

# Non-leptonic Weak Interaction in Magnetized Quark matter

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## Abstract

We investigated the non-leptonic weak interaction in magnetic field. We discussed an improvement of previous method to analytical work out the rate for weak field case. Our result easily goes over to field-free limit. Then we calculated the reaction rate in strong magnetic field where the charged particles are confined to the lowest Landau level. A strong magnetic field strongly suppressed the rate, which will be foreseen to affect viscous dynamics in SQM. We also derived a few approximation formulae under given conditions that can be conveniently applied.

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## 1 Introduction

The composition of comparable number of u, d, and s-quarks are known as strange quark matter (SQM) would be stable or metastable configuration of hadronic matter. The bulk SQM is the  $\beta$ -equilibrium system determined by such a series of weak process:  $u + d \rightarrow s + u$ ,  $d \rightarrow u + e + \bar{\nu}_e$ ,  $s \rightarrow u + e^- + \bar{\nu}_e$  [1]. There has been a lot of interest in the study of the reaction rate and their astrophysical relevance [2, 3, 4, 5, 6, 7]. In the interior of neutron stars, the semi-leptonic reaction is devoted to the cooling of stars while the non-leptonic one to the damping of instability of rapidly rotating stars. We here concern about the non-leptonic process. The previous work [2, 3] were carried out under the usual situation of zero magnetic field. However, the strength of the surface magnetic field of

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a pulsar is typically of order  $10^{12}G$  which concluded from the observation data. Some magnetars are observed to have magnetic field of  $10^{14} \sim 10^{15}G$ . Considering the flux throughout the stars as a simple trapped primordial flux, the internal magnetic field may go up to  $10^{18}G$  or even more [4, 5]. In spite of the fact that we do not know yet any appropriate mechanism to produce more intense field, the scalar virial theorem indeed allows the field magnitude to be as large as  $10^{20}G$ . Therefore, it is advisable to study the effect of the magnetic field on strange quark matter including the calculation of the rate of the reactions. The quark Urca process have been discussed in a few of works[8, 9]. Here we focus on the non-leptonic process leading to bulk viscosity in strange quark matter.

$$u(1) + d \rightarrow u(2) + s \quad (1)$$

In magnetic fields, a classical charged particle will have a transverse cycle motion. In quantum mechanism, the transverse motion is quantized into Landau levels. The quantization effect is important when the magnetic strength is equal or larger than the critical value  $B_m^{(c)} = m_i^2 c^3 / (q_i \hbar)$  defined by equating the cyclotron energy  $qB/(mc)$  to  $mc^2$ . where  $m_i$  and  $q_i$  denoted the mass and charge (absolute value) of the particle [10, 11].  $\hbar, k$  and  $c$  denotes the Planck constant and Boltzman constant and velocity of light respectively, which are taken to be units below. We have considered a wide range of magnetic fields in our study, from "low" to "very high" magnetic fields ( $B \geq 10^{19}G$ ) called respectively "weak" and "strong" field. Since u-quark mass in magnitude of order is smaller than d-quark and s-quark, we assume quantization effect on u-quark is important but that of other flavors are negligible[4]. In the weak field strength situation, we deal with the calculation of the rate through some approximations: (i) the unaffected matrix element; (ii) the approximating free-particle motion direction. In degeneracy, the strong magnetic fields, i.e.  $2qB > p_{F_i}^2$ , force u-quark to occupy the lowest Landau ground state, where  $p_{F_i}$  is the Fermi momentum, we should use the exact solution of Dirac equation in magnetic field to evaluate the reaction rate.

This paper is organized as follows. We solve the the dirac equation in part2. We consider the case of weak field in section 3. And then give the reaction rate in strong field in section 4. Finally, the results are discussed and a summery is made in section 5.

## 2 The solution of Dirac function in magnetic field

We first solve Dirac equation in the presence of a magnetic field  $\mathbf{B}$ . Let the uniform magnetic field  $\mathbf{B}$  along the z-axis, and we choose the asymmetric *Landau gauge*

$$\mathbf{A} = (0, Bx, 0) \quad (2)$$

so that the four-dimensional polarized wave function can be expressed in term of stationary states in the normalization volume  $V = L_x L_y L_z$ . In the magnetic field, the u-quark wave function for fermions ultra-relativistically reads

$$\psi_+(t, \mathbf{x}) = \frac{\exp[-iEt + ip_y y + ip_z z]}{\sqrt{2E(E+m)L_y L_z}} \begin{pmatrix} (E+m)I_{\nu;p_z}(x) \\ 0 \\ p_z I_{\nu;p_z}(x) \\ -i\sqrt{2\nu q B} I_{\nu-1;p_z}(x) \end{pmatrix} \quad (3)$$

for spin up, and

$$\psi_-(t, \mathbf{x}) = \frac{\exp[-iEt + ip_y y + ip_z z]}{\sqrt{2E(E+m)L_y L_z}} \begin{pmatrix} 0 \\ (E+m)I_{\nu-1;p_z}(x) \\ i\sqrt{2\nu q B} I_{\nu;p_z}(x) \\ -p_z I_{\nu-1;p_z}(x) \end{pmatrix} \quad (4)$$

for spin down cases. where the energy  $E = \sqrt{p_z^2 + m^2 + 2\nu q B}$ , and  $\nu$  is the Landau level.

It is degenerate and can be expressed as other quantum numbers such as,  $l$  is the orbital quantum number and  $s$  is the spin quantum number.

$$\nu = l + \frac{1}{2}(1 - s) \quad (5)$$

$q$  is the charge of u-quark. In Eqn(3), (4),

$$\xi = \sqrt{qB} \left(-x + \frac{p_y}{qB}\right) \quad (6)$$

and

$$I_{\nu;p_z} = \left(\frac{qB}{\pi}\right)^{\frac{1}{4}} \exp\left(-\frac{\xi^2}{2}\right) \times \frac{1}{\sqrt{2^\nu \nu!}} H_\nu(\xi) \quad (7)$$

where  $H_\nu$  is Hermite polynomial.

### 3 The rate in weak magnetic field

If the matrix element for the reaction(1) is determined, we can express the rate per volume of reaction(1) as[2, 3]:

$$\Gamma(u_1 d \rightarrow s u_2) = \frac{36}{2} [\prod_i \int \frac{d^3 p_i}{(2\pi)^3 2E_i}] |M_s|^2 S (2\pi)^4 \delta^4(P_1 + P_d - P_2 - P_s) \quad (8)$$

where the phase space integrals are to be calculated over all particle states, the statistical distribution function  $S = f_1 f_d (1 - f_2) (1 - f_s)$ , the quark in equation(1) are described by Fermi-Dirac distributions in the form

$$f_i(E_i) = [1 + \exp(\frac{E_i - \mu_i}{T})]^{-1}, i = 1, 2, d, s \quad (9)$$

where  $\mu_i$  are the chemical potential.

We consider the reaction (1) when the magnetic field is not strong enough to force the quark into the lowest Landau level. Previous study [5] shows that the matrix element for the weak process remains unaffected and only the phase factor is modified.

The matrix element summed over final spins and averaged over initial spins is given by [2]:

$$|M_s|^2 = 64 G_F^2 \sin^2 \theta_c \cos^2 \theta_c (P_1 \cdot P_d) (P_2 \cdot P_s) \quad (10)$$

here  $P_i = (E_i - \mathbf{p}_i)$  is the four-momentum of the quark  $i$  and  $G_F = 1.166 \times 10^{-11} Mev^{-2}$  is the Fermi constant,  $\theta_c$  is the Cabibbo angle ( $\cos^2 \theta_c = 0.948$ ). We neglect the masses of up and down quarks, then  $E_1 = p_1, E_2 = p_2, E_d = p_d, E_s = (p_s^2 + m_s^2)^{1/2}$ . so the four-momentum products can be written as[2, 3]

$$(P_1 \cdot P_d) (P_2 \cdot P_s) = E_1 E_2 E_d E_s (1 - \cos \theta_{1d}) (1 - \frac{p_s}{E_s} \cos \theta_{2s}) \quad (11)$$

where  $\theta_{ij}$  denotes the angle between quarks  $i$  and  $j$ .

Now we can calculate the rate of weak process in the weak magnetic field by replacing the u-quark phase space factor[5]

$$2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \rightarrow \frac{qB}{(2\pi)^2} \sum_{\nu=0}^{\nu_{max}} (2 - \delta_{\nu,0}) \int dp_z \quad (12)$$

so the reaction rate can be write as:

$$\Gamma(u_1 d \rightarrow s u_2) = \frac{18}{(2\pi)^6} G_F^2 \sin^2 \theta_c \cos^2 \theta_c (eB)^2 \sum_{\nu_1=0}^{\nu_{max}} (2 - \delta_{\nu_1,0}) \sum_{\nu_2=0}^{\nu_{max}} (2 - \delta_{\nu_2,0}) \int p_d^2 dp_d p_s^2 dp_s \int dp_{1z} \int dp_{2z} S \delta(E_1 + E_d - E_2 - E_s) I \quad (13)$$

where

$$I = \int \left( \prod_i^{d,s} \right) d\Omega_i (1 - \cos \theta_{1d}) \left( 1 - \frac{p_s}{E_s} \cos \theta_{2s} \right) \delta^3(\mathbf{p}_1 + \mathbf{p}_d - \mathbf{p}_2 - \mathbf{p}_s) \quad (14)$$

As we known ,the angle between two vector is a function of the respective inclinations and azimuth angle in spherical coordinates, ie.

$$\begin{aligned} \cos \theta_{1d} &= \cos \theta_1 \cos \theta_d + \sin \theta_1 \sin \theta_d \cos(\varphi_1 - \varphi_d) \\ \cos \theta_{2s} &= \cos \theta_2 \cos \theta_s + \sin \theta_2 \sin \theta_s \cos(\varphi_2 - \varphi_s) \end{aligned} \quad (15)$$

In magnetic field,the angles of the polarized quarks, $\theta_1, \theta_2, \varphi_1, \varphi_2$  vary with Landau level ( $\nu_1, \nu_2$ ) and z-component momentum ( $p_{1z}, p_{2z}$ ) . Therefore the integrals and summations in Eqn(12) become very difficult due to the coupling of variables  $\nu$  and  $p_z$  into the integrated function. We need to make a improvement on chakrabaty's approach. Fortunately, we can approximately regard the angles as independent variables for the weak field situation and then the integrals and summations decouples each other, because the kinetic quark direction should have only a small deviation from the field-free case,although the modification of the absolute value of the momentum considered under the situations.It completely coincides with free-particle's matrix approximation described by Eqn(10).

We immediately have:

$$2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \longrightarrow \frac{qB}{(2\pi)^3} \sum_{\nu=0}^{\nu'_{max}} (2 - \delta_{\nu,0}) \int' dp_z \int' d\Omega \quad (16)$$

where  $\sum'$  and  $\int'$  denote the summation and integral independent of the direction angles. In this approximation , which may be called free-particle direction average(FDA). Eqn(13) and Eqn(14) read:

$$\begin{aligned} \Gamma(u_1 d \rightarrow s u_2) &= \frac{18}{(2\pi)^8} G_F^2 \sin^2 \theta_c \cos^2 \theta_c (eB)^2 \sum_{\nu_1=0}^{\nu_{1max}} (2 - \delta_{\nu_1,0}) \sum_{\nu_2=0}^{\nu_{2max}} (2 - \delta_{\nu_2,0}) \\ &\int p_d^2 dp_d p_s^2 dp_s \int dp_{1z} \int dp_{2z} S \delta(E_1 + E_d - E_2 - E_s) I' \end{aligned} \quad (17)$$

where

$$I' = \int \left( \prod_i^{1,2,d,s} \right) d\Omega_i (1 - \cos \theta_{1d}) \left( 1 - \frac{p_s}{E_s} \cos \theta_{2s} \right) \delta^3(\mathbf{p}_1 + \mathbf{p}_d - \mathbf{p}_2 - \mathbf{p}_s) \quad (18)$$

Since  $\mu_i \gg T$  ,only those fermions whose momenta lie close to their respect Fermi surface can take part in the reaction. We use the method in [6, 7]to complete partially the

integral of  $I'$  through

$$\delta^3(\mathbf{p}_1 + \mathbf{p}_d - \mathbf{p}_2 - \mathbf{p}_s) = \int \frac{d^3\mathbf{x}}{(2\pi^3)} \exp(i\mathbf{p} \cdot \mathbf{x}) \quad (19)$$

can be written as:

$$I' = \frac{2^7 \pi^2}{p_{F_1} p_{F_2} p_{F_d} p_{F_s}} \int \frac{dx}{x^2} \left[ \prod_i^{1,2,d,s} \sin(p_{F_i} x) + a \sin(p_{F_1} x) \sin(p_{F_d} x) f(p_{F_2} x) f(p_{F_s} x) + \right. \\ \left. \sin(p_{F_2} x) \sin(p_{F_s} x) f(p_{F_1} x) f(p_{F_d} x) + a \prod_i^{1,2,d,s} f(p_{F_i} x) \right] \quad (20)$$

$$\equiv \frac{2^7 \pi^2}{p_{F_1} p_{F_2} p_{F_s}} J \quad (21)$$

where  $a = \frac{p_{F_s}}{\mu_s}$ ,  $f(p_{F_i} x) = \cos(p_{F_i} x) - \frac{\sin(p_{F_i} x)}{p_{F_i} x}$  and the integral  $J$  is defined through  $Eqn(20)$  and  $Eqn(21)$ , can be calculated numerically.

The net rate of transforming d-quark into s-quark is [2]

$$\Gamma(d \rightarrow s) = [1 - \exp(\frac{\mu_d - \mu_s}{T})] \Gamma(u_1 d \rightarrow s u_2) \quad (22)$$

Using the method in [7, 12] and substitute  $Eqn(17)$ , (18) and (21) into (22) we can get

$$\Gamma(d \rightarrow s) = \frac{3}{2\pi^6} G_F^2 \sin^2 \theta_c \cos^2 \theta_c (qB)^2 \sum_{\nu_1=0}^{\nu_{1max}} (2 - \delta_{\nu_1,0}) \sum_{\nu_2=0}^{\nu_{2max}} (2 - \delta_{\nu_2,0}) \\ \frac{\mu_d^2 \mu_s}{\sqrt{\mu_1^2 - 2\nu_1 qB} \sqrt{\mu_2^2 - 2\nu_2 qB}} \Delta\mu (\Delta\mu^2 + 4\pi^2 T^2) J \quad (23)$$

where

$$\nu_{imax} = \text{Int}(\frac{\mu_i^2}{2qB}), i = 1, 2, \Delta\mu = \mu_d - \mu_s \quad (24)$$

when the  $B \rightarrow 0$  the sum can be replaced by integral of  $\nu$  and then the result goes over to the expression:

$$\Gamma(d \rightarrow s) = \frac{6}{\pi^6} G_F^2 \sin^2 \theta_c \cos^2 \theta_c \mu_d^2 \mu_u^2 \mu_s \Delta\mu (\Delta\mu^2 + 4\pi^2 T^2) J \quad (25)$$

It is just the field-free case.

## 4 The rate of weak process in strong magnetic field

Now we consider strong magnetic field effect on the non-leptonic weak interaction in this section. In the case of strong magnetic field that  $B \geq B_m^{(u)}$ , all u-quarks occupy the lowest

Landau ground state with the u-quark spins pointing in the direction of the magnetic field. We treat other flavors in this process as free particle which do not affected by the magnetic field. The matrix element for the reaction reads:

$$M = \frac{G_F \sin \theta_c \cos \theta_c}{\sqrt{2}} \int \bar{\psi}_2 \gamma_\mu (1 - \gamma_5) \psi_d \bar{\psi}_s \gamma^\mu (1 - \gamma_5) \psi_1 d^4x \quad (26)$$

d,s quark treat as free particle ,then we get

$$\psi_i = \frac{1}{V^{\frac{1}{2}}} \exp(-iP_i \cdot r) U_i \quad (27)$$

$$U_i = \sqrt{\frac{E_i + m_i}{2E_i}} \begin{pmatrix} 1 \\ 0 \\ \frac{p_{z_i}}{E_i + m_i} \\ \frac{p_{x_i} + ip_{y_i}}{E_i + m_i} \end{pmatrix} \quad (28)$$

where  $P_i$  denotes four-dimensional momentum,  $i = d, s$ .

Consider  $\nu = 0$ , the wave function Eqn(3)(4) of u-quark can be written as

$$\psi_1 = \frac{1}{\sqrt{L_y L_z}} \exp(-iE_1 t + ip_{y_1} y_1 + ip_{z_1} z_1) \left(\frac{qB}{\pi}\right)^{\frac{1}{4}} \exp\left[-\frac{qB}{2} \left(-x + \frac{p_{y_1}}{qB}\right)^2\right] U_1 \quad (29)$$

$$\psi_2 = \frac{1}{\sqrt{L_y L_z}} \exp(-iE_2 t + ip_{y_2} y_2 + ip_{z_2} z_2) \left(\frac{qB}{\pi}\right)^{\frac{1}{4}} \exp\left[-\frac{qB}{2} \left(-x + \frac{p_{y_2}}{qB}\right)^2\right] U_2 \quad (30)$$

where

$$U_1 = \frac{1}{\sqrt{2E_1(E_1 + m_{u_1})}} \begin{pmatrix} E_1 + m \\ 0 \\ p_{z_1} \\ 0 \end{pmatrix} \quad (31)$$

$$U_2 = \frac{1}{\sqrt{2E_2(E_2 + m_{u_2})}} \begin{pmatrix} E_2 + m \\ 0 \\ p_{z_2} \\ 0 \end{pmatrix} \quad (32)$$

for spin up.

We now calculate the matrix elements squared and summed over the initial state and average over the final state. We use the Eqn(29), Eqn(32) and Eqn(33) to express corresponding matrix element:

$$|M_s|^2 = [\bar{U}_2 \gamma_\mu (1 - \gamma_5) U_d \bar{U}_s \gamma^\mu (1 - \gamma_5) U_1] [\bar{U}_2 \gamma_\mu (1 - \gamma_5) U_d \bar{U}_s \gamma^\mu (1 - \gamma_5) U_1]^\dagger$$

And then immediately get:

$$|M_s|^2 = \frac{1}{2E_1^2 E_2^2 E_d E_s} (E_1 - p_{z_1})^2 (E_2 + p_{z_2})^2 (E_d + p_{z_d}) (E_s - p_{z_s}) \quad (33)$$

Here we set  $m_d = m_u = 0$ , and then  $p_{z_1} < 0, p_{z_2} > 0$ , otherwise  $|M_s|^2 = 0$ . Carrying out the integral, we obtain

$$|M|^2 = \frac{G_F^2 \sin^2 \theta_c \cos^2 \theta_c}{2V L_x^2 (L_y L_z)^3} (2\pi)^3 |M_s|^2 \exp\left[-\frac{(p_{y_1} - p_{y_2})^2 - (p_{x_d} - p_{x_s})^2}{2qB}\right] \delta(E_2 + E_s - E_1 - E_d) \delta(p_{y_1} + p_{y_d} - p_{y_2} - p_{y_s}) \delta(p_{z_1} + p_{z_d} - p_{z_2} - p_{z_s}) \quad (34)$$

The rate per volume of the reaction is given by:

$$\Gamma(u_1 d \rightarrow s u_2) = \frac{n_1 n_d}{2} \int \frac{V d^3 \mathbf{p}_d}{(2\pi)^3} \int \frac{V d^3 \mathbf{p}_s}{(2\pi)^3} \int_{-\frac{qBL_x}{2}}^{\frac{qBL_x}{2}} \frac{L_y}{2\pi} dp_{y_1} \int_{-\frac{qBL_x}{2}}^{\frac{qBL_x}{2}} \frac{L_y}{2\pi} dp_{y_2} \int_{-\infty}^{\infty} \frac{L_y}{2\pi} dp_{z_1} \int_{-\infty}^{\infty} \frac{L_y}{2\pi} dp_{z_2} |M|^2 f_1 f_d (1 - f_2) (1 - f_s) \quad (35)$$

where the factor  $n_1 = n_d = 6$  comes from 2 spins and 3 colors. Since only left-hand helicity states of  $u_1 - quark$  couples to the  $W^-$  ( $W^-$  is the mediate of the reaction), a factor of  $\frac{1}{2}$  is shown in the Eqn(35). The integrals over  $dp_{y_1}$  and  $dp_{y_2}$  can be carried out using the y-component momentum delta function. The integrals over  $dp_{z_1}$  and  $dp_{z_2}$  are converted into  $dE_1$  and  $dE_2$  respectively[4, 5].

Since  $\mu_s \gg T$ , only those momenta lie close to their respective Fermi surfaces can take part in the reaction. As the z-component momentum conservation, we can get

$$p_{F_1} + p_{F_d} \cos \theta_d - p_{F_2} - p_{F_s} \cos \theta_s = 0 \quad (36)$$

We approximately set  $p_{F_u} = p_{F_d} = p_{F_s}$  near the equilibrium. Then we can get  $\cos \theta_d - \cos \theta_s = 2$ , so we can take out the integral over d-quark and s-quark's momentum space.

Substituting Eqn(35) into Eqn(22), we obtain

$$\Gamma(d \rightarrow s) = \frac{G_F^2 \sin^2 \theta_c \cos^2 \theta_c (qB)}{4\pi^5} \exp\left[\frac{(2\mu_u)^2 - (\mu_d + p_{F_s})^2}{2qB}\right] (3\mu_s + 6\mu_u - \mu_d) \mu_d^2 \Delta \mu (\Delta \mu^2 + 4\pi^2 T^2) \quad (37)$$

where  $\Delta \mu = \mu_d - \mu_s, p_{F_s} = \sqrt{\mu_s^2 - m_s^2}$ .



## 5 Discussion and Conclusion

We have done calculations of the rate of non-leptonic quark weak process in magnetic field and give the analytic solutions under weak-field and strong-field approximation. Based on these results , we express the net rate of the transforming d-quark to s-quark in a unified form:

$$\Gamma(d \rightarrow s) = \Gamma_k(n_b, qB) \Delta\mu (\Delta\mu^2 + 4\pi^2 T^2) \quad (38)$$

where  $k$  takes 0,  $L$ ,  $H$  denotes zero-field, weak-field and strong-field cases respectively. In according with the formula(25), the result go over to the field-free case when the magnetic field strength vanishes. We thus have:

$$\Gamma_0(n_b) = \frac{16}{5} G_F^2 \sin^2 \theta_c \cos^2 \theta_c \left(\frac{n_b}{\pi}\right)^{\frac{5}{3}} \quad (39)$$

meanwhile for  $2q_u B < \mu_u^2$ , the  $\Gamma_L$  reads simply from Eqn(23)

$$\Gamma_L(n_b, qB) = \frac{144}{\pi^4} G_F^2 \sin^2 \theta_c \cos^2 \theta_c q_u B n_b J \frac{\nu_m^{\frac{3}{2}}}{3\nu_m^{\frac{1}{2}} + 4(\nu_m - 1)^{\frac{3}{2}} + 8\nu_m^{\frac{3}{2}}} \quad (40)$$

where  $J$  as a function of  $\mu$  define in figure(1). We find  $\Gamma_L$  has a small deviation from field-free case. The comparison is made in figure(2).

The simplified formula of  $\Gamma_H(n_b, qB)$  need slightly complicated calculations for  $2q_u B \geq \mu_u^2$

In the referred SQM ( $\mu_i \gg m_i$ , only u-quark polarized) here , the number density of the components read:

$$n_{d,s} = \frac{\mu_{d,s}^3}{\pi^2} \quad (41)$$

$$n_u = \frac{3q_u B \mu_u}{2\pi^2} \quad (42)$$

the  $\beta$ -equilibrium

$$\mu_d = \mu_s = \mu, \mu_u = \mu - \mu_e \quad (43)$$

the charge neutrality

$$2n_u - n_d - n_s - 3n_e = 0 \quad (44)$$

the baryon number density conservation

$$n_b = \frac{1}{3}(n_d + n_u + n_s) \quad (45)$$

should be satisfied. Then we solve these equation numerically to obtain the chemical potential of the quarks with respective magnetic field.

Figure(3) shows that the chemical potentials of quarks are nearly equal under this situation. Combines Eqn(41)(42)(44) and (45) in the approximation of  $\mu_u \approx \mu_d = \mu_s = \mu$ ,  $\mu$  can be solved analytically through the algebraic equation

$$\mu^3 + \frac{3}{4}q_u B \mu - \frac{\pi^2}{2}n_b = 0 \quad (46)$$

and then  $\Gamma_H(n_b, qB)$  is arrived at

$$\Gamma_H = \frac{G_F^2 \sin^2 \theta_c \cos^2 \theta_c (qB)}{4\pi^5} \left[ \frac{q_u B}{(-6n_b \pi^2 + \sqrt{36n_b^2 \pi^4 + (q_u B)^3})^{\frac{1}{3}}} - (-6n_b \pi^2 + \sqrt{36n_b^2 \pi^4 + (q_u B)^3})^{\frac{1}{3}} \right]^3 \quad (47)$$

Figure(4) give a comparison of Eqn(47) with Eqn(37). They fit well each other with a small error.

The formula (47) will reduce to

$$\Gamma_H = \frac{24G_F^2 \sin^2 \theta_c \cos^2 \theta_c}{11\pi^3} q_u B n_b \quad (48)$$

for  $2qB \sim \mu^2$ , and tend to the limit

$$\Gamma_H = \frac{16\pi G_F^2 \sin^2 \theta_c \cos^2 \theta_c n_b^3}{(q_u B)^2} \quad (49)$$

for  $2qB \gg \mu^2$ .

Figure(5) give a comparison of Eqn(47) with Eqn(48) and Eqn(49). We find they will be some good approximations in future realistic application.

Under the consideration that u-quark is polarized but effect of other flavors are negligible, we investigate the influence of magnetic field on the non-leptonic rate although the result for the weak field case has a small deviation from the field-free case, we give an analytical treatment of the weak reaction which can be extended to the calculations of other reaction process. However, the strong magnetic field can extremely suppress the rate. It is possible to lead the decrease of bulk viscosity in magnetized SQM. This may have serious implications for compact star and pulsar dynamics. In fact, we should also distinguish d-quark and s-quark in calculations to obtain refining result for applications. It is our future works.

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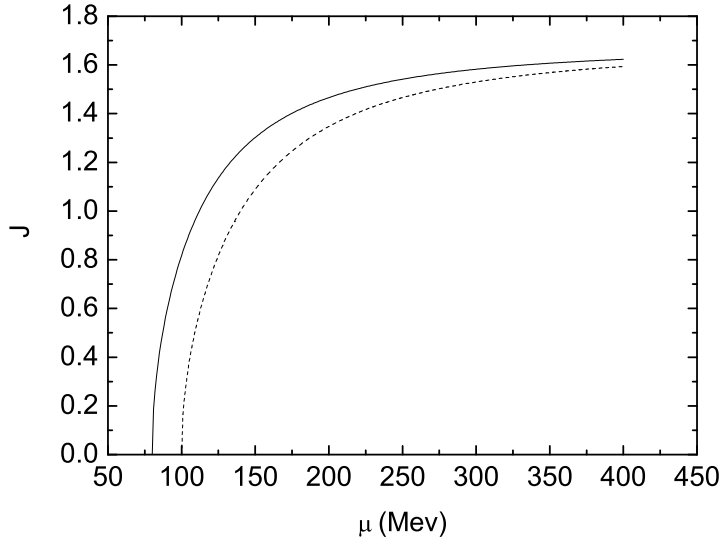


Figure 1: numerical result of  $J$ , given by Eqn(20) and (21), show as a function of  $\mu$  for various value of the parameters  $m_s$ . The solid curve is for  $m_s = 80\text{MeV}$ , The dot curve is for  $m_s = 100\text{MeV}$ .

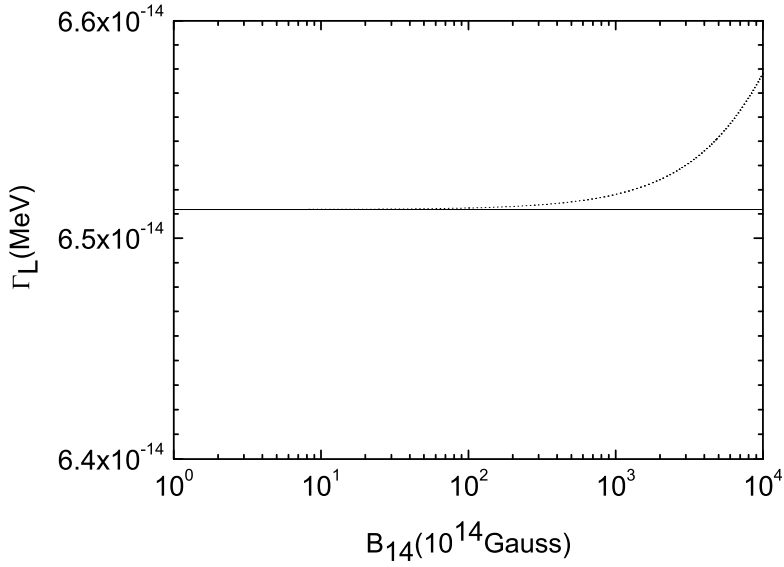


Figure 2: Derivation of  $\Gamma_L$  from  $\Gamma_0$  with  $B$  when  $n_b = 0.2\text{fm}^{-3}$ .

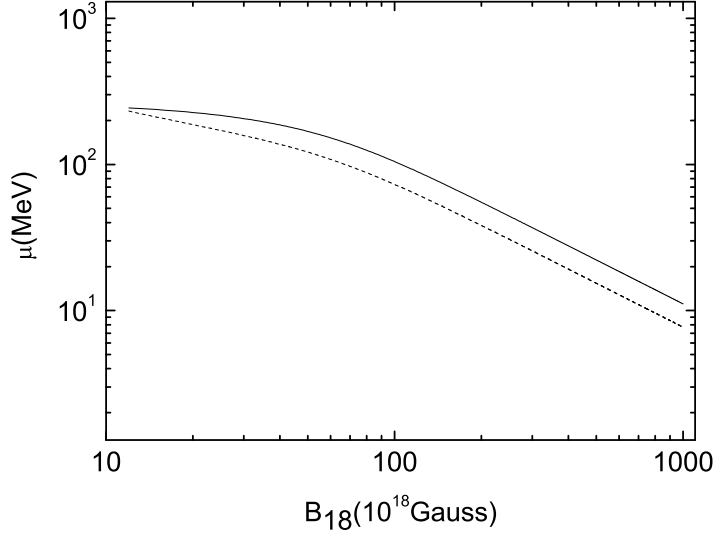


Figure 3: numerical result of  $\mu$  as a function of  $\mathbf{B}$  which get from Eqn(46)(47)(48). The dot curve is for  $\mu_u$  when  $n_B = 0.2 fm^{-3}$ . The solid curve is for the case when  $\mu = \mu_d = \mu_s$ .

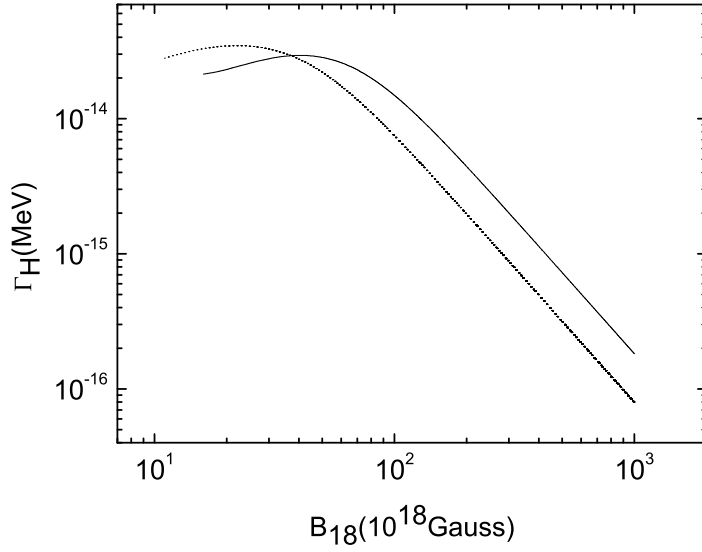


Figure 4:  $\Gamma_H$  as a function of  $B$  when  $n_b = 0.2 fm^{-3}$ . The solid curve is for the result of Eqn(37) and the dot curve is for the result of Eqn(47).

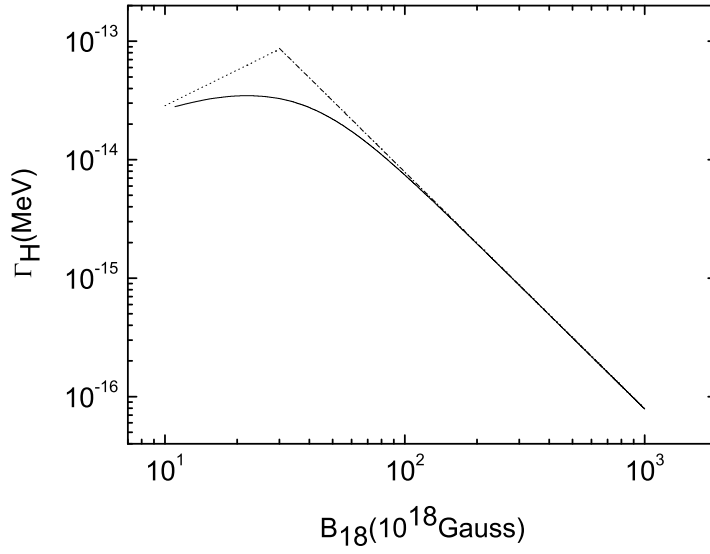


Figure 5:  $\Gamma_H$  as a function of  $B$  when  $n_b = 0.2 fm^{-3}$ . The solid curve is for the result of Eqn(47) and the dot curve is for the result of Eqn(48), the dash dot curve is for the result of Eqn(49).