Helium and Deuterium Abundances as a Test for the Time Variation of the Baryonic Density, Fine Structure Constant and the Higgs Vacuum Expectation Value

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ABSTRACT

We use the semi-analytic method of Esmailzadeh et al. (1991) to calculate the abundances of Helium and Deuterium produced during Big Bang nucleosynthesis assuming the fine structure constant and the Higgs vacuum expectation value may vary in time. We analyze the dependence on the fundamental constants of the nucleon mass, nuclear binding energies and cross sections involved in the calculation of the abundances. Unlike previous works, we do not assume the chiral limit of QCD. Rather, we take into account the quark masses and consider the one-pion exchange potential, within perturbation theory, for the proton-neutron scattering. However, we do not consider the time variation of the strong interactions scale but attribute the changes in the quark masses to the temporal variation of the Higgs vacuum expectation value. Using the observational data of the helium and deuterium, we put constraints on the variation of the fundamental constants between the time of nucleosynthesis and the present time.

1. Introduction

Big Bang Nucleosynthesis (BBN) offers the deepest reliable probe of the early universe. Predictions of the abundances of the light elements D,^{3}He, ^{4}He and ^{7}Li synthesized at the end of the 'first three minutes' are in good overall agreement with the primordial abundances inferred from observational data, which validates the Standard Big Bang Nucleosynthesis (SBBN). BBN also provides powerful constraints on possible deviations from the standard cosmology and on new theories on physics beyond the Standard Model (SM) (Sarkar 1996). Among these theories, there are those in which some of the dimensionless ratios of fundamental constants do vary in time like string-derived field theories (Wu and Wang

1986; Maeda 1988; Barr and Mohapatra 1988; Damour and Polyakov 1994; Damour et al. 2002a,b), related brane-world theories (Youm 2001a,b; Palma et al. 2003; Brax et al. 2003), and (related or not) Kaluza-Klein theories (Kaluza 1921; Klein 1926; Weinberg 1983; Gleiser and Taylor 1985; Overduin and Wesson 1997). On the other hand, recent astronomical data (Webb et al. 1999, 2001; Murphy et al. 2001a,b, 2003) suggest a possible variation of the fine structure constant $\alpha = e^2/\hbar c$ at the 10^{-5} level over a time-scale of 10 billion years. However, other recent independent analysis of similar data (Martínez Fiorenzano et al. 2003; Quast et al. 2004; Bahcall et al. 2004; Srianand et al. 2004; Grupe et al. 2005; Chand et al. 2006) found no variation. On the other hand, measurements of molecular hydrogen (Ivanchik et al. 2002, 2003, 2005) reported a variation of the proton to electron mass $\mu = \frac{m_p}{m_e}$. This fact motivated more general discussions of possible variations of other constants. Calmet and Fritzsch (2002) and Langacker et al. (2002) have studied the implication of gauge unification for the time variation of α while Olive et al. (2002) explored a super-symmetric version of the dynamical Bekenstein model (Bekenstein 1982) in order to produce a large change in α in the redshift range $z = 0.5 - 3.5$ and still be consistent with the constraints on $\Delta\alpha/\alpha$ from the results of high precision limits on the violation of equivalence principle by a fifth force.

On the other hand, there are many non-SBBN models which introduce new free parameters in addition to the baryon density parameter, or equivalently the baryon asymmetry $\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = 2.74 \times 10^{-8}\Omega_B h^2$. Most known of these models are those which assume either a non-standard contribution to the total density, or a lepton asymmetry. The first possibility affects the expansion rate of the universe $S \equiv \frac{H'}{H} = \sqrt{\frac{\rho'}{\rho}}$ and can be restated in terms of 'equivalent' number of extra neutrinos $\Delta N_\nu = N_\nu - 3$. Simple analytic fits to BBN and the cosmic microwave background radiation (CMBR) data provide the following bound: $0.85 < S < 1.15$ (Barger et al. 2003a,b; Steigman 2005, 2006). As regards the lepton asymmetry, observational data do not imply that is should be connected to the 'tiny' baryon

asymmetry η_B, and it could be large enough to perturbe SBBN predictions . Moreover, a small asymmetry between electron type neutrinos and antineutrinos can have a significant impact on BBN since the ν_e affect the interconversion of neutrons to protons changing the equilibrium neutron-to-proton ratio from $(n/p)_{eq}^0 = e^{-\frac{\Delta m}{T}}$ to $(n/p)_{eq} = (n/p)_{eq}^0 e^{-\xi_e}$. In consequence, the ^{4}He abundance changes. In contrast the D abundance is insensitive to $\xi_e \neq 0$. Consistent with the BBN and CMBR data, values of ξ_e in the range $-0.1 < \xi_e < 0.3$ are permitted (Barger et al. 2003a; Steigman 2005, 2006). In our analysis, however, we shall not consider these non-SBBN scenarios, but attribute any non-SBBN issue to time-variation of fundamental constants.

The density of baryonic matter $\Omega_B h^2$ can be estimated using the WMAP data from the CMBR (Spergel et al. 2003, 2006). From the observed WMAP baryon density, the predicted abundances are highly consistent with the observed D but not with ^{4}He and ^{7}Li. However, any change in the value of the fundamental constants would work its way into the value of the abundances of the various light elements and the question we address is whether or not existing observations of the primordial abundances suggest any change in the values of the fundamental constants at the time of BBN.

BBN is sensitive to a number of fundamental dimensionless parameters including the fine structure constant α , Λ_{QCD}/M_{Plank} and m^q/Λ_{QCD} where m^q is the quark mass and Λ_{QCD} is the strong scale determined by the position of the pole in the perturbative QCD running coupling constant. Several authors have studied the dependencies of the BBN parameters on the fundamental constants. The dependence of the primordial abundances on the fine structure constant has been evaluated by Bergström et al. (1999) and improved by Nollett and Lopez (2002). Yoo and Scherrer (2003) analyze the effects of the time variation of the higgs vacuum expectation value $< v >$ on BBN and the cosmic microwave background radiation (CMBR). On the other hand, Müller et al.

(2004) calculate the abundances as function of the Planck Mass M_P, α, $<v>$, electron mass m_e, nucleon decay time, deuterium binding energy (ϵ_D) and neutron-proton mass difference ($\Delta m = m_n - m_p$). Moreover, they study the dependence of the last three quantities as functions of the fundamental couplings and masses. Kneller and McLaughlin (2003) study the dependence of the primordial abundances on ϵ_D and Δm. These papers (Yoo and Scherrer 2003; Müller et al. 2004; Kneller and McLaughlin 2003) use the results of chiral perturbation theory (Beane and Savage 2003) to adress the dependence of the deuterium binding energy with the higgs vacuum expectations value and Λ_{QCD}. However, the dependence of the deuterium binding energy ϵ_D on $<v>$ was estimated from an approximated linear dependence of ϵ_D on the pion mass m_π, while the exact limits on the relative change of $<v>$ would depend on the details of such dependence. On the other hand, Campbell and Olive (1995) and Ichikawa and Kawasaki (2002) study the effects of variation of fundamental constants on BBN in the context of a dilaton superstring model. Finally, limits on cosmological variations of α, Λ_{QCD} and m^q from optical quasar absorption spectra, laboratory atomic clocks and from BBN have been established by Flambaum and Shuryak (2002); Flambaum et al. (2004). For computing the deuterium binding energy ($\epsilon_D \approx 2.225\text{MeV}$) they apply quantum mechanics perturbation theory. This factor is very significant in influencing the reaction rate of $p + n \rightarrow d + \gamma$ which is the first and most crucial step in BBN.

The BBN abundances can be computed using numerical (Wagoner 1973; Kawano 1992) and analytical (Esmailzadeh et al. 1991; Mukhanov 2003) methods. In a previous work (Landau et al. 2006), we used the semi-analytic method of Esmailzadeh et al. (1991) to calculate the abundances of the light elements produced during BBN assuming that the gauge coupling constants may vary in time. We considered the chiral limit of QCD when analyzing the nucleon masses, binding energies and the cross sections. Deviations between standard cosmology calculations and observational data could be interpreted as resulting

from variations in G_F the Fermi constant, α the fine structure constant and Λ_{QCD} the strong interactions scale. The semi-analytical method allows us to obtain semi-analytic dependencies of the primordial abundances on the fundamental constants, which otherwise must be computed using numerical codes.

On the other hand, in the standard model, a variation of the Fermi constant implies a variation of the vacuum expectation value (vev) of the Higgs field (Dixit and Sher 1988):

$$G_F = \frac{\alpha_{weak}(M_W)}{\sqrt{2}M_W^2} = \frac{1}{\sqrt{2} < v >^2} \tag{1}$$

Here M_W is the mass of the W-boson, and $< v > \approx 250\text{GeV}$ is the vev of the Higgs field. Within the QCD chiral limit, the quark masses, which are also proportional to the Higgs vev through the relation $m^q = Y_{Yukawa} < v >$, are neglected. Therefore, the logical and consistent step to follow is to go beyond the chiral limit and take the variation of $< v >$ as affecting the Fermi constant as well as the quark masses. We analyze the nucleon masses, the nuclear binding energies and the cross sections dependence within quantum mechanics perturbation theory. The objective of this paper is to study such variations as model independent as possible. Therefore, we consider the one-pion exchange potential as the perturbation on the p-n scattering responsible for the formation of the deuterium. This perturbation potential varies in time if the pion mass changes in time which leads to a time variation of the deuterium binding energy. The pion mass also depends on the Higgs vev through the Gell-Mann-Oakes-Renner relation: $m_\pi^2 = m^q \frac{|<\bar{q}q>|}{f_\pi^2} \simeq m^q \Lambda_{QCD}$, where $< \bar{q}q >$ is the quark condensate and f_π is the pion decay constant. In order to determine the dependence of the deuterium binding energy on the fundamental constants, we use the square well model to approximate the attractive strong interaction potential of the deuterium and fit current scattering data to get estimates for the depth and width of the well. On the other hand, we will not discuss the effect of Λ_{QCD} variation on the

QCD-determined quantities such as the quark condensate or the width and depth of the square well. The reason for this is that we lack a complete theory for these quantities, and, especially, because of the absence of p-n scattering data in the far past. For the same reason, we will not consider changes in the Yukawa couplings either. On the other hand, the effect of of Λ_{QCD} variation on the abundances of the light elements was analyzed in a previous work (Landau et al. 2006). Thus, we will limit ourselves in this paper to study the dependence on α and $< v >$ for the physical quantities, such as binding energies, nucleon masses and cross sections involved in the BBN calculations. Our treatment of the deuterium binding energy is similar to the one performed by of Flambaum and Shuryak (2002). However, there are some technical differences in the wave function normalization which we describe in section 4. Furthermore, we go one step further in calculating the effects of such variations in the fundamental constants on the primordial abundances of D and ^4He. This is one of the advantages of using the semi-analytical method and it also allows us to compare with observational data in order to put bounds on the variation of α and $< v >$.

On the other hand, the concordance between the WMAP estimates and SBBN has been investigated by many authors (Cyburt et al. 2003; Romano et al. 2003; Cuoco et al. 2004; Cyburt 2004; Coc et al. 2004a,b). From the WMAP baryon density , the predicted abundances are highly consistent with the observed D but not with ^{4}He and ^{7}Li. They are produced more than observed. Such discrepancy is usually ascribed to non reported systematic errors in the observations of ^{4}He and ^{7}Li. Indeed, more realistic determinations of the ^{4}He uncertainty implies a baryon density in line with the WMAP estimate (Cyburt 2004; Olive and Skillman 2004). On the other hand, Richard et al. (2005) have pointed out that a better understanding of turbulent transport in the radiative zones of the stars is needed for a better determination of the ^{7}Li abundance. In our previous work, we obtained results consistent with variation of fundamental constants when considering all data.

However, discarding the ^{7}Li data we obtained no variation. Therefore, we suspect that the possible non reported systematic uncertainties are "hidden" within a setup involving variation of the fundamental constants. Thus, until better estimations of the systematic errors of ^{7}Li are reported, we will only consider the D and ^{4}He data.

Even though the WMAP estimate of the baryon density is the most accurate one, it is still affected by degeneracies with other cosmological parameters (Spergel et al. 2003, 2006). On the other hand, this quantity can be also determined combining data from galaxy surveys (SDSS, 2dF) and x-ray satellites (Chandra, XMM-Newton, ROSAT, ASCA) (Landau et al. 2006). In this work, we consider a weighted mean between the WMAP estimate and Landau et al. (2006) estimate for $\Omega_b h^2$, and, furthermore, we shall compute the dependence of binding energies, cross sections and abundances on this parameter. Finally, we shall use observational data from D and ^{4}He to estimate the variations in time of α and $< v >$ and a possible deviation of $\Omega_b h^2$ from its considered value. We also compare our results with other non-SBBN models, where a non-standard expansion rate and an electron-neutrino asymmetry were considered. Finally, we would like to emphasize that the approach in this work is phenomenological and the results we get are model independent.

The paper is organized as follows. In section 2 we present the notations used and summarize the steps which one follows in the semi-analytic approach to calculate the abundances. In section 3 we calculate the dependence of the abundances on α, $< v >$, $\Omega_b h^2$ and the deuterium binding energy ϵ_D. In section 4, we express the dependence of the deuterium binding energy on the Higgs vev within the square well model. Results of comparing theoretical prediction with observational values are presented in section 5, where we also compare with other non-standard BBN models results. Conclusions are presented in section 6.

2. Preliminaries

The method of Esmailzadeh et al. (1991) consists of calculating the different abundances between fixed points or stages. One solves the equations for the light elements only for one element in each stage. For the other elements (say, the ith), it is necessary to solve the quasi static equilibrium (QSE) equation ($\dot{Y}_i \approx 0$), where Y_i is the abundance of the ith element relatively to baryons, considering only the most important rates of production and destruction. On the other hand, we should also calculate the final temperature of each stage. We show in Table 1 the different stages and their corresponding equations, to which is added also the conservation of the neutron number (further details are given in Esmailzadeh et al. (1991); Landau et al. (2006)).

Table 1: Stages and equations. n refers to neutron, p to proton, d to deuterium, T to tritium, 3 to ^{3}He and α to ^{4}He

Stage	Equations	Final temperature
Until the weak interaction freeze-out		
Until the production of ^{4}He becomes efficient	$\dot{Y}_n = -2\dot{Y}_\alpha - Y_n[n]$ $\dot{Y}_d = \dot{Y}_3 = \dot{Y}_T = 0$	$2\dot{Y}_\alpha \sim Y_n[n]$
Until the production of deuterium dominates rate of change of neutrons	$\dot{Y}_n = -2\dot{Y}_\alpha$ $\dot{Y}_d = \dot{Y}_3 = \dot{Y}_T = 0$	$Y_n = Y_d$
Deuterium final abundance	$\dot{Y}_d = -2\dot{Y}_\alpha$ $\dot{Y}_n = \dot{Y}_3 = \dot{Y}_T = 0$	$T_9 \to 0$

Since we are considering changes in the Higgs vev ($< v >$) and the fine structure constant (α), we need to find expressions for the nucleon masses and binding energies in terms of these quantities. For the P-N mass splitting we have

$$Q = \Delta m = m_n - m_p = \Delta^\alpha m + \Delta^{\rho-w} m \tag{2}$$

where $\Delta^\alpha m$ is the contribution of the electromagnetic energy, and thus $\frac{\delta \Delta^\alpha m}{\Delta^\alpha m} = \frac{\delta \alpha}{\alpha}$. On the other hand, $\Delta^{\rho-w} m$ is due to ρ-w mesons mixing and known to be proportional to $\frac{m_s^2}{m_u + m_d}$ (Christiansen et al. 1991a,b). Therefore, $\frac{\delta \Delta^{\rho-w} m}{\Delta^{\rho-w} m} = \frac{\delta m^q}{m^q} = \frac{\delta <v>}{<v>}$. Thus, we get:

$$\frac{\delta Q}{Q} = -0.587 \frac{\delta \alpha}{\alpha} + 1.587 \frac{\delta <v>}{<v>} \tag{3}$$

We need also to know the dependence of the variation of the nuclear mass of an element ${}^A_Z X$ in terms of the changes in $<v>$ and α. This can be estimated using $M(X) = Z m_p + N m_n - \epsilon_X$ where ϵ_X is the binding energy for the element X, and we have:

$$\frac{\delta \epsilon_x}{\epsilon_x} = \frac{\epsilon_C}{\epsilon_x} \frac{\delta \alpha}{\alpha} \tag{4}$$

where $\epsilon_C = \frac{Z}{4\pi\epsilon_0} \frac{e^2}{R}$ is the electromagnetic contribution. The radius of the nucleus $(R \sim 1.2 A^{\frac{1}{3}} fm)$ is considered as a strong interaction effect and, thus, taken to be constant in our analysis. The change in the neutron decay rate in terms of the changes in α and $<v>$ can be expressed as follows (Ichikawa and Kawasaki 2002; Landau et al. 2006):

$$\frac{\delta \tau}{\tau} = -3.838 \frac{\delta \alpha}{\alpha} - 4.793 \frac{\delta <v>}{<v>} \tag{5}$$

where we have used $\frac{\delta G_F}{G_F} = -2 \frac{\delta <v>}{<v>}$. For the thermonuclear reaction rates dependence on α, we take the phenomenological expressions of tables IV and V in Landau et al. (2006).

Since BBN is very sensitive to ϵ_D, we should go further than equation (4) to evaluate the changes in ϵ_D in terms of $\delta <v>$. We will give our expressions for the different stages in terms of $\frac{\delta <v>}{<v>}, \frac{\delta \epsilon_D}{\epsilon_D}, \frac{\delta \alpha}{\alpha}$ and $\frac{\delta \Omega_B h^2}{\Omega_B h^2}$. In section 4 we find an estimate for $\frac{\delta \epsilon_D}{\epsilon_D}$ in terms of $\frac{\delta <v>}{<v>}$, and thus we can give then the final expressions in terms of $\frac{\delta <v>}{<v>}$, $\frac{\delta \alpha}{\alpha}$ and $\frac{\delta \Omega_B h^2}{\Omega_B h^2}$.

3. Abundances and their dependence on α and $<v>$ in the different stages

The ratio X of the number of neutrons to the total number of baryons in the first stage until the freeze-out of weak interactions ($T > 9.1 \times 10^9$K) can be expressed as follows (Bernstein et al. 1989):

$$X(y = \infty) = \int_0^\infty dy' e^{y'} \frac{1}{1 + e^y} \left(y'\right)^2 e^{-K\left(y'\right)} = 0.151 \tag{6}$$

where $K(y) = b\left[\frac{4}{y^3} + \frac{3}{y^2} + \frac{1}{y} + \left(\frac{4}{y^3} + \frac{1}{y^2}\right) e^{-y}\right]$. Only $b = 255\frac{M_{pl}}{\Delta m^2 \tau}\sqrt{\frac{45}{43\pi^3}}$ depends on the fundamental constants through τ and Δm, so we get:

$$\frac{\delta X(y = \infty)}{X(y = \infty)} = -0.52\frac{\delta b}{b} \tag{7}$$

Using equations 3 and 5 we obtain:

$$\frac{\delta X(y = \infty)}{X(y = \infty)} = 1.385\frac{\delta \alpha}{\alpha} - 0.842\frac{\delta <v>}{<v>} \tag{8}$$

In the second stage, after weak interactions freeze out, neutrons decay freely until the rate of production of ^4He becomes efficient (9.1×10^9K $>$ T $> 0.93 \times 10^9$K). Thus we have:

$$Y_n = X(y = \infty) \ e^{-t/\tau} = 0.151e^{-0.2/T_9^2} \tag{9}$$

where T_9 is the temperature evaluated in units of $10^9 K$.

The abundance of deuterium follows its equilibrium value and we assume the reactions $[npd\gamma]$ and $[d\gamma np]$ dominate for its production and destruction. Taking $\Omega_B h^2 = 0.0223$ we can calculate the final temperature of this stage by setting $\dot{Y}_n = 0$ and thus $2\dot{Y}_n = -Y_n[n]$. We find $T_9^f = 0.93$ and get the abundances $Y_p = 0.76$ and $Y_n = 0.12$. In order to calculate the dependence of the final temperature on the fundamental constants, we derive the equation $2\dot{Y}_n = -Y_n[n]$ with respect to the fundamental constants to get:

$$\frac{\delta T_9}{T_9} = 0.065\frac{\delta \Omega_B h^2}{\Omega_B h^2} + 0.055\frac{\delta \alpha}{\alpha} - 0.119\frac{\delta <v>}{<v>} + 1.195\frac{\delta \epsilon_D}{\epsilon_D}$$

and thus we get the relative variations of the nucleons abundances for this stage as follows:

$$\frac{\delta Y_n}{Y_n} = 0.030\frac{\delta \Omega_B h^2}{\Omega_B h^2} + 2.159\frac{\delta \alpha}{\alpha} - 2.005\frac{\delta <v>}{<v>} + 0.553\frac{\delta \epsilon_D}{\epsilon_D}$$

$$\frac{\delta Y_p}{Y_p} = -0.009\frac{\delta \Omega_B h^2}{\Omega_B h^2} - 0.682\frac{\delta \alpha}{\alpha} + 0.634\frac{\delta <v>}{<v>} - 0.174\frac{\delta \epsilon_D}{\epsilon_D}$$

In order to compute the final abundance of helium, we notice that once ^4He production becomes efficient (i.e. $2\dot{Y}_\alpha = Y_n[n]$), neutrons combine to form α-particles, and the production of the latter is dominated by $[dTn\alpha]$ and $[pT\gamma\alpha]$. One gets for the temperature of the ^4He freeze-out the following equation:

$$2Y_n \left(Y_p\frac{[npd\gamma]}{Y_\gamma[d\gamma np]}\right)^2 [ddpT] = \frac{1}{\tau} \tag{10}$$

where $Y_p = 0.76$, $Y_n = 0.151e^{-0.2/T_9^2}$ and τ is the neutron decay constant. Numerically we find $T_9^\alpha = 0.915$ which is lower than the final temperature of the previous stage and larger than the final temperature of the next one. For the final helium abundance we find $Y_\alpha^f = 2Y_n = 0.238$. As before, deriving equation 10 with respect to ϵ_D, $<v>$ and α we find:

$$\frac{\delta T_9^\alpha}{T_9^\alpha} = 0.061\frac{\delta \Omega_B h^2}{\Omega_B h^2} + 0.049\frac{\delta \alpha}{\alpha} - 0.113\frac{\delta <v>}{<v>} + 1.149\frac{\delta \epsilon_D}{\epsilon_D}$$

Since $Y_\alpha^f = 2Y_n$ we get the relative variation of the helium abundance as:

$$\frac{\delta Y_\alpha^f}{Y_\alpha^f} = 0.029\frac{\delta \Omega_B h^2}{\Omega_B h^2} + 2.182\frac{\delta \alpha}{\alpha} - 2.042\frac{\delta <v>}{<v>} + 0.549\frac{\delta \epsilon_D}{\epsilon_D}$$

In the following 'neutron cooking' stage, corresponding to 0.93×10^9K $>$ T $>$ 0.766×10^9K, the neutron abundance can be expressed as follows:

$$Y_n = \left(\frac{1}{Y_n^0} + 2\int_{t_{initial}}^t \left(Y_p\frac{[npd\gamma]}{Y_\gamma[d\gamma np]}\right)^2 [ddpT]dt\right)^{-1} \tag{11}$$

where the initial condition is given by the final values of the previous stage: $Y_n^0 = 0.12$ at $T_9^0 = 0.93$.

Putting $Y_n = Y_d$ as the condition which determines the final temperature of this stage, we find $\frac{[npd\gamma]}{Y_\gamma[d\gamma np]} = 1$. With Y_p freezed at 0.76, we get numerically:

$$T_9^f = 0.766 \tag{12}$$

$$Y_n = 6.4 \times 10^{-4} = Y_d \tag{13}$$

Again, the condition $(Y_n = Y_d)$ allows the calculation of the relative change of the final temperature:

$$\frac{\delta T_9^f}{T_9^f} = 0.031\frac{\delta\Omega_B h^2}{\Omega_B h^2} - 0.021\frac{\delta\alpha}{\alpha} + 0.020\frac{\delta < v >}{< v >} + 1.041\frac{\delta\epsilon_D}{\epsilon_D}$$

and we get numerically:

$$\frac{\delta Y_d}{Y_d} = \frac{\delta Y_n}{Y_n} = -1.095\frac{\delta\Omega_B h^2}{\Omega_B h^2} + 1.865\frac{\delta\alpha}{\alpha} - 0.075\frac{\delta < v >}{< v >} - 2.275\frac{\delta\epsilon_D}{\epsilon_D}$$

In the last stage $(T < 0.766 \times 10^9 \text{K})$, we notice that the dominant term in the time derivative of Y_d is the production of Tritium, i.e. $Y_d Y_d[ddpT]$. We find:

$$Y_d = \left(\frac{1}{Y_d^0} + 2\int_{t_{initial}}^t [ddpT]dt\right)^{-1} \tag{14}$$

with the initial value $Y_d^0 = 6.4 \times 10^{-4}$ at $T_9^0 = 0.766$.

We can obtain the final abundance of deuterium by setting the temperature equal to zero and we find the abundance numerically equal to $Y_d^f = 2.41 \times 10^{-5}$. Again, we can numerically evaluate the relative change of Y_d^f:

$$\frac{\delta Y_d^f}{Y_d^f} = -1.072\frac{\delta\Omega_B h^2}{\Omega_B h^2} + 2.318\frac{\delta\alpha}{\alpha} - 0.049\frac{\delta < v >}{< v >} - 2.469\frac{\delta\epsilon_D}{\epsilon_D}$$

We summarize the results that we obtained in table 2.

4. The dependence of deuterium binding energy ϵ_D on the Higgs vev $< v >$

As we said before, the deuterium binding energy ϵ_D is the most significant factor that can influence the BBN reactions rates, and its variation was discussed in

Table 2: Final abundances and their relative variations in terms of the relative variations of the fundamental constants, $\frac{\delta Y_i^f}{Y_i^f} = A\frac{\Omega_B h^2}{\Omega_B h^2} + B\frac{\delta\alpha}{\alpha} + C\frac{\delta<v>}{<v>} + D\frac{\delta\epsilon_D}{\epsilon_D}$

Y_i^f	A	B	C	D	Abundance
D	-1.072	2.318	-0.049	-2.469	2.41×10^{-5}
^4He	0.029	2.182	-2.042	0.549	0.238

Flambaum and Shuryak (2002, 2003); Dmitriev and Flambaum (2003); Dmitriev et al. (2004). Indeed, the equilibrium concentration of deuterons and the inverse reaction rate depend exponentially on it. Moreover, the deuterium is a shallow bound level ($\epsilon_D \approx 2.225\text{MeV}$). Therefore the relative variation of the deuterium binding energy ϵ_D is much larger than the relative variation of the strong interaction potential which we neglect in our work. In order to give an estimate for the relative variation of ϵ_D, we should compute, within perturbation theory, the correction to ϵ_D due to the perturbation which might change in time. Thus we write $\epsilon_D = \epsilon_D^0 + \Delta E$, where ϵ_D^0 is the unperturbed binding energy and we consider it a QCD-determined quantity which does not change in time. As to ΔE, we know (Weinberg 1990, 1991) that the one-pion exchange potential represents the first approximation to the perturbation on the strong interaction potential, and it has the form:

$$V^Y = \frac{f^2}{4\pi}\frac{e^{-m_\pi r}}{r} \tag{15}$$

where $\frac{f^2}{4\pi} \sim 0.08$, $m_\pi \sim 140\text{MeV}$ is the pion mass. We simplify the strong interaction potential by a square well model with width a and depth V_0. These two parameters can be determined by fitting the square well 'theoretical' expressions involving these two parameters to the p-n scattering data. According to the shape-independent effective range theory (Bethe 1949; Bethe and Longmire 1950) all the binding and low energy

scattering properties of the potential are determined by just two parameters which can be determined experimentally: the scattering length $a_t = 5.50 \times 10^{-13}$cm and the effective range $r_t = 1.72 \times 10^{-13}$cm (Schiff 1968). The corresponding values for the square well are: the depth $V_0 = 35.5$MeV and the width $a = 2.03 \times 10^{-13}$cm $= 0.0103$MeV^{-1}. We consider the width and the depth as QCD-determined parameters and assume they do not change in time.

Now, we have in the square well model

$$
\begin{aligned}
\Delta E &= \frac{f^2 A^2}{4\pi\xi^2} \int_0^{\xi a} \frac{\sin^2 x}{x} e^{-\frac{m_\pi}{\xi} x} dx \\
&+ \frac{f^2 B^2}{4\pi\beta^2} \int_{\xi a}^\infty \frac{e^{-\left(\frac{m_\pi}{\xi} + \frac{2\beta}{\xi}\right)x}}{x} dx
\end{aligned} \tag{16}
$$

where:

$$
\xi = \sqrt{m_N(V_0 - \epsilon_D)} = 176.76 \text{ MeV} \tag{17}
$$

$$
\beta = \sqrt{m_N \epsilon_D} = 45.71 \text{ MeV} \tag{18}
$$

$$
A = \sqrt{\frac{\xi^2}{\frac{a}{2} - \frac{sin(2\xi a)}{4\xi} + \frac{sin^2(\xi a)}{2\beta}}} = 1393 \text{ MeV}^{3/2} \tag{19}
$$

$$
B = -\frac{\beta}{\xi} \sin(\xi a) e^{\beta a} A = -559 \text{ MeV}^{3/2} \tag{20}
$$

and m_N is the reduced mass of the two-nucleon system.

Whence,

$$
\Delta E = 0.203 \text{ MeV} \tag{21}
$$

On the other hand, the change due to the variation of the nucleon mass in the Yukawa potential V^Y is negligible compared with the change due to the variation of m_π. Since $\frac{\delta m_\pi}{m_\pi} = \frac{1}{2}\frac{\delta <v>}{<v>}$ as mentioned before, we get, evaluating numerically the integrals, the following:

$$
\frac{\delta\Delta E}{\Delta E} = -0.896\frac{\delta m_\pi}{m_\pi} = -0.448\frac{\delta <v>}{<v>} \tag{22}
$$

and so,

$$\frac{\delta \epsilon_D}{\epsilon_D} = -0.041 \frac{\delta < v >}{< v >} \tag{23}$$

These values are one order of magnitude lower than those obtained by Flambaum and Shuryak (2002). The difference arises from the fact that these authors did not consider the continuity of the wave function and its derivative on the boundary of the square well. This results in differences between the normalization factor of the wave functions which propagate into the binding energy first order perturbation. Moreover, the values of a and V_0 they consider are different from this work.

Hence, the final expressions for the relative variations of the helium and deuterium abundances are:

$$\frac{\delta Y_d^f}{Y_d^f} = -1.072 \frac{\delta \Omega_B h^2}{\Omega_B h^2} + 2.318 \frac{\delta \alpha}{\alpha} + 0.052 \frac{\delta < v >}{< v >} \tag{24}$$

$$\frac{\delta Y_\alpha^f}{Y_\alpha^f} = 0.029 \frac{\delta \Omega_B h^2}{\Omega_B h^2} + 2.182 \frac{\delta \alpha}{\alpha} - 2.044 \frac{\delta < v >}{< v >} \tag{25}$$

These results are summarized in table 3

Table 3: Abundances and their dependence on fundamental constants, $\frac{\delta Y_i^f}{Y_i^f} = A \frac{\delta \alpha}{\alpha} + B \frac{\delta <v>}{<v>} + C \frac{\delta \Omega_B h^2}{\Omega_B h^2}$

Y_i^f	Abundance	A	B	C
^{2}H	2.41×10^{-5}	2.318	0.052	-1.072
^4He	0.238	2.182	-2.044	0.029

5. Results

We can now compare the theoretical predictions of the abundances of ^4He and D obtained in the last section with the observational data. The equations (24, 25) are of the form $(i = 1(D), 2(^4He))$

$$\frac{\delta Y_i^f}{Y_i^f} = A_i \frac{\delta \Omega_B h^2}{\Omega_B h^2} + B_i \frac{\delta \alpha}{\alpha} + C_i \frac{\delta <v>}{<v>} \tag{26}$$

and we take the assumption that the difference $\frac{\delta Y_i^f}{Y_i^f}$ is due to a change in the considered fundamental constants: $\frac{\delta Y_i}{Y_i} = \frac{Y_i^{obs} - Y_i^{SBBN}}{Y_i^{SBBN}}$, where Y_i^{SBBN} and Y_i^{obs} are the theoretical and observed abundances respectively. In table 4, the theoretical abundances Y_i^{SBBN} are given for $\Omega_B h^2 = 0.0223$ with their errors resulting from the uncertainty in the values of the parameters involved. In table 5, the observational data of helium and deuterium are stated with their measured errors.

Table 4: Theoretical abundances in the standard model with the WMAP estimate $\Omega_B h^2 = 0.0223$

Nucleus	$Y_i^{SBBN} \pm \delta Y_i^{SBBN}$
^2H	$(2.51 \pm 0.37) \times 10^{-5}$
^4He	0.2483 ± 0.0021

As regards the consistency of the D and ^4He data, we follow the treatment of Landau et al. (2006) and increase the observational error by a factor Θ. The values of Θ are 2.4 for D, 2.33 for ^4He.

The results of solving the system of equations (26) with the given data are shown in table 6. These results are consistent within $1 - \sigma$ with no variation of the fundamental constants. On the other hand, the results considering variation of one fundamental constant

Table 5: Observational abundances

Nucleus	$Y_i^{obs} \pm \delta Y_i^{obs}$	Reference
D	$(1.65 \pm 0.35) \times 10^{-5}$	Pettini and Bowen (2001)
D	$(2.54 \pm 0.23) \times 10^{-5}$	O'Meara et al. (2001)
D	$\left(2.42^{+0.35}_{-0.25}\right) \times 10^{-5}$	Kirkman et al. (2003)
D	$(3.25 \pm 0.3) \times 10^{-5}$	Burles and Tytler (1998a)
D	$\left(3.98^{+0.59}_{-0.67}\right) \times 10^{-5}$	Burles and Tytler (1998b)
D	$\left(1.6^{+0.25}_{-0.30}\right) \times 10^{-5}$	Crighton et al. (2004)
^{4}He	0.244 ± 0.002	Izotov and Thuan (1998)
^{4}He	0.243 ± 0.003	Izotov et al. (1997)
^{4}He	0.2345 ± 0.0026	Peimbert et al. (2000)
^{4}He	0.232 ± 0.003	Olive and Steigman (1995)

only are shown in table 7. These results are consistent within $3 - \sigma$ with no variation of the fundamental constants.

Fig. 1.— The full line shows the theoretical probability of the residuals, while the dotted line shows the empirical probability. Deviations of $\Omega_B h^2$, α and $< v >$ with respect to their mean values are considered
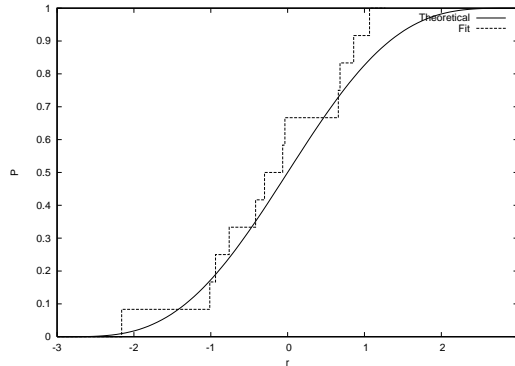
Table 6: Results of the analysis using the square well model considering joint variation of all constants

Realtive variation	Value	σ
$\frac{\delta\Omega_B h^2}{\Omega_B h^2}$	-0.011	0.054
$\frac{\delta\alpha}{\alpha}$	-0.032	0.072
$\frac{\delta<v>}{<v>}$	-0.011	0.078

Table 7: Results of the analysis using the square well model considering variation of each constant only

Realtive variation	Value	σ
$\frac{\delta\Omega_B h^2}{\Omega_B h^2}$	-0.013	0.056
$\frac{\delta\alpha}{\alpha}$	-0.015	0.006
$\frac{\delta<v>}{<v>}$	0.017	0.007

In order to check the goodness of our fit, we performed a Kolmogorov-Smirnov (K-S) test (see figure 5). When considering variation in $\Omega_B h^2$, α and $< v >$ altogether, we have a probability of 79% to obtain a worse fit. However, we consider the results of the K-S test only indicative, since even though the considered data are independent the residuals are not.

Finally, it is interesting to compare our results with other non-standard BBN models. In particular, a non-standard expansion rate and an electron-neutrino asymmetry were considered by several authors (Barger et al. 2003a; Steigman 2005, 2006). While D is more sensitive to the baryon density ($\Omega_b h^2$), the effect of a non zero ν_e asymmetry is more strong for ^{4}He than for the other relic nuclides. This is similar to the effect of $< v >$ as follows

from equations 24 and 25, whereas both D and ^{4}He are sensible to changes in α. In the papers cited above, the authors considered the following cases: i) adding only one free parameter (either ΔN_ν or ξ_e) and ii) adding two free parameters (ΔN_ν and ξ_e) besides the baryon density. They consider the WMAP data from the CMBR togheter with ^{4}He and D abundances. In the first case, the results are consistent within $1 - \sigma$ with $\xi_e \neq 0$ and $\Delta N_\nu \neq 0$. In the second case, the results are consistent within $1 - \sigma$ with $\xi_e = 0$ and $\Delta N_\nu = 0$. These results are similar to ours in that considering only one free parameter besides the baryon density is consistent with non standard physics within $1 - \sigma$ whereas considering two free parameters is consistent within $1 - \sigma$ with the SBBN model. However, it is important to remind that most of the theories where the fundamental constants may vary in cosmological time scales, predict joint variation of constants.

6. Conclusions

In this work, we assumed that the discrepancy between SBBN estimation for ^4He and D and their observational data is due to a change in time for the fundamental constants: the Higgs vev $< v >$, the fine structure constant α. We analyzed the dependence of the ^4He and D abundances on these fundamental constants within perturbation theory and on deviations with respect to the mean value of the baryonic density. Furthermore, we compared them with the observational data. We find that varying fundamental constants may not solve, in our case, the discrepancy between the theoretical SBBN and the observed data considered in this work. We hope this work stimulates further research in this interesting subject.

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