

# Chaplygin gas may prevent big trip

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This paper deals with the study of the accretion of a generalized Chaplygin gas with equation of state  $p = -A/\rho^\alpha$  onto wormholes. We have obtained that when dominant energy condition is violated the size of wormhole increases with the scale factor up to a given plateau. On the regime where the dominant energy condition is satisfied our model predicts a steady decreasing of the wormhole size as generalized Chaplygin gas is accreted. Our main conclusion is that the big trip mechanism is prevented in a large region of the physical parameters of the used model.

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Several astronomical and cosmological observations, ranging from distant supernovae Ia [1] to the cosmic microwave background anisotropy [2], indicate that the universe is currently undergoing an accelerating stage. It is assumed that this acceleration is due to some unknown stuff usually dubbed dark energy, with a positive energy density  $\rho > 0$  and with negative pressure  $p < -(1/3)\rho$ .

There are several candidate models for describing dark energy, being the cosmological constant,  $\Lambda$ , by far, the simplest and most popular candidate [3]. Other interesting models are based on considering a perfect fluid with given equation of state like in quintessence [4], K-essence [5] or generalized Chaplygin gas [6, 7, 8, 9, 10] models. Note that there are also other candidates for dark energy based on brane-world scenarios [12] and modified 4-dimensional Einstein-Hilbert actions [13], where a late time acceleration of the universe may be achieved, too.

One of the peculiar properties of the resulting cosmological models is the possibility of occurrence of a cosmic doomsday, also dubbed big rip [14]. The big rip appears in models where dark energy particularizes as the so-called phantom energy for which the dominant energy condition is violated, so that  $p + \rho < 0$ . In these models the scale factor blows up in a finite time because its cosmic acceleration is even larger than that induced by a positive cosmological constant. In these models every component of the universe goes beyond the horizon of all other universe components in finite cosmic time. It should be noted that the condition  $p + \rho < 0$  is not enough for the occurrence of a big rip, i.e., if one consider an universe filled with phantom generalized Chaplygin gas, one can avoid the big rip [15] (see also [16]). Other peculiar properties of phantom energy are that it can make the exotic substance that fuel wormholes [17], triggering the possibility of occurrence of a big trip [18, 19, 20], i.e. if there is a wormhole in an universe filled with phantom energy, due to processes of phantom energy accretion onto the wormhole, the size of this wormhole increases in such way that the wormhole can engulf the universe itself before it reaches the big rip singularity, at least relative to an asymptotic observer. Then, the following question arises: Does an universe filled with generalized phantom Chaplygin gas which avoids the big rip singularity also escape from the big trip? We have found in the present paper that for a large range of the Chaplygin parameters no big trip is predicted by our model, though there still exists sufficient room in the physically allowed parameter space to not exclude the possibility for the occurrence of such a strange causality disruption. Thus, generalized Chaplygin gas can be seen to have several interesting potential properties, as it avoids big rip [15], it may be used as the stuff to construct wormholes [21], it prevents the universe to be engulfed by a black hole [23], and, finally it may also circumvent the big trip problem.

We starts by first reviewing the accretion formalism first considered by Babichev, Dokuchaev and Eroshenko [22] (see also [24]), generalizing it to the case of wormholes. Throughout this paper we shall use natural units so that  $G = c = 1$ . The Morris-Thorne static space-time metric of one wormhole is given by [25]

$$ds^2 = -e^{\Phi(r)} dt^2 + \frac{dr^2}{1 - \frac{K(r)}{r}} + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

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where  $\Phi(r)$  is the shift function and  $K(r)$  is the shape function. We model the dark energy in the wormhole by the test perfect fluid with a negative pressure and an arbitrary equation of state  $p(\rho)$ , with the energy-momentum tensor

$$T_{\mu\nu} = (p + \rho) u_\mu u_\nu + p g_{\mu\nu}, \quad (2)$$

where  $p$  is the pressure,  $\rho$  is the energy density, and  $u^\mu = dx^\mu/ds$  is the 4-velocity with  $u^\mu u_\mu = -1$ . The zeroth (time) component of the energy-momentum conservation law  $T^{\mu\nu}{}_{;\nu} = 0$  can then generally be written as

$$0 = \frac{d}{dr} \left[ e^{\Phi(r)} (p + \rho) \frac{dt}{ds} \frac{dr}{ds} \right] + e^{\Phi(r)} (p + \rho) \left[ \Phi'(r) + \frac{K'(r)r - K(r)}{2r^2 \left(1 - \frac{K(r)}{r}\right)} + \frac{2}{r} \right] \frac{dt}{ds} \frac{dr}{ds}. \quad (3)$$

This expression should now be integrated. The integration of Eq. (3) gives then,

$$u M^{-2} r^2 \left(1 - \frac{K(r)}{r}\right)^{-1} (p + \rho) \sqrt{u^2 + 1 - \frac{K(r)}{r}} = C, \quad (4)$$

where  $u = dr/ds$ , and  $M$  is the exotic mass of the wormhole which, following the procedure of Ref. [22], has been introduced to render the integration constant  $C$  to have the dimensions of an energy density (note that we are using natural units), and, without any loss of generality for our present purposes, we have adhered to the case where  $\Phi' = 0$ .

Another integral of motion can be derived by using the projection of the conservation law for energy-momentum tensor along the four-velocity, i.e. the flow equation  $u_\mu T^{\mu\nu}{}_{;\nu} = 0$ . For a perfect fluid, this equation reduces to

$$u^\mu \rho_{;\mu} + (p + \rho) u^\mu{}_{;\mu} = 0. \quad (5)$$

The integration of Eq. (5) gives the second integral of motion that we shall use in what follows

$$M^{-2} r^2 u \left(1 - \frac{K(r)}{r}\right)^{-1/2} e^{\int_{\rho_\infty}^{\rho} \frac{d\rho}{p+\rho}} = -A, \quad (6)$$

where  $u < 0$  in the case of a fluid flow directed toward the wormhole, and  $A$  is a positive dimensionless constant. Eq. (6) gives us the energy flow induced in the accretion process. From Eqs. (4) and (6) one can easily get

$$(p + \rho) \left(1 - \frac{K(r)}{r}\right)^{-1/2} \sqrt{u^2 + 1 - \frac{K(r)}{r}} e^{-\int_{\rho_\infty}^{\rho} \frac{d\rho}{p+\rho}} = C_2, \quad (7)$$

where  $C_2 = -C/A = \tilde{A} (p(\rho_\infty) + \rho_\infty)$ , with  $\tilde{A}$  a positive constant.

The rate of change of the exotic mass of wormhole due to accretion of dark energy can be derived by integrating over the surface area the density of momentum  $T_0^r$ , that is [26]

$$\dot{M} = - \int T_0^r dS, \quad (8)$$

with  $dS = r^2 \sin\theta d\theta d\phi$ . Using Eqs. (2), (6) and (7) this can be rewritten as [20]

$$\dot{M} = -4\pi D M^2 \sqrt{1 - \frac{K(r)}{r}} [p(\rho_\infty) + \rho_\infty], \quad (9)$$

with the constant  $D = A\tilde{A} > 0$ . For the relevant asymptotic regime  $r \rightarrow \infty$  where the big trip occurs, the rate  $\dot{M}$  reduces to

$$\dot{M} = -4\pi M^2 D (p + \rho). \quad (10)$$

We see then that the rate for the wormhole exotic mass due to accretion of dark energy becomes exactly the negative to the similar rate in the case of a Schwarzschild black hole, asymptotically. For current quintessence models, the use of a scale factor  $R = R_0 (1 + \sqrt{6\pi\rho_0}(1+w)(t-t_0))^{2/[3(1+w)]}$  for  $w < -1$  (which corresponds to the solution of the

general equation  $-3H(1+w) = 2\dot{H}/H$ , with  $\dot{\phantom{x}} = d/dt$  and  $H = \dot{R}/R$  for  $w=\text{constant}$  [11]), gives arise to a singular behaviour of the size of the wormhole before reaching the big rip, that is the big trip.

We shall derive now the expression for the rate  $\dot{M}$  in the case of a generalized Chaplygin gas. This can be described as a perfect fluid with the equation of state [10]

$$p = -A_{ch}/\rho^\alpha, \quad (11)$$

where  $A_{ch}$  is a positive constant and  $\alpha > -1$  is a parameter. In the particular case  $\alpha = 1$ , the equation of state (11) corresponds to a Chaplygin gas. The conservation of the energy-momentum tensor implies

$$\rho = \left( A_{ch} + \frac{B}{R^{3(1+\alpha)}} \right)^{1/(1+\alpha)}, \quad (12)$$

with  $B \equiv (\rho_0^{\alpha+1} - A_{ch}) R_0^{3(\alpha+1)}$ ,  $R \equiv R(t)$  is the scale factor and the subscript ‘‘0’’ means initial value. Now, from the Friedmann equation we can get

$$\dot{R} = \sqrt{\frac{8\pi}{3}} R \left( A_{ch} + \frac{B}{R^{3(1+\alpha)}} \right)^{1/[2(1+\alpha)]}. \quad (13)$$

Hence, from Eqs. (10)-(13) we obtain

$$M = \frac{M_0}{1 - DM_0 \sqrt{\frac{8\pi}{3}} \left[ \rho^{\frac{1}{2}} - \rho_0^{1/2} \right]}, \quad (14)$$

For the case where the dominant energy condition is preserved, i.e.  $B > 0$ , we obtain that  $M$  decreases with time and tends to a constant value. On the other hand,  $M$  is seen to decrease more rapidly as parameter  $\alpha$  is made smaller. If the dominant energy condition is assumed to be violated, i.e.  $B < 0$ , as phantom energy is assumed to require [15], then  $M$  increases with time, with  $M$  tending to maximum, nonzero constant values. Making  $|B|$  or  $\alpha$  smaller, makes the evolution quicker.

When time goes to infinity, then the exotic mass of wormhole approaches to

$$M = \frac{M_0}{1 - DM_0 \sqrt{\frac{8\pi}{3}} \left( A_{ch}^{\frac{1}{2(1+\alpha)}} - \rho_0^{1/2} \right)}, \quad (15)$$

that is a generally finite value both for  $B > 0$  and  $B < 0$ . Thus, at first sight it could be thought that, unlike what happens in phantom quintessence models, the presence of a generalized Chaplygin gas precludes the eventual occurrence of the big trip phenomenon. However, such a conclusion cannot be guaranteed as the size of the wormhole throat could still exceed the size of the universe during its previous evolution. Note that for e.g. wormholes with zero tidal force, one can consider that the exotic matter is confined into an arbitrarily small region around the wormhole throat, then the radius of wormhole throat becomes roughly proportional to its mass. That is to say, the question remains whether the wormhole would grow eventually rapidly enough or not to engulf the universe during the evolution to its final classically stationary state. That question should be settled down before reaching a conclusion on the possibility of the big trip in these models. Actually, in order for avoiding a big trip, the following two conditions are also required: (i)  $R \neq M$  along the entire evolution, and (ii) that  $N(R) = \dot{R}/M$  be always an increasing function along that evolution. The first of these conditions implies that the function

$$f(M) = M - MM_0 D \sqrt{\frac{8\pi}{3}} \left[ \left( A_{ch} + \frac{B}{M^{3(1+\alpha)}} \right)^{\frac{1}{2(1+\alpha)}} - \rho_0^{1/2} \right] - M_0, \quad (16)$$

be nonvanishing everywhere. We analyze this question by taking the zeros of the second derivative

$$f''(M) \equiv \frac{d^2 f(M)}{dM^2} = \sqrt{6\pi} DB \frac{M_0}{M^{3(1+\alpha)+1}} \left( A_{ch} + \frac{B}{M^{3(1+\alpha)}} \right)^{\frac{-2\alpha-1}{2(1+\alpha)}} \left[ 1 - 3(1+\alpha) + \frac{3B(2\alpha+1)}{2M^{3(1+\alpha)}} \left( A_{ch} + \frac{B}{M^{3(1+\alpha)}} \right)^{-1} \right]. \quad (17)$$

Now, by taking into account that physically,

$$M > M_0 > \left(-\frac{B}{A_{ch}}\right)^{\frac{1}{3(1+\alpha)}}, \quad (18)$$

The second inequality meaning that, during its evolution, the radius of the universe should be larger than its initial size. Eq. (17) can be reduced to imply for the zeros

$$1 - 3(1 + \alpha) + \frac{3B(2\alpha + 1)}{2M^{3(1+\alpha)}} \left(A_{ch} + \frac{B}{M^{3(1+\alpha)}}\right)^{-1} = 0, \quad (19)$$

whose solution would read  $M^{3(1+\alpha)} = -B/[2(2 + 3\alpha)A_{ch}]$ . Nevertheless, from condition (18) we have that  $0 < 2(2 + 3\alpha) < 1$  which, in turns, implies that there could be at most three crossing points, as this interval allows for values of  $\alpha$  within the range of the generalized Chaplygin gas models, i.e.  $\alpha > -1$ . For  $\alpha$ -values outside the interval  $0 < 2(2 + 3\alpha) < 1$ , but still inside the Chaplygin range, one might expect just two points at most. It follows from this analysis that a big trip could in principle happens.

As to condition (ii), we obtain from Eqs. (10) and (13) that

$$N(R) = -\frac{1}{BM_0^2 D \sqrt{6\pi}} R^{3(1+\alpha)+1} \rho^{\frac{2\alpha+1}{2}} \left\{ 1 - M_o D \sqrt{\frac{8\pi}{3}} \left[ \rho^{\frac{1}{2}} - \rho_0^{1/2} \right] \right\}^2. \quad (20)$$

From Eq. (20) it follows that  $N(R_0) \geq 1$  because the size of the wormhole must be quite smaller than that of the universe initially. Thus, an always increasing  $N(R)$  should necessarily imply that the scale factor increased more rapidly than  $M$  did, so preventing any big trip to occur. By differentiating  $N(R)$  with respect to  $R$  and taking into account that  $B < 0$  for Chaplygin phantom, it can be checked that  $N(R)/dR > 0$  in the general case that  $\alpha$  does not reach values sufficiently close to  $-1$ ; that is, it follows from Eq. (15) that inside the interval

$$-1 < \alpha < \frac{\ln A_{ch}}{\ln \left( \sqrt{\frac{3}{8\pi}} \frac{1}{M_o D} + \rho_0^{1/2} \right)^2} - 1, \quad (21)$$

a big trip would still take place.

On the other hand, the question still remains on what happens with the grown-up wormhole once it has reached its maximum, final size. Since the wormhole size tends to become constant at the final stages of its evolution and it is rather a macroscopic object, it would be subjected to chronology protection [27]. In fact, one expected that vacuum polarization created particles which catastrophically accumulated on the chronology horizon of the wormhole making the corresponding renormalized stress-energy tensor to diverge and hence the wormhole would disappear.

It would be quite interesting to probe the region of parameter space ( $\alpha, A_{ch}, H_0, \Omega_K, \Omega_\phi$ ) allowed by current observations in order to determine whether there exist any allowed sections leading to a big trip. However, all available analysis [28, 29, 30, 31] are restricted to the physical region where no dominant energy condition is violated. Therefore, the section described by the interval implied by Eq. (21) necessarily is outside the analyzed regions. One had to extend the investigated domains to include values of parameter  $A_{ch} > 1$  to probe the parameters space relative to the big trip. In any events the range of  $\alpha$  values compatible with a big trip has been seen to be extremely narrow and hence the occurrence of the big trip phenomenon in the generalized Chaplygin gas model appears to be highly unlikely.

In this paper we have studied the accretion of a generalized Chaplygin gas onto a wormhole. First, we have reviewed the accretion formalism originally considered by Babichev, Dokuchaev and Eroshenko [22] for the case of a wormhole [20]. We have then applied such a formalism to the generalized Chaplygin gas model. The evolution of exotic mass with the accretion of Chaplygin dark energy has been first considered for the case that the dominant energy condition is satisfied. It has been seen that in that case the mass decreases with cosmic time.

If accretion involves Chaplygin phantom energy, then  $M$  increases from its initial value, tending to reach a plateau as cosmic time goes to infinity. It is obtained that for a wide region of the Chaplygin parameters no big trip is predicted, contrary to what happens in quintessence and K-essence dark energy models. However, as far as the Chaplygin regime tends to match the quintessence regime, but presumably still within the Chaplygin region, the possibility for a big trip at a finite time in the future is not excluded. Finally, we also argued that the fate of the final wormhole is to be destabilized by quantum vacuum processes. To conclude, the generalized Chaplygin gas has several very interesting features and may circumvent several singularities that appear in the usual quintessence models. Whether or not the above features can be taken to imply that Chaplygin gas is a more consistent component than usual quintessential or K-essence dark energy component with  $w < -1$  is a matter that will depend on both the intrinsic consistency of the models and the current observational data and those that can be expected in the future.

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