

# Interpreting Cosmological Vacuum Decay

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The cosmological vacuum decay scenario recently proposed by Wang and Meng [1] is rediscussed. From thermodynamic arguments it is found that the  $\epsilon$  parameter quantifying the vacuum decay rate must be positive in the presence of particle creation. If there is no particle creation, the proper mass of Cold Dark Matter (CDM) particles is necessarily a time dependent quantity, scaling as  $m(t) = m_o a(t)^\epsilon$ . By considering the presence of baryons in the cosmological scenario, it is also shown that their dynamic effect is to alter the transition redshift  $z_*$  (the redshift at which the Universe switches from decelerating to accelerating expansion), predicting values of  $z_*$  compatible with current estimates based on type Ia supernova. In order to constrain the  $\Omega_m - \epsilon$  plane, a joint statistical analysis involving the current supernovae observations, gas mass fraction measurements in galaxy clusters and CMB data is performed. At 95% c.l. it is found that the vacuum decay rate parameter lies on the interval  $\epsilon = 0.11 \pm 0.12$ . The possibility of a vacuum decay into photons is also analyzed. In this case, the energy density of the radiation fluid scales as  $\rho_r = \rho_{r0} a^{-4+\epsilon}$ , and its temperature evolution law obeys  $T(t) = T_o a(t)^{\epsilon/4-1}$ .

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## I. INTRODUCTION

There is nowadays significant observational evidence that the expansion of the Universe is undergoing a late time acceleration [2, 3, 4, 5, 6]. This, in other words, amounts to saying that in the context of Einstein's general theory of relativity some sort of *dark energy*, constant or that varies only slowly with time and space, dominates the current composition of the cosmos (see, e.g., [6] for some recent reviews on this topic). The origin and nature of such an *accelerating field* constitutes a completely open question and represents one of the major challenges not only to cosmology but also to our current understanding of fundamental physics.

Among many possible alternatives, the simplest and most theoretically appealing possibility for dark energy is the energy density stored on the true vacuum state of all existing fields in the Universe, i.e.,  $\rho_\Lambda = \Lambda/8\pi G$ , where  $\Lambda$  is the cosmological constant. From the observational side, flat models with a relic cosmological term ( $\Lambda$ CDM) seems to be in agreement with almost all cosmological observations, which makes them an excellent description of the observed universe. From the theoretical viewpoint, however, the well-known cosmological constant problem, i.e., the unsettled situation in the particle physics/cosmology interface, in which the cosmological upper bound ( $\rho_\Lambda \lesssim 10^{-47} \text{GeV}^4$ ) differs from theoretical expectations ( $\rho_\Lambda \sim 10^{71} \text{GeV}^4$ ) by more than 100 orders of magnitude, originates an extreme fine-tuning problem

[7] or makes a complete cancellation (from an unknown physical mechanism) seem more plausible.

In this regard, a phenomenological attempt at alleviating such a problem is allowing  $\Lambda$  to vary<sup>1</sup>. Cosmological scenarios with a time-varying or dynamical  $\Lambda$  were independently proposed almost twenty years ago in Refs. [8] (see also [9]). Afterward, a number of models with different decay laws for the variation of the cosmological term were investigated in Ref. [10] and the confrontation of their predictions with observational data has also been analyzed by many authors [11]. It is worth mentioning that the most usual critique to these  $\Lambda(t)$ CDM scenarios is that in order to establish a model and study their observational and theoretical predictions, one needs first to specify a phenomenological time-dependence for  $\Lambda$ . In this concern, an interesting step towards a more realistic decay law was given recently by Wang & Meng in Ref. [1]. Instead of the traditional approach, they deduced a new decay law from a simple argument about the effect of the vacuum decay on the cold dark matter (CDM) expansion rate. Such a decay law is similar to the one originally obtained in Ref. [12] from arguments based on renormalization group and seems to be very general, having many of the previous attempts as a particular case and being capable of reconciling  $\Lambda(t)$ CDM models with an initially decelerated and late time accelerating universe, as indicated by current SNe Ia observations [4].

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<sup>1</sup> Strictly speaking, in the context of classical general relativity any additional  $\Lambda$ -type term that varies in space or time should be thought of as a new *time-varying field* and not as a cosmological constant. Here, however, we adopt the usual nomenclature of time-varying or dynamical  $\Lambda$  models.

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The aim of the present paper is twofold: first, to interpret thermodynamically the process of cosmological vacuum decay, as suggested in Ref. [1]. From thermodynamic considerations, it is shown that such a process leads to two different effects, namely, a continuous creation of particles and an increasing in the mass of CDM particles given by  $m(t) = m_o a(t)^\epsilon$ , where  $a(t)$  is the cosmological scale factor and  $\epsilon$  is the parameter quantifying the decay vacuum rate; second, to analyze the dynamic modifications in the original Wang-Meng cosmic scenario by introducing explicitly the baryonic component. As we shall see, the presence of baryons alters considerably the accelerating redshift  $z_*$ , that is, the redshift at which the Universe switches from deceleration to acceleration. In order to constrain the parametric space  $\Omega_m - \epsilon$ , we also perform a statistical analysis involving three sets of observables, namely, the latest Chandra measurements of the X-ray gas mass fraction in 26 galaxy clusters, as provided by Allen et al. [5], the so-called “gold” set of 157 SNe Ia, recently published by Riess et al. [4], and the measurement of the CMB shift parameter, as given by WMAP, CBI, and ACBAR [3]. Finally, we extend the treatment of Ref. [1] to a scenario in which the vacuum energy decays into photons. In this case, it is found that the temperature evolution law of radiation is modified to  $T = T_o a(t)^{\epsilon/4-1}$ .

## II. VACUUM DECAY INTO CDM

Let us first consider the Einstein field equations

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = \chi \left[ T^{\mu\nu} + \frac{\Lambda}{\chi} g^{\mu\nu} \right], \quad (1)$$

where  $R^{\mu\nu}$  and  $R$  are, respectively, the Ricci tensor and the scalar curvature,  $T^{\mu\nu}$  is the energy-momentum tensor of matter fields and CDM particles, and  $\chi = 8\pi G$  ( $c = 1$ ) is the Einstein’s constant. Note that according to the Bianchi identities, the above equations implies that  $\Lambda$  is necessarily a constant either if  $T^{\mu\nu} = 0$  or if  $T^{\mu\nu}$  is separately conserved, i.e.,  $u_\mu T^{\mu\nu};_\nu = 0$ . In other words, this amounts to saying that (i) vacuum decay is possible only from a previous existence of some sort of non-vanishing matter and/or radiation, and (ii) the presence of a time-varying cosmological term results in a coupling between  $T^{\mu\nu}$  and  $\Lambda$ . For the moment, we will assume a coupling only between vacuum and CDM particles, so that

$$u_\mu \mathcal{T}^{\mu\nu};_\nu = -u_\mu \left( \frac{\Lambda g^{\mu\nu}}{\chi} \right);_\nu, \quad (2)$$

or, equivalently,

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = -\dot{\rho}_v, \quad (3)$$

where  $\rho_m$  and  $\rho_v$  are the energy densities of the CDM and vacuum, respectively, and  $\mathcal{T}^{\mu\nu} = \rho_m u^\mu u^\nu$  denotes the energy-momentum tensor of the CDM matter.

As commented earlier, the traditional approach for  $\Lambda(t)$ CDM models was first to specify a phenomenological decay law and then establish a cosmological scenario (see, e.g., [10, 11]). Here, however, we follow the arguments presented in Ref. [1], in which a decay law is deduced from the effect it has on the CDM evolution. The qualitative argument is the following: since vacuum is decaying into CDM particles, CDM will dilute more slowly compared to its standard evolution,  $\rho_m \propto a^{-3}$ . Thus, if the deviation from the standard evolution is characterized by a positive constant  $\epsilon$ , i.e.,

$$\rho_m = \rho_{m0} a^{-3+\epsilon}, \quad (4)$$

Eq. (3) yields

$$\rho_v = \tilde{\rho}_{v0} + \frac{\epsilon \rho_{m0}}{3-\epsilon} a^{-3+\epsilon}, \quad (5)$$

where  $\rho_{m0}$  is the current CDM energy density and  $\tilde{\rho}_{v0}$  stands for what is named in Ref. [1] “the ground state value of the vacuum”. As discussed there, such a decay law seems to be the most general one, having many of the previous phenomenological attempts as a particular case.

## III. THERMODYNAMICS OF VACUUM DECAY

Let us now investigate some thermodynamic features of the decaying vacuum scenario described in the last section. As discussed in Ref. [13], the thermodynamic behavior of a decaying vacuum system is simplified if one assumes that the chemical potential of the vacuum component is zero, and also if the vacuum medium plays the role of a condensate carrying no entropy, as happens in the two fluid description employed in superfluid thermodynamics. In this case, the thermodynamic description require only the knowledge of the particle flux,  $N^\alpha = nu^\alpha$ , and the entropy flux,  $S^\alpha = \sigma u^\alpha$ , where  $n = N/a^3$  and  $\sigma = S/N$  are, respectively, the concentration and the specific entropy (per particle) of the created component.

It is clear from last Section that in the Wang-Meng description the two component are changing energy, but it is not clear where the vacuum energy is going to or, in other words, where the CDM component is storing the energy received from the vacuum decay process. In principle, since the energy density of the cold dark matter is  $\rho = nm$ , there are two possibilities:

(i) the equation describing concentration,  $n$ , has a source term while the proper mass of CDM particles remains constant;

(ii) the mass  $m$  of the CDM particles is itself a time-dependent quantity while the total number of CDM particles,  $N = na^3$ , remains constant.

The case (i) seems to be physically more realistic, and coincides exactly with the description presented in Ref.

[13]. However, for the sake of completeness, in what follows we consider both cases.

### A. Case I: Vacuum decay into CDM particles

In this case, there is necessarily a source term in the current of CDM particles, that is,  $N^{\alpha}_{;\alpha} = \psi$ . In terms of the concentration it can be written as

$$\dot{n} + 3\frac{\dot{a}}{a}n = \psi = n\Gamma, \quad (6)$$

where  $\psi$  is the particle source ( $\psi > 0$ ), or a sink ( $\psi < 0$ ), and we have written it in terms of a decay rate,  $\Gamma$ . Since  $\rho = nm$  we find from (4) that  $n = n_o a^{-3+\epsilon}$ . Inserting this result into the above equation it follows that

$$\Gamma = \epsilon \frac{\dot{a}}{a}. \quad (7)$$

The vacuum decay and the associated particle creation rate are the unique sources of irreversibility. Thermodynamically, the overall energy transfer from the vacuum to the fluid component may happens in several ways. In the most physically relevant case it has been termed adiabatic decaying vacuum [13](see also [14] for more applications of adiabatic decay processes in cosmology). In this case, several equilibrium relations are preserved, and, perhaps, more important, the entropy of the created particles increases but the specific entropy (per particle) remains constant ( $\dot{\sigma} = 0$ ). This means that

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N} = \Gamma. \quad (8)$$

On the other hand, from Eq. (7) we see that the total number of particles scales as a power law

$$N(t) = N_o a(t)^\epsilon, \quad (9)$$

whereas the second law of thermodynamics,  $\dot{S} \geq 0$ , implies that  $\epsilon \geq 0$ , as should be expected. To close the connection with the Wang-Meng scenario we need to show that the vacuum energy density follows naturally from the thermodynamic approach. Actually, for an adiabatic vacuum decay process one may write (see Eqs. (8) and (19) of Ref. [13])

$$\dot{\rho}_v = -\beta\psi, \quad (10)$$

where the phenomenological parameter  $\beta$  is defined by

$$\beta = \frac{\rho + p}{n}. \quad (11)$$

Finally, by considering that the CDM medium is pressureless, Eq. (10) can be rewritten as

$$\dot{\rho}_v = -nm\epsilon \frac{\dot{a}}{a}, \quad (12)$$

or still,

$$\dot{\rho}_v = -\rho_{m_o}\epsilon a^{-4+\epsilon}\dot{a}, \quad (13)$$

whose integration reproduces expression (5) previously derived by Wang and Meng [1]. Beyond the independent derivation of the decaying vacuum energy density, the interesting point here is that the sign of the ‘‘coupling constant’’,  $\epsilon$ , is constrained by the second law of thermodynamics.

### B. Case II: Variable Mass Particles

In this case, *there is no creation of CDM particles*, which means that the concentration satisfies the equation

$$\dot{n} + 3\frac{\dot{a}}{a}n = 0, \quad (14)$$

whose solution is  $n = n_o a^{-3}$  which implies that  $N(t) = \text{constant}$ . Naturally, if CDM particles are not being created, the unique possibility is an increasing in the proper mass of CDM particles. Actually, since  $\rho = nm$ , Eqs. (4) and (14) imply that the mass of the CDM particles scales as

$$m(t) = m_o a(t)^\epsilon, \quad (15)$$

where  $m_o$  is the present day mass of CDM particles (compare with expression (9)). Note that this approach for the vacuum decay process leads to a VAMP<sup>2</sup>-type scenario, in which the interaction of CDM particles with the dark energy field imply directly in an increasing of the mass of CDM particles (see, e.g., [15] and references therein for more about VAMP models). To complete our thermodynamic approach for the vacuum decay, a similar treatment for the case in which the vacuum decays only into photons is briefly presented in Appendix A.

## IV. OBSERVATIONAL ASPECTS

In this Section we study some observational aspects of the cosmological scenario discussed above. The Friedmann equation for this modified  $\Lambda(t)$ CDM cosmology reads

$$\left(\frac{H}{H_o}\right)^2 = \left[\Omega_b a^{-3} + \frac{3\Omega_m}{3-\epsilon} a^{-3+\epsilon} + \tilde{\Omega}_{v_o}\right], \quad (16)$$

where  $\Omega_b$  and  $\Omega_m$  are, respectively, the baryon and CDM density parameters and  $\tilde{\Omega}_{v_o}$  is the density parameter associated with ‘‘the ground state of vacuum’’. Note that

<sup>2</sup> VArIable Mass Particles

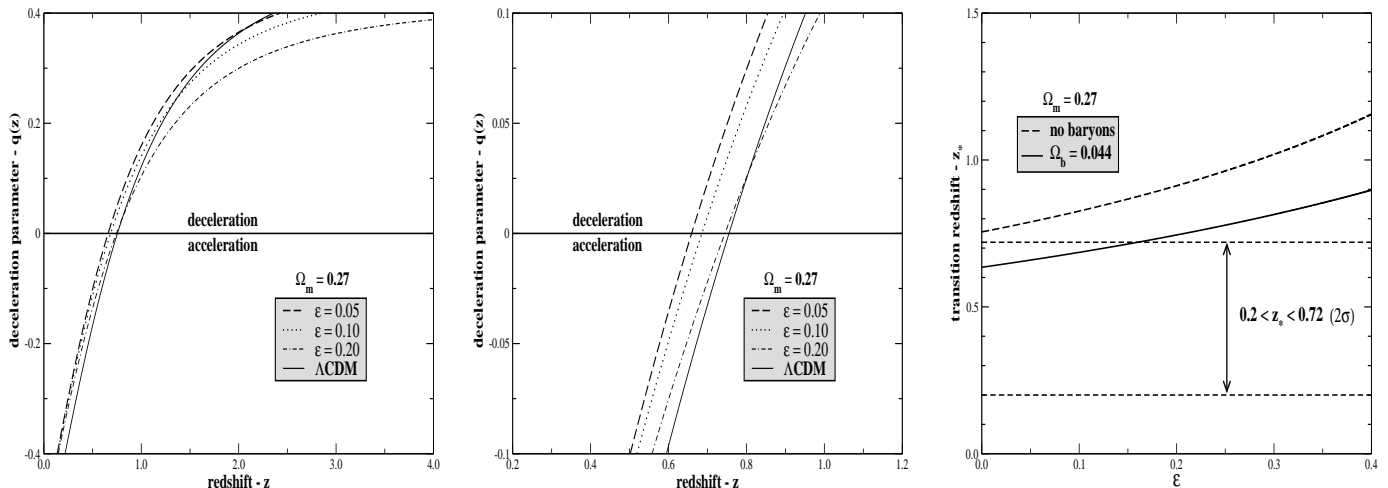


FIG. 1: Effect of baryons on the transition epoch. **a)** The deceleration parameter as a function of redshift for some selected values of  $\epsilon$ . In all curves a baryonic content corresponding to  $\simeq 4.4\%$  of the critical density has been considered. **b)** A closer look at Panel (a). **c)** The transition redshift  $z_*$  as a function of the decay rate parameter  $\epsilon$ . The two cases displayed correspond to the scenario discussed in Ref. [1] (“no baryons”) and the scenario proposed in this paper ( $\Omega_b = 0.044 \pm 0.004$ ). The horizontal dashed lines stand for the  $2\sigma$  interval  $0.2 \leq z_* \leq 0.72$ , as provided by SNe Ia observations [4]. Note that the unique way to make vacuum decay models compatible with the SNe Ia interval for  $z_*$  is to consider explicitly the presence of baryons (see Eq. 16). In particular, from this analysis we find  $\epsilon \leq 0.16$ .

unlike Eq. (6) of Ref. [1], the above Friedmann equation has an additional term which accounts for the baryon contribution to the cosmic expansion. The presence of such a term – redshifting as  $(1+z)^3$  – is justified here since the vacuum is assumed to decay only into CDM particles.

### A. Transition epoch

Although subdominant at the present stage of cosmic evolution, the baryonic content may be important for reconciling  $\Lambda(t)$ CDM models with some current cosmological observations. As an example, let us consider the transition redshift,  $z_*$ , at which the Universe switches from deceleration to acceleration or, equivalently, the redshift at which the deceleration parameter vanishes. From Eq. (16), it is straightforward to show that the deceleration parameter, defined as  $q = -a\ddot{a}/\dot{a}^2$ , now takes the following form

$$q(a) = \frac{3}{2} \frac{\Omega_b a^{-3} + \Omega_m a^{-3+\epsilon}}{\Omega_b a^{-3} + \frac{3\Omega_m}{3-\epsilon} a^{-3+\epsilon} + \tilde{\Omega}_{vo}} - 1, \quad (17)$$

where we have set  $a_o = 1$ .

Two important aspects concerning the above equation should be emphasized at this point. First, note that the presence in Eq. (17) of a non-null density parameter associated with the ground state of vacuum makes possible a transition deceleration/acceleration, as indicated by current SNe Ia observations [4]. As well discussed in Ref.

[1], in most of the cases,  $\Lambda(t)$ CDM models without such a term predict a universe which is either always accelerating or always decelerating from the onset of matter domination up to today. Second, note also that, due to the presence of the baryons, the transition epoch is delayed relative to previous cases (including the standard  $\Lambda$ CDM model), which seems to be in better agreement with recent results indicating  $z_* = 0.46 \pm 0.13$  at  $1\sigma$  [4].

To better visualize the effect of baryons on the transition epoch, we show in Fig. 1a the behavior of the deceleration parameter as a function of redshift [Eq. (17)] for selected values of the parameter  $\epsilon$ . In agreement with WMAP estimates [3] we also assume  $\Omega_m = 0.27 \pm 0.04$  and  $\Omega_b = 0.044 \pm 0.004$ . The best fit  $\Lambda$ CDM case (the so-called “concordance model”) is also showed for the sake of comparison. Note that at late times ( $z = 0$ ), since  $\epsilon$  is a positive quantity, the standard  $\Lambda$ CDM scenario always accelerates faster than  $\Lambda(t)$ CDM models, with the condition for current acceleration being  $\tilde{\Omega}_{vo} > \frac{\Omega_b}{2} + \frac{3\Omega_m(1-\epsilon)}{6-2\epsilon}$ . A closer look at the results shown in Fig. 1a is displayed in Fig. 1b. In Fig. 1c we show the transition redshift  $z_*$  as a function of the parameter  $\epsilon$ , which is obtained from the expression

$$\Omega_b(1+z_*)^3 + \left[ \frac{3-3\epsilon}{3-\epsilon} \right] \Omega_m(1+z_*)^{3-\epsilon} - 2\tilde{\Omega}_{vo} = 0. \quad (18)$$

Two different cases are shown. The scenario of Ref. [1] (no baryons – dashed line) and the model presented here (solid line), in which the baryonic content accounts for  $\sim 4.4\%$  of the critical density. As physically expected

(due to the attractive gravity associated with the baryonic content),  $z_*$  is always smaller in the latter scenario than in the former. In particular, by considering the  $2\sigma$  interval  $0.2 \lesssim z_* \lesssim 0.72$  [4] (horizontal dashed lines) we find  $\epsilon \lesssim 0.16$ , which is in full agreement with the results of the statistical analysis performed in the next Section.

### B. SNe Ia, Clusters and CMB Constraints

In order to delimit the parametric space  $\Omega_m - \epsilon$  we perform in this Section a joint statistical analysis involving three complementary sets of observations. We use to this end the latest Chandra measurements of the X-ray gas mass fraction in 26 galaxy clusters, as provided by Allen et al. [5] along with the so-called “gold” set of 157 SNe Ia, recently published by Riess et al. [4], and the estimate of the CMB shift parameter [3],  $R \equiv \Omega_m^{1/2} \Gamma(z_{\text{CMB}}) = 1.716 \pm 0.062$  from WMAP, CBI, and ACBAR [3], where  $\Gamma(z)$  is the dimensionless comoving distance and  $z_{\text{CMB}} = 1089$ . In our analysis, we also include the most recent determinations of the baryon density parameter, as given by the WMAP team [3], i.e.,  $\Omega_b h^2 = 0.0224 \pm 0.0009$  and the latest measurements of the Hubble parameter,  $h = 0.72 \pm 0.08$ , as provided by the HST key project [16] (we refer the reader to [17] for more details on the statistical analysis).

In Fig. 2 we show the results of our statistical analysis. Confidence regions (68.3%, 95.4% and 99.7%) in the plane  $\Omega_m - \epsilon$  are shown for the particular combination of observational data described above. Note that, although the limits on the parameter  $\epsilon$  are very restrictive, the analysis clearly shows that the model presented here constitutes a small but significant deviation from the standard  $\Lambda$ CDM dynamics. The best-fit parameters for this analysis are  $\Omega_m = 0.27$  and  $\epsilon = 0.11$ , with the relative  $\chi^2_{\text{min}}/\nu \simeq 1.12$  ( $\nu$  is defined as degrees of freedom). Note that this value of  $\chi^2_{\text{min}}/\nu$  is similar to the one found for the so-called “concordance model” by using SNe Ia data only, i.e.,  $\chi^2_{\text{min}}/\nu \simeq 1.13$  [4]. At 95.4% c.l. we also found  $\Omega_m = 0.26 \pm 0.05$  and  $\epsilon = 0.11 \pm 0.12$ .

## V. CONCLUSION

In this paper we have slightly modified and interpreted several features of the decaying vacuum scenario recently proposed by Wang and Meng [1]. A baryonic component has been explicitly introduced, and we have seen that it has an important dynamic effect, namely, the transition epoch from a decelerating/accelerating regime is delayed relative to the one predicted by the original Wang-Meng scenario (including the standard  $\Lambda$ CDM model). The importance of the baryonic contribution cannot be neglected because it reconciles the decaying vacuum scenario with the recent observations [4] (see figure 1, panel c). However, other details of the radiation and matter

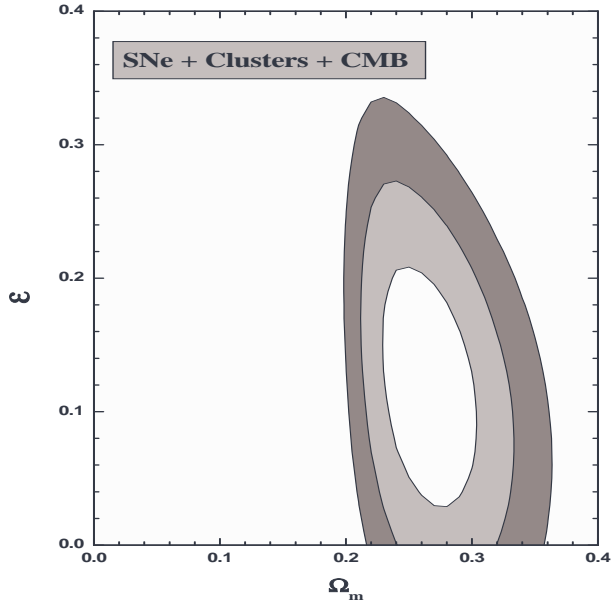


FIG. 2: The plane  $\Omega_m - \epsilon$  for the  $\Lambda(t)$ CDM scenario. The curves correspond to confidence regions of 68.3%, 95.4% and 99.7% for a joint analysis involving SNe Ia, Clusters and CMB data. The best-fit parameters for this analysis are  $\Omega_m = 0.27$  and  $\epsilon = 0.11$ , with reduced  $\chi^2_{\text{min}}/\nu \simeq 1.12$ .

dominated phases are not modified. This is easily verified by computing the value of the redshift  $z$  for which  $\rho_b = \rho_m$ . For the present values of the density parameters,  $\Omega_{m0} \sim 0.3$  and  $\Omega_{b0} = 0.04$ , one finds  $z \simeq 10^{1/\epsilon}$ . Therefore, for  $\epsilon \simeq 0.11$  (the best-fit found in this paper), we obtain  $z \simeq 10^{10}$ . In other words, after this redshift, the Universe is still radiation dominated but the baryons are already subdominant in comparison to the CDM component.

We have also discussed some thermodynamic aspects of such a scenario assuming that the baryonic component is identically conserved. In particular, if CDM particles are produced by the decaying vacuum, we shown that the sign of the coupling parameter,  $\epsilon$ , is restricted by the second law of thermodynamics to assume only positive values. In this case, the total number of CDM particles is a time-dependent function given by  $N(t) = N_0 a^\epsilon$ . However, VAMP-type scenarios - VARIable mass particles - are also possible when the total number of particles remains constant. In this case, the mass scales as  $m(t) = m_0 a^\epsilon$ , that is, the energy of the vacuum decay process is totally transformed in mass of the the existing particles. Naturally, if photons are produced, the temperature law of radiation must also be affected. This case has been discussed with some detail in the Appendix A.

## APPENDIX A: VACUUM DECAY INTO RADIATION

In this Appendix we briefly discuss how the Wang-Meng treatment can be extended to the case of radiation. Now, the energy conservation law reads

$$\dot{\rho}_r + 4H\rho_r = -\dot{\rho}_v, \quad (\text{A1})$$

where  $\rho_r$  is the radiation energy density. By considering that radiation will dilute more slowly compared to its standard evolution,  $\rho_m \propto a^{-4}$ , and that such a deviation is characterized by a positive constant  $\alpha$  we find

$$\rho_r = \rho_{ro}a(t)^{-4+\alpha}, \quad (\text{A2})$$

where  $\rho_{ro}$  is the present day energy density of radiation. For an adiabatic vacuum decay the equilibrium relations are preserved [13, 14], as happens with the Stefan law,  $\rho_r = aT^4$ . As a consequence, one may check that the product  $Ta^{1-\alpha/4}$  remains constant and, as such, this implies that the new temperature law scales with redshift as

$$T = T_o(1+z)^{1-\alpha/4}. \quad (\text{A3})$$

By inserting (A2) into (A1) it follows that

$$\rho_v = \tilde{\rho}_{vo} + \frac{\alpha\rho_{ro}}{4-\alpha}a^{-4+\alpha}, \quad (\text{A4})$$

which should be compared with Eq. (5) describing a decaying vacuum energy density into cold dark matter. Note that the ratio between the vacuum and radiation energy densities are:

$$\frac{\rho_v}{\rho_r} = \frac{\tilde{\rho}_{vo}}{\rho_{ro}}a^{4-\alpha} + \frac{\alpha}{4-\alpha}. \quad (\text{A5})$$

The first term is asymptotically vanishing at early times whereas the second one is smaller than unity. Therefore, a radiation dominated stage is always guaranteed in this kind of scenarios.

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