030}_{-0.041}$ (Spergel et al. 2006) into $w_0 = -\Omega_\Lambda^{0*} = -0.762^{+0.030}_{-0.041}$. We note that the evolution of the equation of state of this UDE fluid is given by

$$w = \frac{w_0}{(1 + w_0)a^{-3} - w_0} \,, \tag{10}$$

making it indeed behave as CDM ($w \sim 0$) at early times ($a \sim 0$) and as a cosmological constant much later ($a \to \infty$).

4   SOUND SPEED AND BACKGROUND DYNAMICS

The simplest UDE model just described has a null square sound speed $c_s^2 = 0$ but that will cease to be the case in the context of more general models. Indeed, it is a simple matter to show, using (3) and (4) that

$$\dot{\rho} = \frac{3}{4\pi G}\mathrm{H}\frac{d\mathrm{H}}{dt} \,, \tag{11}$$

$$\dot{p} = -\frac{1}{4\pi G}\frac{d}{dt}\left[\left(\frac{\ddot{a}}{a} + \frac{1}{2}\mathrm{H}^2\right)\right] \,, \tag{12}$$

implying a sound speed of

$$c_s^2 \equiv \frac{dp}{d\rho} = \frac{1}{3\mathrm{H}}\frac{d}{d\mathrm{H}}\left[\mathrm{H}^2\left(q - \frac{1}{2}\right)\right] \,, \tag{13}$$

where $q \equiv -\ddot{a}/(a\mathrm{H}^2)$ is the usual deceleration parameter. Let us assume that H is a decreasing function of cosmic time. The sign of the sound speed squared c_s^2 today will be determined by the way q is evolving. If it is evolving sufficiently fast (towards negative values) $c_s^2 > 0$; otherwise $c_s^2 < 0$. On the other hand, the evolution of q is linked to how fast the transition from dark matter to dark energy occurs for quartessence. If it is steep enough (faster than in ΛCDM, that is), c_s^2 will be positive (negative otherwise). Therefore, the sign of c_s^2 is connected to the background dynamics.

5   EXTENDING THE SIMPLEST UDE MODEL

A straightforward generalization of the simplest UDE model leads to $w = -A\rho^{-(1+\alpha)}$ with a constant $\alpha > 0$, i.e. the generalized Chaplygin gas. It is a simple matter to show that the sound speed square of this fluid is given by

$$c_s^2 \equiv \frac{dp}{d\rho} = -\alpha w \tag{14}$$

and is, in fact, positive for $\alpha > 0$. This means that the transition from dark matter to dark energy is fast enough to make $c_s^2 > 0$. As we mentioned before, one interesting characteristic of the Chaplygin gas is that it has a non-zero minimal density. No matter how much you expand it, ρ will never drop below a certain value: $\rho_m = A^{1/(1+\alpha)}$. When ρ reaches this value, however, the pressure will be exactly $-\rho_m$, that is, $w = -1$; it behaves as a cosmological constant.

6   ONE-TO-ONE MAP BETWEEN UDE AND QUINTESSENCE

Let us now consider the simplest quintessence model with $p_Q = w_Q\rho_Q$ with constant w_Q plus pressureless CDM and make a *one-to-one* map into a unified dark energy model with density $\rho = \rho_Q + \rho_{cdm}$ and pressure $p = p_Q + p_{cdm} = p_Q$ (see also Kamenshchik, Moschella & Pasquier 2001; Bertolami, Sen, Sen & Silva 2004). If the density perturbations are small then the sound speed of this fluid is uniform and is given by

$$c_s^2 \equiv \frac{dp}{d\rho} = \frac{dp_Q}{d\rho_Q}\frac{d\rho_Q}{d\rho} \tag{15}$$

$$= w_Q\frac{1}{1 + (\kappa/(1 + w_Q))a^{3w_Q}} \,,$$

where $\kappa = \rho_{cdm}^0/\rho_Q^0 \approx 0.3 - 0.4$. The equation of state of this fluid has the following simple form,

$$\rho = \frac{p}{w_Q} + \rho_{cdm}^0\left(\frac{p}{w_Q\rho_Q^0}\right)^{1/(1+w_Q)} \,. \tag{16}$$

If we the dominant energy condition ($w \equiv p/\rho \geq -1$) to be valid at all times then we must have $w_Q \geq -1$. In this case $c_s^2 < 0$ at all times.

7   NON-LINEAR EFFECTS

General relativity is a non-linear theory of gravity which means that, unlike in the electromagnetic case, the superposition principle is not valid. However, in many situations we may neglect non-linear terms and effectively linearize the field equations. In cosmology, this is often possible for large enough scales. However, this is in general not the case in the context of perfect fluid UDE models.

7.1   Case with $c_s^2 > 0$

In this section we illustrate in the simplest possible manner how non-linearities can affect the evolution of the Universe *even on large cosmological scales*, focusing on the particular example of the Chaplygin gas. Here, one should bear in mind than the Chaplygin gas has a minimum density ρ_m attainable and consequently a minimum pressure $p_m = -\rho_m$ (corresponding to a maximum in modulus).

Consider a spherical region of radius R with an average density $\langle\rho\rangle$. If the energy density is *uniformly* distributed then we clearly have $\langle p\rangle = -A\langle\rho\rangle^{-\alpha}$. However, if the density is not uniformly distributed then, in general, this will no

longer be valid. Take the case in which the region with $R_1 < r < R$ has a smaller density than the region with $r < R_1$. In fact, let us assume that $\rho(r < R_1) = N\langle\rho\rangle$, where N is some factor higher than 1, and $\rho(R_1 < r < R) = \rho_m = A^{1/(1+\alpha)}$ so that $p(R_1 < r < R) = p_m = -\rho_m$. The sum of the masses inside the two regions divided by the entire volume still has to be $\langle\rho\rangle$ by construction which implies that N is equal to

$$N = \left(\frac{R}{R_1}\right)^3 + \left(1 - \left(\frac{R}{R_1}\right)^3\right)\frac{\rho_m}{\langle\rho\rangle}. \tag{17}$$

Now, while the average density inside R is still the same as in the uniform case, the average pressure $\langle p\rangle$ *is not*. It is simple enough to show that

$$\langle p\rangle = \left(\frac{R_1}{R}\right)^3 (N\langle\rho\rangle)^{-\alpha} - \left(1 - \left(\frac{R_1}{R}\right)^3\right)\rho_m \sim -\rho_m, \tag{18}$$

where the approximation is valid for large N (small R_1/R). If $N \gg 1$, the real average pressure $\langle p\rangle$, is considerably larger (in modulus) than $A\langle\rho\rangle^{-\alpha}$, the modulus of the average pressure of the Chaplygin gas *in the absence of perturbations*. Recall that for the Chaplygin gas, the lower the density, the bigger the pressure (in modulus). This is why the pressure in the lower density region will dominate the average pressure inside R.

Hence, the non-linear collapse will make $\langle p\rangle = p_m = -\rho_m$ early on, and will thus anticipate and slowdown the transition from dark matter to dark energy, thus mimicking the background evolution in the simplest UDE model with $\alpha = 0$ (or, equivalently, the ΛCDM scenario). The small scale-structure of the Chaplygin gas is interfering with the evolution of the Universe on large scales.

Of course, this is an oversimplified picture: we have not taken into account the dynamical effects of pressure gradients. In high density regions the pressure will be significantly smaller (in modulus) than average. Still, we need to take into account that gravitational collapse will only be effective on a given scale λ if $\lambda \gtrsim c_s H^{-1}$. However, note that once the perturbations become non-linear pressure will not influence significantly the subsequent dynamics.

7.2 Case with $c_s^2 < 0$

Let us recall what a negative c_s^2 means; remember that well inside the horizon, linear theory describes the evolution of a perturbation as a wave:

$$\ddot{\delta} - c_s^2 \nabla^2 \delta \simeq 0, \tag{19}$$

so that $\delta_k \propto \exp{(i(wt - \mathbf{k} \cdot \mathbf{r}))}$ where $w^2 = c_s^2 k^2$ (although this is a linear result, we will still use it to guide us into the non-linear realm). This makes the interpretation of the sign of the sound speed c_s^2 very straightforward. If $c_s^2 > 0$, δ will oscillate as an acoustical wave, acting against the formation of voids and dense regions. On the other hand, if $c_s^2 < 0$, the opposite will happen: density perturbations in collapsing regions and voids get amplified.

Let us now apply to the UDE model of Section 6 the same reasoning we did to the non-uniform Chaplygin gas and focus on a background of negative $c_s^2 < 0$ (the relevant case). High density regions will tend to behave as pressure-less matter with $c_s^2 \approx 0$. On the other hand, low density regions with $c_s^2 < 0$ will tend to become increasingly emptier.

There is, however, a major difference to the Chaplygin gas case: there is not a positive minimum density underdense regions cannot go below (except if $w = -1$) and, consequently, ρ can be arbitrarily close to zero. This has the interesting consequence that the average pressure may be close to zero at all times so that the Universe may never start to accelerate (despite the linear theory prediction), something which is clearly inconsistent with current observational evidence.

8   CONCLUSIONS

In this paper we have shown that there is a one-to-one map between any dark energy model and a perfect fluid quartessence model. Note, however, that in general these models do not share the same underlying physics. Also, although these models are equivalent in the absence of perturbations, this will in general not be the case in the presence of density fluctuations. In particular, if the perturbations are small then the evolution of c_s^2 for the quartessence model is fully coupled to the large-scale dynamics of the universe. However, we have shown that non-linear effects are of capital importance and should be included in quantitative treatments of perfect fluid UDE. In fact, small scale structure may alter the background evolution of the Universe making the global behaviour of quartessence quite different from the one predicted by linear theory. In particular, we have found that if $c_s^2 > 0$ then the transition from the dark matter to the dark energy dominated epoch may be anticipated with respect to linear expectations, leading to a background evolution similar to that of standard ΛCDM models. On the other hand, if $c_s^2 < 0$ the transition from the decelerating to the accelerating phase may never happen. There is one obvious consequence of these results: if future observations turn out to be incompatible with the background evolution predicted in the context standard ΛCDM models then that will also constitute a serious challenge to perfect fluid UDE scenarios.

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