

Semilocal Cosmic String Networks

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We report on a large scale numerical study of networks of semilocal cosmic strings in flat space in the parameter regime in which they are perturbatively stable. We find a population of segments with an exponential length distribution and indications of a scaling network without significant loop formation. Very deep in the stability regime strings of superhorizon size grow rapidly and “percolate” through the box. We believe these should lead at late times to a population of infinite strings similar to topologically stable strings. However, the strings are very light; scalar gradients dominate the energy density and the network has thus a global texture-like signature. As a result, the observational constraints, at least from the temperature power spectrum of the CMB, on models predicting semilocal strings, should be closer to those on global textures or monopoles, rather than on topologically stable gauged cosmic strings.

I. INTRODUCTION

The improvement in observational data during the last years, especially on Cosmic Microwave Background (CMB) radiation, has shown that topologically stable cosmic strings cannot be the dominant component creating the primordial fluctuations that seed large scale structure as we see it in the Universe today. Strings could be a subdominant component [1] but there is as yet no evidence for their existence (see [2] for a candidate string lensing event that turned out to be two galaxies). On the other hand, from the theoretical point of view, the formation of topological defects via the Kibble mechanism [3] is inevitable in theories that allow stable defects. The formation of cosmic strings after inflation appears to be generic both in supersymmetric grand unified theories breaking down to the Standard Model [4, 5] and in brane inflation models within the framework of fundamental superstrings [6]. Therefore, models that do not predict topologically stable defects at the end of inflation are somewhat appealing. Recent work [7] suggests that a particular type of non-topological defects, called *semilocal* strings, could form in the early universe both after D -term inflation [8] and after brane inflation with $D3/D7$ -branes [9, 10].

The semilocal model [11] is a minimal extension of the Abelian Higgs model by a global $SU(2)$ symmetry. With two equally charged Higgs fields and only one $U(1)$ gauge field, there are not enough gauge degrees of freedom to cancel all scalar gradients, even outside the string cores, unless the scalar windings are correlated. Semilocal (SL) strings are stable for sufficiently large gauge coupling [11, 12], but due to their non-topological character they appear after the phase transition as open segments. These are closely related to electroweak dumbbell configurations [13, 14, 15] but whose ends have long-range interactions that resemble those of global monopoles [16], with a force roughly independent of distance. Therefore the network of SL strings has several features that are

not present in networks of topologically stable Abrikosov-Nielsen-Olesen (ANO) strings [17] and their cosmological evolution could be quite different [18, 19]. Semilocal string segments can either contract and eventually disappear or grow to join a nearby segment and form a longer string. Closed loops can be formed by intercommutations, and also if the two ends of a segment join. The obvious question is if at some point this joining of segments can form infinitely long strings. If so, the networks present characteristics intermediate between topological ANO strings and global defects – both of which show scaling behaviour in an expanding universe [17] – and we would like to understand their cosmological impact.

The purpose here is to report on a large scale numerical study of SL string networks. The non-topological nature of the SL strings does not permit a Nambu-Goto approach and our results are based on classical field theory simulations. Our basic results are twofold. We find that SL strings will, in the right parameter range, form a network that shares features with a network of ANO strings due to the existence of fast growing, extremely long percolating strings, which we believe would lead to an infinite string component at late times (we use the word “percolating” in a non-technical sense, to mean that the strings span the simulation box). On the other hand, the energy density is dominated by scalar gradients, like in models for global defects, even in the case of dense string networks deep in the stable regime. As the energy density plays a crucial role for the observable cosmological consequences, like the effect on the temperature power spectrum of the CMB, the signature of a SL string network is thus likely to be very similar to that of global defects.

II. SEMILOCAL MODEL

The action in dimensionless units is given by [19]

$$S = \int d^4x \left[|D_\mu \Phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \beta (\Phi^\dagger \Phi - 1)^2 \right], \quad (1)$$

where $D_\mu = \partial_\mu - iA_\mu$ and $\Phi^T = (\phi_1, \phi_2)$, is a complex $SU(2)$ doublet. The only parameter of the model is the ratio of the scalar and gauge masses, $\beta = m_{\text{scalar}}^2/m_{\text{gauge}}^2$ (more generally, $\beta \propto \lambda/q^2$, where q is the gauge coupling and λ the quartic coupling).

ANO flux tubes embedded in the SL model (e.g. with $\phi_2 \equiv 0$) are classical solutions of (1) but they are not topologically stable because the vacuum manifold (the three-sphere $|\phi_1|^2 + |\phi_2|^2 = 1$) is simply connected. A string with $\phi_2 \equiv 0$ can decay into configurations with $\phi_2 \neq 0$ and stability is a dynamical question. It was shown in [11, 12] that infinite straight vortices are stable if $\beta < 1$ (neutrally stable at $\beta = 1$) for any winding number. For $\beta > 1$ the bare embedded ANO strings with unit winding are unstable toward spreading of the magnetic flux. (A new class of solutions carrying persistent currents was found [20] whose stability is under investigation).

The string core carries $U(1)$ magnetic flux. We use this magnetic field to identify the strings in numerical simulations, as observing the zeros of scalar fields does not provide an unambiguous detection method for non-topological defects [19]. A SL string can be continuously deformed into “skyrmion-like” configurations; an excited string may have no scalar zeroes at the core but nevertheless it is a linear concentration of energy and for cosmological applications it should be considered a string-like defect. Here we follow the strategy of [22, 23]: we separately obtain numerically the profile of the scalar and magnetic field of the straight ANO string for several values of β and use this information to analyze the data from simulations. If the magnetic field strength at a given point exceeds a certain fraction f of the maximum magnetic field of the corresponding ANO string core, the point is considered to be part of a SL string. The string core fattens with decreasing values of β , therefore we define the length of a string to be the number of lattice points belonging to the string divided by the cross section of the corresponding ANO string. We will discard blobs whose length is less than a cutoff (set to be order of the width of a string).

III. SIMULATIONS

Previous numerical studies of SL strings have tested their stability [21] and formation rate in two [22] and three [23] dimensions, showing that formation rates can reach a third of the rate of topologically stable strings at the lowest values of β simulated (which was $\beta = 0.05$). Based on these studies, we work under the assumption

that the SL string network forms and we utilise present high performance computing technology to achieve much longer dynamical range to study the evolution of the network.

The discretized field equations were derived from (1) using techniques from Hamiltonian lattice gauge theories [24] and solved using the staggered leapfrog algorithm. Damping terms $\eta\dot{\phi}$ and $\eta\dot{A}_\mu$ are introduced into the scalar and gauge field equations, respectively. We set the lattice spacing to be $\Delta x = 1.0$ (the string cores are at least three points wide) and the time step $\Delta t = 0.2$.

During the simulation total energy densities are calculated and strings are monitored via their magnetic energy. We study two main string observables as a function of time: the distribution of strings as a function of length $n(l, t)$ and a length scale $\xi(t) = \sqrt{V/L(t)}$ where V is the volume and $L(t) = \int_0^\infty n(l, t) dl$ is the total string length. Linear growth of ξ as a function of time is referred to scaling of the string network. With periodic boundary conditions and our choice of lattice spacing the time required for two signals emitted from the same point and travelling in opposite directions to interfere with each other is half of the side of the grid. This sets the maximum time the simulations can probe reliably. If a string spanning the box forms before that time, from the point of view of an observer in the box it would appear coming from superhorizon scales.

We are interested in the regime $\beta \leq 1$. Fattening of the scalar core sets a lower bound $\sqrt{\beta} = 0.2$ that we can examine numerically; for smaller values of β scalar cores of different strings start to overlap in the initial configurations used. We tested several choices of f , the threshold of the magnetic field that decides if a point belongs to a string. For $0.2 \leq f \leq 0.4$ we do not observe any significant discrepancy in the length scale ξ of the network.

Based on earlier works [15, 22] we do not expect the formation of the network to be very sensitive upon the initial conditions. Two different strategies for setting the initial conditions were considered:

a) One strategy consists of setting all fields (scalars and gauge) and gauge momenta to zero but giving the scalar field momenta some velocity obtained from a set of Gaussian random variables with zero mean. The configuration of uncorrelated velocities is then smoothed by averaging the velocities over nearest neighbours s times.

b) The second strategy is closer to the Vachaspati-Vilenkin simulations [25] where all field momenta and gauge fields are set to zero, but random phases are assigned to the scalar field, which in turn are smoothed out by averaging over neighbouring points s times.

Note also that both types of initial configurations satisfy Gauss’ law. A set of simulations was performed in simulation boxes of size 384^3 using both kinds of initial conditions varying the parameter s from 10 to 40. We monitored the length scale ξ and found that the simulations did not show qualitatively different behaviour, and quantitatively, the deviations lay within the statistical

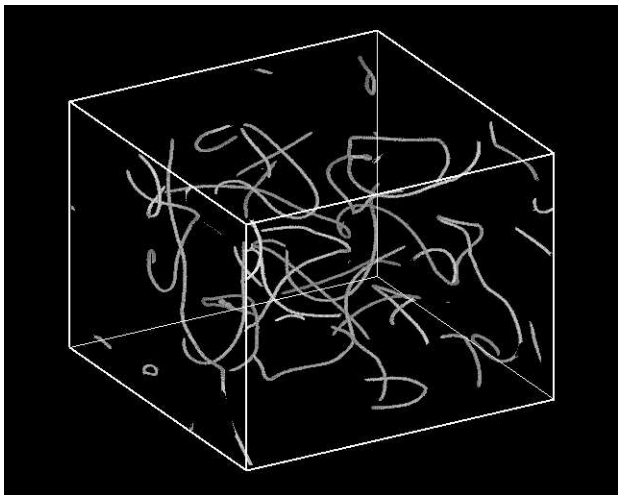


FIG. 1: A snapshot of the simulation box (512^3) for $\sqrt{\beta} = 0.2$, $\eta = 0.05$ at $t = 250$.

errors obtained by repeating the simulations with several (of order ten) different initial configurations. Moreover, case b) has typically enough energy so that the fields climb up the potential and then the system goes to a state resembling that of case a).

After some transient time, the system forms a network of strings, and the network reaches a scaling regime. This happens for all the initial conditions studied above, the only difference being the time it takes for the simulation to go through the transient regime. Therefore, we chose the most favourable way of creating initial conditions from the point of view of reaching quickly a scaling regime. As the dynamical range of simulations is in any case fairly limited, we seek to minimize the initial transient time. We do not expect to capture all the physical phenomena in the phase transition with this treatment, but neither is the main focus of this study the precise description of the dynamics at the phase transition, but rather the evolution and properties of the network, if formed and persistent, after the phase transition. For instance, the initial conditions used are not thermal, but this is the case for e.g. tachyonic preheating after inflation [26].

In the early universe there would be several sources of dissipation for the string networks, most notably the Hubble damping that decreases inversely proportional to time. In the case of our simulations there is the need to set the damping in such a way that the subsequent evolution of the network proceeds fast but without the fields overshooting. Several choices for η were tested and for the data shown it is either a constant ($\eta = 0.05$), or inversely proportional to time ($\eta = 6/t$). During the first time steps ($t < 30$) the damping is set to $\eta = 0.2$ to go through the initial transient period fast and lose energy efficiently.

IV. RESULTS

Simulations in boxes size 512^3 were performed for a set of ten ($\sqrt{\beta} > 0.3$) or fifteen ($\sqrt{\beta} \leq 0.3$) initial configurations for each value of $\sqrt{\beta}$ ranging from 0.6 to 0.2 (we also verified the disappearance of the string network at the Bogomol'nyi limit, $\beta = 1$). We used initial random velocities with smoothing 20 times ($s = 20$), the fraction $f = 0.25$ for string identification and damping term either constant ($\eta = 0.05$) or time dependent ($\eta = 6/t$).

After the initial transient ($t \sim 30$) a network of string segments is formed. The subsequent evolution of the segments depends strongly on the value of β , but as a general rule strings are longer and their density increases as β decreases. Intercommutations of strings are rare.

For $\sqrt{\beta} = 0.6$ the maximum string length is around half the size of the simulation box. In principle, two strings of that length joining together could form a string spanning the box size, but we have not seen any such event, because the network is very sparse. For $\sqrt{\beta} = 0.4$ we have the first event of a string that spans the simulation box at time = 250. Decreasing β even more leads to longer strings and a more dense network, for $\sqrt{\beta} = 0.3$ approximately one third of the initial configurations lead to the formation of at least one percolating string; for $\sqrt{\beta} = 0.2$ more than a half. Figure 1 shows a snapshot of the simulation box for $\sqrt{\beta} = 0.2$, where there are several strings spanning the size of the simulation box. For $\sqrt{\beta} \leq 0.25$ we find some extremely long strings. Analysis of the length distribution shows a qualitative change around this value of $\sqrt{\beta} \approx 0.25$.

For $\sqrt{\beta} > 0.25$ the segments lie in an exponential distribution. Nevertheless, for $\sqrt{\beta} \leq 0.25$ we find that besides the population of segments in the exponential distribution, we find extremely long strings ($l/\xi = 19$ in Figure 2) which are clearly not part of the exponential. These long strings percolate through the box and are the precursors of infinite strings at late times.

The value of β influences the network in two different ways. First, the initial string density (after discarding the transient) grows with decreasing β , see figure 3. Second, the accretion of disperse magnetic flux into tubes is much more efficient for low β . To try to disentangle the two effects we have tested configurations with different initial defect densities for low β and found that they all quickly approach the same $\xi(t)$, suggesting that flux accretion is the dominant effect that determines the scaling behaviour.

When the string network becomes more dense, there are also occasionally loops forming, but approximately only every third simulation shows a loop for $\sqrt{\beta} \leq 0.4$. The reason might be simply an observational effect, loops disappear fast and are not likely to be seen in snapshots. Moreover, we have not seen any intercommutation in our simulations. On the other hand field theory simulations do not tend to show many loops for ANO string networks either [27]. We do not observe a population of small loops and such a tiny number of loops suggests that they

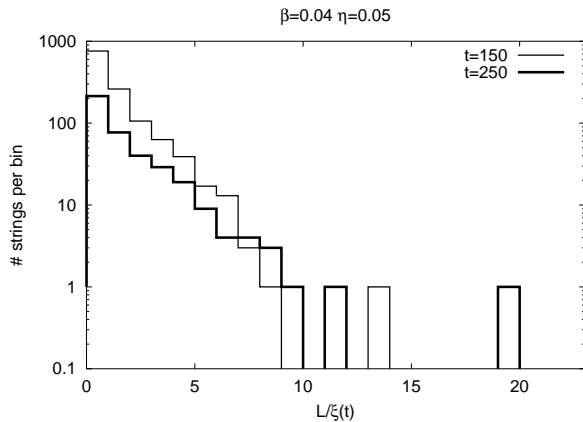


FIG. 2: The string length distribution averaged over 15 simulations for $\sqrt{\beta} = 0.2$, $\eta = 0.05$, at time = 150 (shaded) and 250 (thick line). Up to $l/\xi \sim 10 - 13$ the segments lie on an exponential distribution. The other points are extremely long strings not belonging to the exponential. Those cases are only found for $\sqrt{\beta} \leq 0.25$.

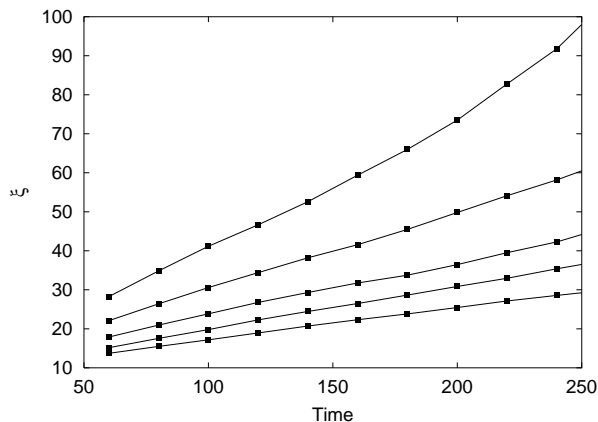


FIG. 3: $\xi(t)$ in a simulation box in a single run with $\sqrt{\beta} = 0.6, 0.5, 0.4, 0.3, 0.2$ (from top to bottom), $\eta \sim t^{-1}$.

cannot have a major effect in the network.

Figure 3 shows the observable ξ as a function of time in simulations where the damping term was set to be $\eta = 6/t$. The slow change in time for small β proves that the string network can persist for a considerably long time and goodness of linear fit to data indicates a scaling network within the range we can probe.

Finally, we monitored the total energy densities in the simulation box. In all cases studied here the scalar gradients dominate the energy density. This is not surprising in the Bogomol'nyi limit, since the string network disappears, but it is the case also even at lowest values of β we were able to probe. The approximate ratios of energy densities (kinetic : potential : magnetic : scalar gradients) are (2 : 1 : 1 : 15) for $\sqrt{\beta} = 0.5$ and (1 : 1 : 1 : 7) for $\sqrt{\beta} = 0.2$.

V. DISCUSSION

Consider the exponential distribution of short segments. At low β the string segments are very light compared to the (global) monopoles at the ends, which are strongly interacting and playing substantial role in the evolution of the segments. This is just the opposite of the usual situation encountered in cosmological scenarios in which the magnetic monopole problem is solved by a subsequent phase transition in which the magnetic flux of the (local) monopoles is confined to strings. In this latter case the monopoles are very light and hardly contribute to the dynamics, which are dominated by the string tension.

If a semilocal segment is much shorter than the average interstring separation it is very likely that the segment will shrink and the monopole and antimonopole at the ends will annihilate each other. In this case one naively expects similar length distributions whether the strings are heavy or light. However, for long segments the difference becomes apparent. In particular, the breaking of a semilocal string into a monopole-antimonopole pair is strongly suppressed [28]. Tunneling is forbidden because there is no configuration that the straight string can tunnel to, as the string configuration has lower energy than the broken configuration. In this respect, semilocal strings at low β resemble topological strings. The break-up instability is the reason why in the usual cases of (light) monopoles connected by strings, the length distribution stays exponentially suppressed throughout. By contrast, in the semilocal case there is not such a suppression of long strings. Long SL strings, once formed, are likely to continue growing by joining.

Thus it would be tempting to argue that, deep in the stability regime, a SL string network could be considered effectively as a collection of global monopoles joined by comparatively light strings. If this picture is valid, the scaling in σ -model studies of global monopoles in an expanding universe [29, 30] provides further support for the scaling behaviour we observed within the limited range we were able to probe. As time progresses, for sufficiently low β we expect the appearance of a population of infinite strings with typical ANO behaviour, loop production, etc. but at a lower density than in the topological case starting from the same initial state.

VI. CONCLUSIONS

In this paper we studied the evolution of semilocal string networks deep in the stability regime using high performance computing to carry out numerical field theory simulations in flat space. Our main results are:

- A population of segments with an initially exponential length distribution. Short segments vanish rapidly and the mean string length grows.
- Appearance of superhorizon strings (much larger than the box size) percolating through the box deep in

the stability regime, $\sqrt{\beta} \lesssim 0.3$. For $\sqrt{\beta} \lesssim 0.25$ these fast growing strings fall outside the exponential distribution.

- No significant population of small loops. In dense networks $\sqrt{\beta} \lesssim 0.4$ about one third of the simulations show a loop of a larger size.

- The string network can persist for a long time and for low β the results are consistent with linear scaling.

- The energy density is dominated by scalar gradients.

These findings suggest that SL networks may have a cosmological signature, in particular in the CMB and large scale structure, closer to that of global defects (monopoles or textures) than to topological strings. Current bounds [31] on global textures in CMB allow about 13% of the signal to come from defects at $\ell = 10$.

The relevance of these results to the cosmological scenarios in [7, 9] would require further study in expanding backgrounds (see also [18]) and the development of analytic characterizations of the network to verify the appearance of an infinite string population at late times, as was done in ref. [32] for the case of dense networks of cosmic string loops. Apart from the expansion, a complete quantum field theoretical study of the conditions at formation is still lacking, as pointed out in [9].

If it is confirmed that the observational signal of the semilocal network is similar to global textures or monopoles in the temperature power spectrum [33], there

exists a potential problem for distinguishing between models via future data. A dedicated study is needed to find out if there is any difference e.g. in polarisation of vector modes, or in the possibility of gravitational waves radiating from oscillating loops [17].

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