Topological Defects in Cosmology*

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Abstract:

The scenario of a cosmology with topological defects is surveyed starting from the field theoretic aspects and ending with a description of large-scale structure formation and magnetic field generation.

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TOPICS

1. Introduction

Classical, non-dissipative solutions - called "lumps" or "defects" - have been studied extensively in the history of physics with various motivations behind their investigation. Initially, they were studied in the context of Scott Russell's waves in hydrodynamics [1], then as "solitons" by mathematical physicists and more recently by particle physicists, as possible new "particles" in the spectrum of non-linear field theories. They have been investigated by condensed matter physicists looking into superconductivity and by astrophysicists studying the formation of galaxies. Even without considering the fluids and condensed matter research, a vast amount of literature [2,3,4,5,6] has accummulated on different aspects of such solutions and the interest continues unabated.

Lumps have been observed in fluids, plasmas and a number of condensed matter systems but, in particle physics, not a single solution of this type has been detected. The magnetic monopole remains elusive and, in fact, the archetypal GUT monopole is perceived to be enough of a cosmological problem that it has to be "inflated" away. Cosmic strings, though astrophysically promising, have yet to be detected and are tightly constrained by the millisecond pulsar and other observations. Heavy domain walls are believed to be cosmological disasters and a particle physics model is considered inadmissible if it predicts them. The question still looms large if there are any classical, non-dissipative solutions in particle physics.

On the other hand, the detection of a defect in a system can give us valuable information about the system. Since the defect is non-perturbative, it gives us information about the non-perturbative structure of the theory. The existence of a topological defect would tell us something about the topology of the theory and with it, other features that would be impossible to glean by perturbative scattering experiments. A famous example is that due to Dirac[7]: the very existence of a monopole would tell us that electric charge is quantized*. The presence of lumps in a system can lead to novel phenomenon: GUT monopoles can catalyze proton decay[8,9], and, strings and textures can lead to galaxy formation and to the generation of primordial magnetic fields[10,11,12,13]. Lumps can also give rise to exotic quantum phenomena such as fermionic zero modes[14] and quantum hair on black holes[15]. The benefits that would be reaped if lumps exist in particle physics seem to far outweigh the doubts one may have about their existence. It is hardly surprising, then, that so much effort has gone in the past several decades to uncover the mysteries of the lump.

In the following notes, we shall merely touch upon certain basic aspects of this immense subject and hope that this can be a starting point for the reader to follow up on the references that have been provided. The choice of topics included here are essentially aspects of the subject that I have been personally involved in together with some basic underlying topics that are the foundations on which the subject has grown.

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* It should also be said that this reason is no longer as compelling as it used to be in view of the fact that electric charge is automatically quantized within the framework of Grand Unified theories.

2. Topological defects: field theory

In this section we will study topological defects as classical solutions in certain field theories [16].

The general criterion for the existence of a d (spacetime) dimensional topological defect in a field theory which exhibits spontaneous symmetry breaking from a group G to a subgroup H is:

$$\pi_{3-d}(G/H) \neq 1 \tag{2.1}$$

where $\pi_n(G/H)$ is the nth homotopy group of the coset space G/H. The cases $d = 0, 1, 2, 3$ correspond to the texture, monopole, string and domain wall. The condition (2.1) leads to topological defects but the theory might also contain non-topological, semilocal and embedded defects whose existence cannot be detected by using (2.1).

It is often more convenient to think in terms of the vacuum manifold, Σ_V, described by the values of the scalar field ϕ that minimize the potential:

$$\Sigma_V = \left\{ \phi : \frac{dV}{d\phi} = 0 \ , \frac{d^2V}{d\phi^2} > 0 \right\} . \tag{2.2}$$

If the surface Σ_V has incontractible surfaces of $3 - d$ dimensions, the theory will contain a d spacetime dimensional topological defect.

A physical justification for the above criterion can be given. Suppose that the configuration of ϕ on a $3 - d$ dimensional surface at spatial infinity (S) is denoted by ϕ_∞. If we assume that there is vanishing energy at infinity, then ϕ_∞ lies on Σ_V. Therefore ϕ_∞ describes a mapping from S to Σ_V. Next let us imagine the case when the image of this mapping is one of the incontractible $3 - d$ dimensional surfaces in Σ_V and denote this image by I. But S is contractible - assuming that space itself has trivial topology* - and so we can continuously shrink this surface to a point. When S shrinks to a point, it must continue to be mapped to a non-trivial surface in Σ_V since I is assumed to be incontractible. But this would mean that ϕ would be multi-valued at the point to which S has been contracted and this is not acceptable. The only way out of this contradiction is that the field ϕ must leave Σ_V at some point in space. However, this means that ϕ cannot remain at the minimum of the potential everywhere and there must necessarily be at least

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* If there are black holes or other gravitational peculiarities present, these arguments would need modification.

one point where there is non-zero potential energy. The location of this potential energy is the location of the topological defect and the energy distribution at this location defines the energy distribution of the defect. Note that this argument shows that the asymptotic configuration of the field ϕ is sufficient to determine the existence of the defect.

The case of the texture is somewhat different since the asymptotic field configuration is not sufficient to determine its presence and at almost all times the field never leaves the vacuum manifold. There is energy in the configuration because the symmetries are global and hence the variations in the field carry gradient energy. The topology can be understood by considering the configuration in *spacetime* since then we can consider *three spheres* in the vacuum manifold that are incontractible. If the model one is considering only contains gauged symmetries, there are no textures since all the gradients in the scalar field can be compensated by gauge fields. Yet one may still have textures in the entire universe and these result in different sectors in the gauge theories and lead to degenerate vacuua [4].

To make things more concrete, we now list the simplest models that give rise to walls, strings, monopoles and textures.

The domain wall solution arises in models in which a discrete symmetry is spontaeously broken. For example, consider the model:

$$S_w = \int d^4x \left[(\partial_\mu \phi)^2 - \lambda(\phi^2 - \eta^2)^2 \right] \tag{2.3}$$

where, ϕ is a real scalar field. The symmetry breaking in this model is $Z_2 \rightarrow 1$ and hence $\pi_0(G/H) \neq 1$. In terms of the vacuum manifold, it is given by $\phi = \pm\eta$ and consists of two disconnected minima. The consequence is a domain wall in the model which interpolates between the two minima. For a static domain wall in the yz plane, the field solution is:

$$\phi = \eta \tanh(\sqrt{\lambda} \, \eta x) \tag{2.4}$$

and the energy per unit area of a wall in the $yz-$plane is:

$$E_w = \frac{8}{3}\sqrt{\lambda}\eta^3 \ . \tag{2.5}$$

The most familiar example of a model with gauge strings [17] is the Abelian-Higgs model:

$$S_s = \int d^4x \left[|(\partial_\mu + ieA_\mu)\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{4}\left(\phi^2 - \frac{\eta^2}{2}\right)^2 \right] \tag{2.6}$$

where, ϕ is a complex scalar field and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Here the symmetry breaking is $U(1) \to 1$ and $\pi_1(G/H) \neq 1$. The vacuum manifold is given by $\phi = \frac{\eta}{\sqrt{2}}e^{i\alpha}$ where α is any phase. Therefore the vacuum manifold is a circle (parametrized by α) and contains incontractible 1 dimensional curves. The corresponding unit winding string solution in cylindrical coordinates (r, θ, z), and along the $z-$axis, is of the form:

$$\phi = \frac{\eta}{\sqrt{2}}f(r)e^{i\theta} \tag{2.7}$$

$$A_\mu = -\frac{v(r)}{er}\partial_\mu\theta \tag{2.8}$$

where, the functions $f(r)$ and $v(r)$ vanish at the origin and go to 1 as $r \to \infty$. There is no known closed form for f and v and they have to be found numerically.

When $8\lambda = e^2$, that is, when the scalar and vector masses are equal, the energy of the vortex can be found analytically using Bogomolnyi's method [18] and the result is:

$$E_s = \pi\eta^2 . \tag{2.9}$$

For other values of the parameters, the energy has to be evaluated numerically [19].

A simple example of the magnetic monopole [20,21] occurs in the model:

$$S_m = \int d^4x \left[|(\partial_\mu + iee^a A_\mu^a)\vec{\phi}|^2 - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \frac{\lambda}{4}\left(\vec{\phi}^2 - \frac{\eta^2}{2}\right)^2 \right] \tag{2.10}$$

where, $\vec{\phi}$ is a triplet of fields, $a = 1, 2, 3$, $F_{\mu\nu}^a$ are the non-Abelian field strengths and $(\epsilon^a)_{ij} = \epsilon_{aij}$ is the usual epsilon symbols. The symmetry breaking is $O(3) \to O(2)$ and this gives magnetic monopoles since $\pi_2(G/H) \neq 1$. Here the minimum of the potential is a two sphere and hence contains incontractible two dimensional surfaces. The monopole configuration can be written down in spherical coordinates:

$$\vec{\phi} = \frac{\eta}{\sqrt{2}}\frac{f(r)}{er}\hat{r} \tag{2.11}$$

$$A_\mu^a = \epsilon_{aij}\hat{r}^i\partial_\mu\hat{r}^j\left(\frac{1 - v(r)}{er}\right) \tag{2.12}$$

where,

$$\hat{r} = (sin\theta \; cos\phi, sin\theta \; sin\phi, cos\theta). \tag{2.13}$$

The functions $f(r)$ and $v(r)$ and the mass of the monopole need to be found numerically. The only exception is in the Prasad-Sommerfeld [22] limit, when the solutions and the mass

of the monopole are known in closed form. This is the limit $\lambda \to 0$ and with e and η held fixed. Then the solution is:

$$f(r) = Cr\coth(Cr) - 1 \tag{2.14}$$

$$v(r) = \frac{Cr}{\sinh(Cr)} \tag{2.15}$$

where, $C = \eta e$. The energy in this limit is:

$$E = \frac{4\pi\eta}{e} \ . \tag{2.16}$$

Finally, the simplest texture[23,24] occurs in a model with symmetry breaking $O(4) \to O(3)$:

$$S_t = \int d^4x \left[(\partial_\mu \Phi)^2 - \lambda(\Phi^2 - \eta^2)^2 \right] \tag{2.17}$$

where, Φ is a column vector of 4 real fields. The vacuum manifold in this case is a three sphere and admits incontractible three dimensional surfaces. The texture solution is time dependent. The usual approach is to assume spherical symmetry and only consider the σ−model limit when $|\Phi| = \eta$. Then, in spherical coordinates[24],

$$\Phi = \eta(\cos\chi, \sin\chi\hat{r}) \tag{2.18}$$

where, $\hat{r}$ is the unit radial vector as given in (2.13). The function $\chi(t, r)$ can also be found analytically:

$$\chi(t, r) = 2\tan^{-1}(-u) \ , \quad u < 0 \tag{2.19a}$$

$$\chi(t, r) = 2\tan^{-1}(+u) + \pi \ , \quad 1 \geq u > 0 \tag{2.19b}$$

$$\chi(t, r) = 2\tan^{-1}(+1/u) + \pi \ , \quad u \geq 1 \tag{2.19c}$$

where, $u \equiv r/t$.

This completes our synopsis of topological defect solutions. The description is far from complete but suffices for applications to cosmology. The reader interested in the classification of defects, defects in particle physics models, exotic defects etc. is referred to the excellent review by Preskill[25] and to the book by Vilenkin and Shellard[6].

3. Embedded defects: field theory

Even if the general criterion for the existence of a topological defect (eq. (2.1)) is not satisfied, the model can still permit the existence of topological defect like solutions[26,27]. These solutions are essentially the topological defects of a smaller theory which are embedded in the bigger theory under consideration. The conditions under which such an embedding can be successfully carried out are not very stringent and so we can expect embedded defects to exist in almost any model.

The existence of a solution in a model does not automatically mean that it is stable and it is in this crucial way that embedded defects differ from their topological counterparts. A single topological defect is stabilized by topology and is separated from the zero topological defect sector by an infinite energy barrier. Embedded defects, however, need not be stable. In fact, mostly they are unstable, sometimes they are metastable and the only known examples of stable embedded defects are semilocal strings[28].

Instead of giving the general arguments for constructing embedded defect solutions, we shall only present some illustrative examples. We shall first construct an embedded domain wall solution as this is the simplest example of an embedded defect and then we will focus on the electroweak model and construct the embedded electroweak string solutions. A limiting case of electroweak strings will give us the semilocal string.

Walls - The most trivial embedded solution is a domain wall embedded in a global $G = U(1)$ model. We express the Higgs field in terms of two real scalar fields ϕ^a, $a = 1, 2$. A Lagrangian that is invariant under the global $U(1)$ rotation and describes static field configurations is,

$$L = \partial_i \phi^a \partial^i \phi^a - \lambda \left(\phi^a \phi^a - \eta^2 \right)^2, \tag{3.1}$$

with i labeling the spatial coordinates.

The first step in constructing the embedded domain wall solution is to identify a Z_2 subgroup of the full symmetry group. Let us consider the Z_2 subgroup defined by the transformation: $(\phi_1, \phi_2) \rightarrow (-\phi_1, \phi_2)$. Any non-zero vacuum expectation value of ϕ_1 will break this Z_2 subgroup completely and so the embedded symmetry breaking is $Z_2 \rightarrow 1$. This symmetry breaking has topological domain walls:

$$\phi_1 = \eta \tanh(\sqrt{\lambda} \, \eta x) \tag{3.2}$$

and so this configuration for ϕ_1 together with $\phi_2 = 0$ is our candidate embedded domain wall solution.

Once we have identified a candidate embedded defect solution, we should check if it extremizes the energy functional. The general conditions for this to be true can be written [26,27] and require setting up a formalism. The idea however is simple: to check that the configuration is a solution, we perturb the configuration and verify that the variation in the energy vanishes to first order in the perturbations. Since we know that the domain wall is a solution to the theory when ϕ_2 is zero, and the directions of ϕ_1 and ϕ_2 are orthogonal, there is no need to perturb ϕ_1 - only perturbations in ϕ_2 might be dangerous. Now we see that ϕ_2 appears quadratically in the energy functional and so the variation in the energy functional vanishes to linear order and the configuration is a solution.

This argument for checking when embedded configurations are solutions can be extended to arbitrary models and defects without much difficulty.

Electroweak strings [29,30,31] -

Consider the Weinberg-Salam [32,33] model of the electroweak interactions. The symmetry breaking is: $SU(2)_L \times U(1)_Y \to U(1)$ and the bosonic sector of the Lagrangian is:

$$L = -\frac{1}{4}G_{\mu\nu a}G^{\mu\nu a} - \frac{1}{4}F_{B\mu\nu}F^{B\mu\nu} + |D_\lambda\phi|^2 - \lambda(\phi^\dagger\phi - \eta^2/2)^2 \tag{3.3}$$

where, ϕ is a complex doublet. The definitions of the field strengths are:

$$G_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc}W_\mu^b W_\nu^c \tag{3.4}$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{3.5}$$

and the covariant derivative is:

$$D_\mu = \partial_\mu - i\frac{g}{2}\tau^a W_\mu^a - i\frac{g'}{2}B_\mu \tag{3.6}$$

where, g and g' are coupling constants and τ^a are the Pauli spin matrices.

The electroweak energy functional follows from the Lagrangian (3.6):

$$E = \int dz \int d^2x \left[\frac{1}{4}G_{ij}^a G_{ij}^a + \frac{1}{4}F_{Bij}F_{Bij} + (D_j\phi)^\dagger(D_j\phi) + \lambda(\phi^\dagger\phi - \eta^2/2)^2\right] \tag{3.7}$$

where, $i, j = 1, 2, 3$ and we have restricted ourselves to the case when there is no time dependence and the time components of all gauge fields vanish. In addition, since we

will only be interested in string solutions, we will only consider configurations that do not depend on the $z-$direction. Then the integration over z can be ignored and we can think in terms of the energy per unit length of the string.

The first step is to choose a $U(1)$ subgroup of the full symmetry group. We choose this to be the $U(1)$ subgroup generated by

$$\mathcal{T}^3 = -cos^2\theta_w\tau^3 + sin^2\theta_w\mathbf{1} = diag(-\cos 2\theta_w, 1) \; . \tag{3.8}$$

(Note that $\mathcal{T}^3$ is the generator corresponding to the $Z-$boson (see (3.11) below)).

Now the candidate embedded string solution may be written down:

$$\phi_{emb} = \frac{\eta}{\sqrt{2}}f_{vor}(r)e^{i\mathcal{T}^3\theta}\phi_0. \tag{3.9}$$

where, we take,

$$\phi_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \; . \tag{3.10}$$

Here, (r, θ) are polar coordinates. In addition, we want that the covariant derivatives vanish at infinity and so we take

$$Z_\mu = cos\theta_w W_\mu^3 - sin\theta_w B_\mu = [A_\mu]_{vor} \tag{3.11}$$

where, $[A_\mu]_{vor}$ is defined by eq. (2.8). All the other fields in the model are taken to vanish.

One can check that the static configuration (3.9), (3.11) extremizes the energy functional and hence is a solution [26].

Different choices of the ("embedded") subgroup lead to other string solutions. The choice that we now consider is the subgroup sitting entirely in the $SU(2)$ factor of the electroweak model and generated by: $\mathcal{T}_\alpha \equiv sin\alpha \; \tau^1 + cos\alpha \; \tau^2$, where α is some constant. Then the corresponding embedded string solution is:

$$\phi_{emb} = f_{vor}(r)e^{i\mathcal{T}_\alpha\theta}\varphi_0 \; , \quad sin\alpha W_i^1 + cos\alpha W_i^2 = (A_i)_{vor} \; , \tag{3.12}$$

and all other orthogonal combinations of gauge fields vanish.

The one parameter family of string solutions in (3.12) is called the $W(\alpha)$ string since the flux in the string is purely in the $SU(2)$ sector. Furthermore, by a global gauge transformation, any single string solution in the family - that is, a string with any value

of α - may be transformed into the string configuration with $\alpha = 0$. Explicitly, this gauge transformation is:

$$\phi' = exp\left[-i\frac{(1+\tau^3)}{2}\alpha\right]\phi \tag{3.13}$$

together with a corresponding transformation of the gauge fields. This does not, however, mean that if there are many different string solutions of different α present, they can be gauge transformed to another multi-string configuration with all strings having the same value of α. The simplest way to see that α is a non-trivial parameter is to consider a loop of W string such that α runs from 0 to 2π as we go around the loop. The winding of α around the loop is a discrete number and cannot be altered by any non-singular gauge transformation. Hence, a loop with varying α is not gauge equivalent to one with a constant value of α.

One may also see that strings of different α are distinct by comparing the directions of the field strengths in group space in each of the strings. Of course, the field strengths are only gauge covariant and not gauge invariant and so one must first parallel transport the gauge field of one string to the location of the other string and then take the scalar product of the field strengths. This leads us to the following quantity:

$$\Delta \equiv Tr\left(\tau^a F^a_{ij}(\vec{x}_2;\alpha)P\left[exp(-i\int_{\vec{x}_1}^{\vec{x}_2}\vec{dl}\cdot\vec{W}^b\tau^b)\right]\tau^c F^c_{ij}(\vec{x}_1;\alpha')\right) \tag{3.14}$$

The quantity Δ is a gauge invariant measure of differences in α between strings.

Semilocal strings -

Consider the case $g = 0$ in the electroweak model. Now the symmetry group is $SU(2)_{gl} \times U(1)_Y$ (where gl stands for "global") and it breaks down to a $U(1)_{gl}$ group. The vacuum manifold is still given by the minima of the potential and is a three sphere. However, the model continues to have the electroweak strings as solutions. But the Z-string is the only gauge string since there is only one gauge field in the model.

With the knowledge of embedded defects, the existence of the semilocal string solution is not a surprise, but what is surprising is its stability. The simplest case where one can explicitly check the stability is when $\lambda = e^2$. In this particular case, the method of Bogomolnyi can be used and it is at once obvious that the string minimizes the energy [28]. A more careful analysis [34,35] reveals that the string is only neutrally stable in this case, unstable for larger λ and stable for smaller λ. Subsequent numerical studies [36,37] have confirmed this result.

One way to understand the stability of the semilocal string is by inspecting the processes by which the string solution can destabilize. These processes necessarily require the presence of gradients of the Higgs field for which there can be no compensating gauge fields. Hence, unwinding requires a growth of the gradient energy but accomplishes a decrease of the potential energy. When the coupling constant λ is large, the potential energy is the more important piece in the energy functional and the string prefers to unwind. If λ is small, however, the gradient energy required to unwind the string is prohibitive and so the string is stable.

Topological aspects of semilocal defects have been investigated in Ref. 38, 39 and such defects have been constructed in a wide range of theories in Ref. 40. Some cosmological aspects of semilocal strings have been investigated in Ref. 41.

Next, by considering the case of small but non-zero values of g, it is clear that even the $Z-$string will be metastable for some values of parameters. A plot of the region of parameter space where the string is stable may be found in Ref. 42. The stability of electroweak strings (and other embedded defects) can be considerably enhanced if there are bound states present on the string [43].

4. Cosmological formation of defects

So far we have only discussed topological defects as classical solutions in certain field theories. What relevance can such solutions have for cosmology? Here we will argue that these defects would form during phase transitions in the early universe [44] and, in most cases, would have survived until the present epoch.

Consider a cosmological phase transition occurring at some temperature T. During the phase transition the Higgs field ϕ acquires a vacuum expectation value so that $|\phi| = \eta$. If the vacuum manifold is non-trivial, this still does not fix the location of the vacuum expectation value on the vacuum manifold. For example, if the model is the one with Z_2 symmetry (eqn. (2.3)), we can have $\phi = +\eta$ or $\phi = -\eta$ at any given point in space. Furthermore, the value that is acquired at one point is uncorrelated with the value acquired at some other point provided the points are separated by a distance greater than the correlation distance ξ at the phase transition. Causality necessarily implies that $\xi < t$ where t is the epoch of the phase transition. Hence, after the phase transition, the Higgs field lies on different points of the vacuum manifold at different spatial points. Then, it will happen, just by chance, that the configuration of the field on some asymptotic surface will be topologically non-trivial. (For example, in the Abelian-Higgs model (2.6), there will be closed circuits in space on which the phase of the Higgs field varies from 0 to 2π as each of the circuits is traversed.) When this happens, a defect will necessarily be present somewhere inside the surface.

This existence proof of the formation of defects during cosmological phase transitions relies on two facts: (1) the presence of a defect can be determined solely by looking at the asymptotic field configuration and, (2) the vacuum expectation value is uncorrelated on distances larger than t. This mechanism for the formation of defects is called the "Kibble mechanism".

Realistically, the phase transition is a thermal process and one should study the formation of defects using statistical field theory. Less ambitiously, one should estimate ξ by finding the thermal correlation function during the phase transition and this has been attempted in simple models. A rigorous estimate of, say, the density of defects after a phase transition is not available. However, for cosmological purposes this is not very relevant either. The point is that even if some defects - as few as one per horizon - are produced, the cosmological effects can be very dramatic.

Another mechanism by which defects can be produced is by quantum mechanical nucleation of the defect during an inflationary phase of the universe [45]. A quick way to understand this phenomenon is by thinking of inflation as a strong force that pulls apart any object. On the other hand, the mutual attraction of monopoles to anti-monopoles, the tension in cosmic strings and domain walls, tends to collapse these objects. In terms of a potential, there are two regions - one when there is no defect and the other where there is a defect-anti-defect pair (or loop of string, or shell of domain wall) that are being pulled apart due to inflation. These two regions are minima of an "effective potential" and they are separated by a potential barrier. But then there can be quantum mechanical tunneling from one region to another and if there are no defects to start with, inflation can literally pull a pair out from the vacuum. As this is a quantum mechanical effect, the number density of defects produced in this way is small. However, in the post-inflationary universe, our horizon is only a small patch of the universe and it may well be that our horizon is located in the region which does contain a few defects. If these defects are domain walls or strings, they may still be relevant for cosmology within our universe [46].

What do the defects look like on formation? The following summary is based on numerical simulations to study the formation of topological defects during phase transitions.

Walls: There is one infinite wall and very few smaller walls whose size distribution is exponentially suppressed [47,48]. The fraction of wall energy that resides in the infinite wall is about 87% .

Strings: There are a few infinite strings with close to 80% of the total string length [48]. The remaining string is in loops with a scale invariant distribution:

$$dn(R) = c\frac{dR}{R^4}$$

where, $dn(R)$ is the number density of loops having size between R and $R + dR$ and $c \sim 6$ is a numerical coefficient determined by simulations [48]. The loops will collapse, radiate energy and disappear but the infinite strings will survive as they are protected by topology.

Monopoles: The number density of monopoles is $\sim \xi^{-3}$ where ξ is the correlation distance after the phase transition [49]. For a second order phase transition $\xi \sim T^{-1}$ and for a first order phase transition it is given by the typical size of the bubbles when they collide. The radius of bubbles at collision could be anywhere between T^{-1} and the Hubble distance $\sim t$.

Embedded defects: The Kibble mechanism does not directly apply to the formation of embedded defects since the point (1) above is not met and, in this case, a full study of the phase transition appears necessary. However, some guesses can be made on the basis of what is known about the formation of topological defects. For example, if the embedded strings are metastable they can end on monopoles. And if the probability for a string to break by the formation of monopole and antimonopole pair is small, one would still expect to form infinite strings. For a larger probability, however, the length distribution of embedded string segments is expected to be exponential[50,51] - that is, the number of long strings is exponentially suppressed. At this time, however, no one has a quantitative understanding of the formation of non-topological defects.

Nucleated defects: For defects that nucleate in de Sitter space, it is possible to find the wave-function for quantum fluctuations of the defect and hence the distribution of various shapes and sizes of nucleated defects[46,52]. Cosmogical consequences of such defects have also been investigated in Ref. 53.

5. Cosmological constraints on domain walls and monopoles

Domain walls and magnetic monopoles are strongly constrained by cosmology. Consider domain walls first [54].

As discussed in the previous section, an infinite domain wall will be formed during a domain wall forming phase transition. The infinite domain wall will move under its own tension and try to straighten out. Immediately after the phase transition, the motion of the wall is damped by friction but as the plasma gets diluted by Hubble expansion, the drag decreases and eventually the motion of the wall is effectively undamped by friction. Hubble expansion is still important. The single domain wall in the universe cannot disappear since it is protected by topology and would be present in the universe if it were ever produced. (Inflation could, however, push the domain wall outside our horizon in which case it would be irrelevant for our observable universe.) Assuming that the domain wall straightens out completely, its area within our horizon is $\sim t^2$ and its mass is σt^2 where σ is the energy per unit area of the wall. Therefore the energy density in the wall is $\sim \sigma/t$ and, for the walls not to dominate the universe today, we require that $\sigma/t < \rho$, where, ρ is the matter density at time t. The energy per unit area of a domain wall is usually (for example, in the model (2.3)) given by $\sigma = \sqrt{\lambda}\eta^3$. Assuming that the coupling constant λ is of order 1, and taking $\rho \sim 3/(32\pi Gt^2)$ the constraint on domain walls gives $\eta \lesssim 10 MeV$.

The scale η is the scale at which the phase transition occurs and hence the only permissible walls are the ones that can form relatively late in the history of the universe. This excludes domain walls that form at the Grand Unification scale or even at the electroweak scale.

Next consider magnetic monopoles. There are several interesting bounds on the number density of monopoles - each involving different physics. Here we will only consider the cosmological bound which in itself is quite severe [49,55,6].

The cosmological bound is that magnetic monopoles should not overclose the universe. This means that the energy density in monopoles ρ_m today should be less than the critical density of the present universe. The smallest possible number density of monopoles at formation (n_f) is one per horizon at that epoch:

$$n_f \sim \frac{1}{t_f^3} \tag{5.1}$$

After formation they quickly become non-relativistic and from then on their energy density decays like that of matter and redshifts as $a(t)^{-3}$ where $a(t)$ is the scale factor of the universe. The energy density of the radiation dominated universe, however, redshifts faster, in proportion to $a(t)^{-4}$. Therefore the ratio of monopole energy density to critical density (Ω_m) at time t is:

$$\Omega_m(t) = \frac{mn_f}{\rho_c(t_f)} \frac{a(t)}{a(t_f)} \tag{5.2}$$

where, m is the mass of the monopole. With $\rho_c = 3/(32\pi Gt^2)$, $a(t) \propto t^{1/2}$ and $m \sim T_f$ where, T_f is the temperature at the phase transition, we find that Ω_m becomes one at a time t_* given by:

$$t_* \sim \left(\frac{10^{19}GeV}{m}\right)^8 10^{-46}s. \tag{5.3}$$

For GUT scale monopoles $(m \sim 10^{16}GeV)$, this epoch occurs at $10^{-22}s$ - well before the matter era. (This justifies our use of $a(t) \propto t^{1/2}$.) Therefore, if monopoles were formed at the GUT epoch, their energy density would completely overwhelm our universe and would overclose it. This is not observed and so some way has to be found to resolve this "monopole overabundance problem"*.

The monopole overabundance problem is head-on in conflict with the philosophy of conventional GUTs which is that the electroweak and strong forces are unified in a Grand Unified symmetry group - a simply connected group - at an energy scale of about $10^{16}GeV$ and that there is no new force that comes into play between the electroweak $(10^2 GeV)$ and the Grand Unification scale. Coupled with standard cosmology, this philosophy implies that the Grand Unified symmetry group broke down to a subgroup with a hypercharge $U(1)$ factor at the Grand Unification scale. But then condition (2.1) is satisfied for the case of monopoles. This means that within conventional GUTs, the monopole overabundance problem must be confronted.

One obvious solution is to relax conventional GUTs and allow for the possibility of a $U(1)$ factor in the Grand Unified group. Then monopoles will never form and there will be no overabundance problem.

If one is unwilling to relax one's philosophy of Grand Unification, there is still a cos-mological solution and another particle-physics solution to the monopole overabundance

* The problem is even more severe when one considers the various other constraints on the present monopole flux - such as the Parker bound or the bound coming from neutron stars[55].

problem. The cosmological solution is to have an inflationary phase during or after the formation of monopoles [56]. The inflationary phase simply dilutes the monopoles until their number density gets so small that they cause no problem. The particle physics solution is due to Langacker and Pi [57] who consider the formation of monopoles at the GUT stage and then another stage when the electromagnetic symmetry gets broken. At this stage the magnetic flux of the monopoles gets confined and the monopoles get connected by strings. The strings bring the monopoles and antimonopoles together, they annihilate, and subsequently the electromagnetic symmetry is restored.

6. Cosmic strings: general properties

Here we will summarize some properties of cosmic strings.

At formation, the string network consists of closed loops and infinite strings. It is estimated that $\sim 80\%$ of the energy in the string network resides in infinite strings [48].

Once formed, the strings move under their own tension and try to straighten out. This motion is damped due to the frictional force of the ambient matter [58,59] and is also slowed due to the Hubble expansion. The frictional force is more important than the Hubble expansion as the matter density is high. With time, however, the matter density gets redshifted and the Hubble expansion dominates the frictional force. The time at which the Hubble expansion drag become comparable to that due to friction is $t_* \sim (G\mu)^{-2} t_{Pl}$. After this time, the frictional force can be ignored [60].

The motion of a string in vacuum with energy density μ is well described by the Nambu-Goto action:

$$S = -\mu \int d\tau d\sigma \sqrt{-g^{(2)}} \, , \tag{6.1}$$

where, $g^{(2)}$ is the determinant of the world-sheet metric defined by

$$g_{ab}^{(2)} = g_{\mu\nu} \partial_a x^\mu(\tau, \sigma) \partial_b x^\nu(\tau, \sigma) \, , \tag{6.2}$$

$a, b = \tau, \sigma$; $g_{\mu\nu}$ is the metric of the background spacetime and $x^\mu(\tau, \sigma)$ are the coordinates of the string world-sheet.

The Nambu-Goto action is valid as long as the radius of curvature of the string is much larger than the thickness of the string [61,62,63]. Also, the action does not include the interactions of strings when they intersect. When there is a collision of a string with another string, it leads to the phenomenon of intercommuting [64,65,66]. In this event, the strings intersect, exchange partners and then again move as given by (6.1).

Another factor that plays an important role in the evolution of the string network is the gradual loss of energy from strings into forms of radiation. An oscillating string is a time dependent solution to a set of field equations and so one would expect that the motion would lead to the radiation of quanta of any fields that couple with the fields that form the string. However, it has been shown that the radiation in these quanta is negligible from the strings that are expected to be of cosmological interest [67]. This is solely due to the fact that the curvature and oscillation frequencies of such strings is very small compared to particle

physics scales and hence the only radiation that can possibly be emitted is into massless particles. A more detailed study then shows that the dominant loss from oscillating gauge strings is to gravitational radiation[68,69]. A loop of size L emits gravitational radiation and loses all its energy in a time:

$$\tau \sim \frac{L}{\Gamma G \mu} \qquad (6.2)$$

where the coefficient Γ is numerically found to be of order 100 for certain family of string loops[69]. (In the case of global strings, the energy loss is dominated by the emission of Goldstone bosons[70,71].)

At times later than t_*, the evolution of the network of strings is governed by string tension, Hubble expansion, intercommuting and gravitational radiation. These four factors make the evolution complicated enough so that no one has a clear picture of what the network looks like at any instant. Progress in this problem has relied on the results of numerical simulations of the string evolution and, recently, an analytical attack is also underway.

Even though the string network is not fully understood, a few features seem to be emerging[72,73,74,75,76,77]. First of all, the network at any instant much later than t_* seems to consist of a few infinite strings (that is, strings that traverse the whole horizon without closing up on themselves) and a large number of tiny loops. The size distribution of the loops is not known but the favoured guess is that the size is given by the gravitational radiation cut-off distance: $l \sim \Gamma G \mu t$. (Loops smaller than this evaporate in less than a Hubble time and would probably not be significant for cosmological purposes.) The distribution of loops in space is not known either but, since the loops are produced at very high velocities ($v \sim 1$) one would expect them to be distributed roughly homogenously even if they are initially produced in a localized region of space (for which there is some visual evidence). The long strings are not smooth but have a lot of irregularities. The scale of the irregularities is guessed to be the same as the size of the loops $\sim \Gamma G \mu t$ at any time t. These irregularities are called "kinks" or "wiggles" in the literature and the long strings are said to be "wiggly".

Cosmic strings can also have the ability to carry persistent electric currents[14]. Such superconducting cosmic strings[78] can have very dramatic cosmological signatures. Other notable varieties of strings include global, non-abelian and Alice strings. We shall not discuss these varieties of strings but the reader can find a description in Ref. 6.

7. Cosmic strings: gravitational properties

Non-superconducting topological defects interact with their environment primarily via gravitational forces. Here we will consider the metric of gauge strings and textures and describe some of the known properties[79].

We first consider the metric of a source with energy-momentum tensor[80,81]

$$T^\nu_\mu = \delta(x)\delta(y)diag(\mu, 0, 0, T) . \tag{7.1}$$

With $T = \mu = \mu_0$ this is the effective energy-momentum tensor of an unperturbed string with string tension μ_0 as seen from distances much larger than the thickness of the string. When[82,83]

$$\mu T = \mu_0^2 \tag{7.2}$$

this also describes the energy-momentum tensor of a wiggly string as seen by an observer who cannot resolve the wiggles on the string.

The gravitational field of the string can be found by solving the linearized Einstein equations with T^ν_μ from (7.1). This gives[80]

$$h_{00} = h_{33} = 4G(\mu - T)ln(r/r_0),$$

$$h_{11} = h_{22} = 4G(\mu + T)ln(r/r_0), \tag{7.3}$$

where, $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ is the metric perturbation, $r = (x^2 + y^2)^{1/2}$ and r_0 is a constant of integration.

For an unperturbed string, $T = \mu = \mu_0$ and we get

$$h_{00} = h_{33} = 0, \quad h_{11} = h_{22} = 8G\mu_0 ln(r/r_0). \tag{7.4}$$

A coordinate transformation brings this metric to a locally flat form,

$$ds^2 = dt^2 - dz^2 - dr^2 - (1 - 8G\mu_0)r^2 d\phi^2 \tag{7.5}$$

It describes a conical space, which is just a Euclidean space with a wedge of angular size $\Delta_0 = 8\pi G\mu_0$ removed and the two faces of the wedge identified. A particle at rest with respect to a straight string experiences no gravitational force, but if the string moves with velocity v_s, then nearby matter gets a boost

$$u_i = 4\pi G\mu_0 v_s \gamma_s \tag{7.6}$$

in the direction of the surface swept out by the string. Here, $\gamma_s = (1 - v_s^2)^{-1/2}$. This effect is responsible for the formation of wakes[84] and for a discontinuous change of the microwave background temperature across a moving string[81,85]. Assuming that the string is perpendicular to the line of sight, the magnitude of the latter effect is

$$\frac{\delta T}{T} = 8\pi G \mu_0 v_s \gamma_s . \tag{7.7}$$

The conical metric also results in the formation of double images of background objects. In the cosmological context, this would lead to the gravitational lensing of background quasars and galaxies[80,81].

Returning now to the wiggly string metric (7.3), we first consider the effect of the wiggles on light propagation[13]. Assuming for simplicity that the direction of propagation is perpendicular to the string, we can write the relevant components of the metric in the form

$$ds^2 = (1 + h_{00})[dt^2 - (dx^2 + dy^2)] \tag{7.8}$$

where we should identify the half-lines $y = \pm 4\pi G \mu x$, $x \geq 0$. The conformal factor $(1 + h_{00})$ does not affect light propagation and can be dropped. Then the resulting metric describes Minkowski space with a deficit angle $8\pi G \mu$, and we conclude that background temperature discontinuities produced by wiggly strings are given by the same equation (7.7) with μ_0 replaced by μ. In contrast to the smooth string, however, the wiggly string also produces a change in the photon temperature when the photon propagates parallel to the string but perpendicular to the velocity of the string.

Next, we study the formation of a wake behind a moving wiggly string. First look at the problem in the rest frame of the string where the particles are flowing past the string with a velocity v_s in the x-direction. The linearized geodesic equations in the metric (8) can be written as

$$2\ddot{x} = -(1 - \dot{x}^2 - \dot{y}^2)\partial_x h_{00}, \tag{7.9}$$

$$2\ddot{y} = -(1 - \dot{x}^2 - \dot{y}^2)\partial_y h_{00}, \tag{7.10}$$

where over-dots denote derivatives with respect to t. We need only work to first order in $G\mu$, in which case (7.10) can be integrated over the unperturbed trajectory $x = v_s t$, $y = y_0$. Then we can transform to the frame in which the string has a velocity v_s. The result for the velocity impulse in the y-direction after the string has passed by is[13]:

$$u_i = -\frac{2\pi G(\mu - T)}{v_s \gamma_s} - 4\pi G \mu v_s \gamma_s \tag{7.11}$$

The second term is the usual velocity impulse due to the conical deficit angle. But, for small velocities, it is the first term that dominates the deflection of particles. The origin of this term can be easily understood. From eqn. (7.3), the gravitational force on a non-relativistic particle of mass m is $F = 2mG(\mu - T)/r$. A particle with an impact parameter r is exposed to this force for a time $\Delta t \sim r/v_s$ and the resulting velocity is $u_i \sim (F/m)\Delta t \sim G(\mu - T)/v_s$.

The metric of a loop of cosmic string can be obtained quite easily in the weak field approximation [86,87]. The result is that, at distances much larger than the size of the loop, the metric is Schwarzschild with mass parameter M equal to the mass of the loop. Singular points ("cusps") on the string world-sheet may have novel gravitational features [87] but a proper treatment of these features requires going beyond the weak gravitational field approximation. (In addition, these singular features do not seem to be very relevant for cosmology since now it is believed that loops are not very important and that the occurrence of cusps on loops is not as generic as it first seemed to be [88].)

8. Structure formation by wiggly cosmic strings

The formation and evolution of long-string wakes and their possible role in structure formation in the scenario where strings are not wiggly have been discussed in Refs.84, 89, 90, 91, 92, 93 and many other papers. The scenario where the strings are wiggly has a shorter history and some of the relevant papers can be found in Refs. 13, 94, 95, 96, 97, 98. Here we will follow Ref. 13.

When a collisionless fluid flows in the wiggly string metric, the gravitational field focusses the fluid particles inwards so that streamlines flowing on either side of the string converge behind the string. Therefore a wake forms behind the string which has twice the density of the ambient fluid. If the fluid is not collisionless, the fluid flow into the wake will be accompanied by the formation of shocks and turbulence. These features of the wake are likely to be important for the formation of structure on galactic scales and for the generation of magnetic fields. However, the details of the wake are not important for the formation of large-scale structure which is what we shall now describe.

Consider a wake formed behing a moving string segment of length $\sim \xi(t_i)$ at time t_i. The distance travelled by the string in one Hubble time is $\sim v_s t_i$, and thus the initial length and width of the wake are $l_i \sim t_i$, $w_i \sim v_s t_i$. We shall first assume that the universe is dominated by cold dissipationless matter. In this case the two opposite streams of matter in the wake overlap, and the mass density is enhanced by a factor of 2 within a wedge with an opening angle $2u_i/v_s$, where u_i is from eqn. (7.11). The average thickness of this wedge is $d_i \sim u_i t_i$. The initial surface density of the wake is

$$\sigma_i \approx 2\rho(t_i)d_i \approx \frac{2}{3}\frac{\mu - T}{v_s t_i} \tag{8.1}$$

where $\rho(t) = (6\pi G t^2)^{-1}$ is the average density of the universe, and its total mass is $M_i \approx \sigma_i l_i w_i \approx (\mu - T)t_i$. Note that M_i is independent of the string velocity v_s. If the string moves faster, the wake is wider, but the surface density is decreased proportionately. We note also that the velocity perturbation (7.11) is produced at distances up to $\sim w_i$ from the plane of the wake. For $r > w_i$, the gravitational field of the string is like that of a stationary rod and $u_i \sim G(\mu - T)t_i/r$.

As the universe expands, the length and width of the wake grow like the scale factor, $a(t) \sim t^{2/3}$, while the total mass of the wake grows by gravitational instability like $M \propto$

$a(t)$. As a result, the wake thickness (defined as the turnaround distance) and surface density evolve like $d \propto a^2(t)$, $\sigma \propto a^{-1}(t)$. At the present time $(t = t_0)$

$$\sigma_0 \approx \frac{\mu - T}{v_s t_0} \left(\frac{t_0}{t_i}\right)^{1/3}. \tag{8.2}$$

Cold-dark-matter wakes can also be formed during the radiation era $(t_i < t_{eq})$, but in this case the gravitational instability sets in only at $t \sim t_{eq}$. It can be shown that the surface density of the resulting wakes is proportional to $(t_i/t_{eq})^{1/2}$. Together with eqn. (8.2) this implies [89] that the most prominent wakes having the largest surface density are the ones formed at $t \sim t_{eq}$.

The fraction of the total mass of the universe accreted onto wakes which were formed at time $\sim t_i$ can be estimated as (for $t_i > t_{eq}$)

$$f \approx \frac{2 w_i d_i z_i}{L^2(t_i)} \approx 8\pi G(\mu - T) z_i \tag{8.3}$$

where z_i is the redshift at t_i. The total mass of dark matter in all wakes is dominated by the wakes formed at $t \sim t_{eq}$,

$$f_{tot} \sim 20 G \mu_0 z_{eq} \sim 0.4 h^2 \mu_6. \tag{8.4}$$

Here, h is the Hubble constant in units of $100 km/s.Mpc$, the universe is assumed to have critical density, $\Omega = 1$, $\mu_6 = G\mu_0/10^{-6}$, and in the last step we have used the values of μ and T from the simulations.

The evolution of the initial velocity perturbation (7.11) can be found from the equation of motion for dark matter particles,

$$\dot{u} + \frac{\dot{a}}{a} u = g \tag{8.5}$$

where, $g = 2\pi G \sigma(t)$ is the gravitational acceleration due to the wake. This gives $u(t) \propto t^{1/3}$. A careful analysis shows that the present velocity perturbation due to a single string impulse is [92]

$$u_0 \approx \frac{2}{5} u_i z_i^{1/2}. \tag{8.6}$$

If the filamentary wakes created by strings were sheet-like, they would have a thickness $\sim u_i t_i z_i^2$. For wakes produced around t_{eq}, this thickness is somewhat larger than the width

of the wake $\sim v_s t_i z_i$ and hence we should not treat the filamentary wake as having planar geometry. Instead the wake should be treated as having linear geometry and the accreting structure will be cylindrical in shape with the possibility of some planar sub-structure. The diameter of the cylindrical structure is characterized by the geometric mean of the previously calculated width and thickness and is $\sim (u_i v_s)^{1/2} z_i^{3/2} t_i$ while the length is $t_i z_i$.

To get a qualitative feel for the appearance of the wakes, we adopt the picture developed in Ref. 72 for the evolution of the string network. The basic idea is that the long strings are moving slowly at speeds ~ 0.2 for about one Hubble time. Then there is an intercommuting somewhere in the network which triggers an instability and speeds up the string to a much higher velocity ~ 0.6. In this way, during every Hubble time period, a string moves slowly for most of the time but the slow motion is followed by a rapid motion that helps maintain the scaling solution in which the distance between strings stays a fixed fraction of the horizon size. In addition, string simulations show [73,72] that the coherence length of strings, beyond which the directions along the string are uncorrelated, is $\xi(t) \approx t$. The inter-string separation $L(t)$ is of the same order of magnitude. In the matter era $L(t) \approx 0.7t$. The rms string velocity on the scale of the smallest wiggles is [73] $(< v^2 >)^{1/2} \approx 0.6$, but the coherent velocity obtained by averaging over a scale ξ is $v_s \sim 0.15$. The average mass per unit length and string tension are (in the matter era) $\mu \approx 1.4\mu_0$, $T \approx 0.7\mu_0$. With these values, the first term in eqn. (7.11) is about ten times larger than the second.

If a string segment moves coherently for more than one Hubble time, the resulting wake will have a variable surface density, with denser parts being the ones formed at earlier times. The straightening of long strings on the scale $\xi \sim t$ occurs mainly due to string intersections. Long strings occasionally self-intersect producing a horizon-size loop which then rapidly collapses into miriads of tiny stable loops. If two different strings intercommute, the highly curved regions near the points of intercommuting develop a high velocity, $v_s \sim 1$, and also shed off a large number of tiny loops as they move. The wakes due to rapidly moving strings have the form of sheets with dimensions $t_i z_i \times t_i z_i \times u_i t_i z_i^2$ while the wakes due to slow strings have a filamentary appearence. As we explained (see below (8.1)), the masses of both types of wakes are comparable, but the surface density in the filamentary wakes is much higher, and we expect filamentary features to be prominent in the large- scale galaxy distribution. In addition to wakes due to long strings, there will also be comet-like wakes produced by rapidly moving small loops [99]. The characteristic scale of the large-scale structure in this scenario is $t_{eq} z_{eq} \sim 10h^{-2} Mpc$. With $h = 0.5$ it is

comparable to the scale suggested by observations [100] ($\sim 25h^{-1}Mpc$).

The wiggliness of the string network implies that the wakes will not be uniform but will have sub-structure on the scale of the wiggles. This scale is expected to be larger than the damping scale due to gravitational radiation from the string network which is $\Gamma G\mu t$, at the time of formation of the wake, where $\Gamma \sim 10^2$ is a numerical factor coming from the rate of gravitational radiation [69]. For the wakes produced at t_{eq}, the sub-structure is on a comoving scale larger than $\sim 1\mu_6 h^{-2}kpc$. We expect that the wakes will fragment into smaller objects due to this sub-structure.

The large-scale velocities predicted at the present time can be found from eqn. (8.6). For sheet-like wakes from rapidly moving strings, it gives $u_0 \sim 300\mu_6 \ h \ km/s$ where we assumed that $t_i \sim t_{eq}$ and $v_s\gamma_s \approx 1$. These velocity perturbations extend over regions of size $(10h^{-2}Mpc)^3$ and may account for the observed large-scale streaming velocities [101]. Reasonable values of u_0 are obtained, e.g., for $h \sim 0.5$, $\mu_6 \sim 4$. We note that in some regions of space the motion of matter can be affected by two or more different strings. The streaming velocity in such regions will typically be enhanced by a factor $\sqrt{n}$ where n is the number of string impulses that the matter experiences [102]. Regions larger than $(10h^{-2}Mpc)^3$ will also get peculiar velocities due to string impulses but the velocity will scale as $1/L$ where L is the size of the region. (This is simply because a string gives a coherent impulse to a region of size L, when its correlation length becomes comparable to L. For large L, this happens later, giving less time for the velocity to grow.) The observational situation on the dependence of peculiar velocity on length scale is quite unclear and it remains to be seen if this predicted fall-off agrees with observation [103].

If the dark matter is cold, wakes formed at all epochs prior to radiation-matter equality will survive and density fluctuations will be present on very small scales too. Albrecht and Stebbins [96,97] argue that this small scale power is excessive and the sheet-like structures formed later would not be prominent. The situation to me does not seem as clear since one could imagine small scale structures themselves clustering into larger scale structures. So the large scale wake could be prominent simply because it rearranges the small scale structure into sheets and filaments.

In a universe dominated by light neutrinos, wake perturbations are damped by neutrino free streaming on co-moving scales smaller than $\lambda_\nu(t) \sim v_\nu(t)t$, where $v_\nu(t) \approx v_{eq}(t_{eq}/t)^{2/3}$ is the rms velocity of neutrinos and $v_{eq} \approx 0.2$. On larger scales the evolution of pertur-

bations is similar to that in cold dark matter. For a cold-dark-matter wake formed at time t_i, all matter initially within a distance $u_i t_i z_i / z$ will be accreted onto the wake by the redshift z. A neutrino wake will go nonlinear at the redshift z_{nl} when the co-moving scale of $\lambda_\nu(t_i)$ becomes less than the distance to which the matter has been swept by the wake[91] : $u_i t_i (a_{nl}/a_i)^2 \approx \lambda_\nu(t_i) a_{nl}/a_i$, where the scale factor $a(t)$ is related to the redshift by $1 + z = a(t_0)/a(t)$. For filamentary wakes, this gives[104] $1 + z_{nl} \approx 4.5 \mu_6 h^2$ independent of z_i. With $h = 0.5$ and $\mu_6 = 4$, we have $z_{nl} \approx 3.5$. For sheet-like wakes, we find $1 + z_{nl} \approx 2 \mu_6 h^2$. Observations do indicate that $z = 2 - 3$ is the epoch of intensive galaxy and quasar formation[105]. Thin wakes of small relativistic loops are strongly suppressed by the neutrino free streaming[106], and it appears that loops play a negligible role in this scenario. Eqn. (8.4) then implies that most of the matter in the universe remains unclustered at the present time[93]. This may explain why dynamical measurements in clusters give values of Ω substantially smaller than 1. By contrast, in the cold dark matter scenario the loops accrete at least as much matter as the wakes, and the voids will be pierced by the long comet-like wakes formed behind relativistically moving loops. The characteristic scale of the large-scale structure in this scenario is $t_{eq} z_{eq} \sim 10 h^{-2} Mpc$. With $h = 0.5$ it is comparable to the scale suggested by observations[100] ($\sim 25 h^{-1} Mpc$). The surface density of the neutrino wakes produced subsequently decreases but the decrease is only $\propto t_i^{-1/3}$. This means that the structure on still larger scales can also be prominent.

Baryonic wakes in a neutrino-dominated universe start collapsing after baryons decouple from radiation, $t > t_{dec}$. However, since baryons constitute only a small fraction of the total density, the growth of these wakes is strongly suppressed. Baryonic wakes could nonetheless be cosmologically significant if the energy output from the primordial stars formed in the wakes might trigger some kind of explosive amplification and lead to preferential galaxy formation along these wakes[90]. They could also explain the existence of quasars at redshifts greater than 3. The scale of baryonic wakes, $t_{dec} z_{dec} \sim 50 h^{-1} Mpc$, is comparable to the largest-scale structure observed in the universe.

A novel outcome of the cosmic string scenario is that it predicts the generation of primordial magnetic fields[13,98,107]. The mechanism by which this happens relies on the fact that the relativistic motion of strings after decoupling of matter and radiation induces vorticity in the baryonic fluid. The vorticity then leads to the generation of primordial magnetic fields.

To put these arguments on a firmer footing it is necessary to establishe generation of

vorticity and then to show that the vorticity will lead to magnetic fields. Fortunately, the second step had been investigated in the 70's and several mechanisms are known by which vorticity can lead to magnetic fields. So the main task that remains is to show that there will be vortical motion and to estimate the vorticity.

An estimate of the Reynold's number for the flow of the baryonic fluid into the string wake shows that it is very high ($\sim 10^{11}$) and hence it is natural to suspect that the flow will be turbulent. A large Reynold's number, however, is not sufficient to guarantee turbulence and one must demonstrate that the flow is unstable. In the case of fluid flow into a cosmic string wake one can actually go further and explicitly describe the mechanism by which vorticity is generated.

Consider the wake formed by a relativistically moving string at the recombination epoch. At this epoch, the sound speed is dropping steeply but the flow of the fluid into the wake is still given by (7.11). Then, for string tensions that are suitable for structure formation, the fluid flow is supersonic and the wake is bounded by strong shocks. On large-scales this shock is uniform but on small scales, the shock is non-uniform because the string is wiggly and the wiggles have highly variable velocities. This is the crucial feature - the wiggly string wakes are bounded by *strong, non-uniform shocks*.

Once we have shown that the scenario has this feature, the presence of vorticity follows. From Euler's equation and steady flow, one finds that the vorticity is related to the gradient of the entropy by the equation [108]:

$$\vec{v} \times \vec{\omega} = -T\vec{\nabla}s \tag{8.7}$$

where, $\vec{v}$ is the fluid velocity, ω the vorticity, T the temperature and s the specific entropy. In the preshock region, the flow is isentropic but, at the shock, the entropy suffers a discontinuous jump. Since the shock is non-uniform, the post-shock entropy is different at different points along the shock. This gives us gradients in the post-shock entropy and a non-zero vorticity.

The presence of vorticity in the baryonic fluid flow means that the protons and electrons are in vortical motion. But we also have ambient photons and neutral particles which interact with the protons and electrons. The next crucial ingredient in the scenario is that the masses of the protons and electrons are different and therefore their interaction times with photons are also different. (The scattering cross-section of either particle with photons

is inversely proportional to the square of the mass of the particle.) Then the stronger interaction of photons with electrons slows them down with respect to the protons and the resulting electric current due to the differential rotation of charges produces a magnetic field. Such scenarios - using the different interaction rates of protons and electrons with photons - were proposed by Harrison [109], Mishustin and Ruzmaikin [110] and others in the 70's.

While the scenario is clear qualitatively, quantitative estimates are more difficult to obtain. The first hurdle is to understand the vorticity in the fluid flow. For this we would need to have a quantitative analysis of the turbulence in the fluid flow and, as far as I know, there is no theoretical recipe for analyzing turbulent flow. But dimensional arguments allow us to estimate the average vorticity as follows. We expect that vorticity will be produced on the scale on which the wake is inhomogenous. Therefore the co-moving scale of this vorticity is $\sim \Gamma G\mu t_i z_i$ where z_i is the redshift at the epoch t_i when the vorticity is generated at a time t_i. The velocity of the fluid is estimated as in (7.11) and so the vorticity is

$$\omega \sim \frac{u}{l} \sim \frac{0.1}{t_i} \ . \tag{8.8}$$

Given the vorticity, the generated magnetic field is estimated from the results of Mishustin and Ruzmaikin [110]

$$B \approx 2\frac{mc^2}{e} \frac{(1+z)^{5/2}}{\Omega^{1/2} H \tau_{e\gamma(0)}}\omega \tag{8.8}$$

where, m is the electron mass, e the electron charge, z the red- shift at which the vortical motion starts, Ω the mean-to-critical density or the baryonic matter, H the Hubble constant, $\tau_{e\gamma}(0)$ the interaction time between electrons and photons at the present epoch and ω is the angular velocity of the eddy. The interaction time between electrons and photons at the present epoch is given by

$$\tau_{e\gamma}^{-1}(0) = \frac{4\sigma_T \rho_\gamma(0)c}{3m} \tag{8.9}$$

where, $\sigma_T \sim 10^{-24} \ cm^2$ is the Thompson scattering cross-section and $\rho_\gamma(0)$ is the present photon energy density. Inserting $\Omega \sim 0.03$, $H \sim 50 \ km/s - Mpc$, $z = 10^3$, and $\omega = 10^{-13} \ s$ gives,

$$B \sim 10^{-14} \ G \ . \tag{8.10}$$

The magnetic field produced due to the vorticity at decoupling can be further amplified by turbulence in the wake and by a galactic dynamo. Such a tiny seed field is all that is needed to generate the observed galactic magnetic field of $10^{-6}G$.

Cosmic strings seem to be uniquely suited for generating magnetic fields via vorticity since they naturally have the two features that seem essential for this mechanism: coherence and strength. The first feature is that the vorticity should be on relatively large scales ($\sim 10kpc$) for which it is essential that the source producing the vorticity should also have this coherence scale. If the source is not lineal , it is difficult to see how this large a scale is obtained. The second point is that the vorticity on these scales has to be relatively large, implying that the source itself has to be undergoing violent motion. Once again, this feature comes up naturally in the string picture but seems hard to get with other sources.

Outlook:

While the cosmic string scenario for structure formation has ingredients that seem to be promising, it is not detailed enough yet to be testable. This is because the problem is doubly difficult - first one has to understand the evolution of the string network and then the evolution of the matter that gravitates around the network. The evolution of the network itself has turned out to be a really sticky problem and an analytical understanding is just beginning to emerge. The flow of matter around the string network promises to be an even more difficult problem since, as we have seen while discussing the generation of magnetic fields, the flow will be non-linear and turbulent. Without going into the details of the flow, one can only make some broad predictions about the *large-scale* structure as we have done above. To make predictions on galaxy scales, it is essential to understand the structure and fragmentation of the wake. Only then will we be able to say something about the galaxy-galaxy correlation function and other quantitative measures that can enable a comparison with observation.

These problems seem so difficult that at times I am tempted to think that it may be simpler to observe cosmic strings or rule them out (as a mechanism for structure formation) on the grounds that they are not observed. The millisecond pulsar observations seem to be a foolproof way to go but for this we have to wait for another decade or so. The microwave background anisotropy measurement by COBE does not confirm or rule out strings. But small-scale measurements - when they become more reliable - could test the cosmic string

scenario*. The direct observation of strings by their gravitational lensing property is also possible but is likely to be effort consuming. On the other hand, this effort seems very worthwhile considering all the exciting outcomes!

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* The situation is muddied by the interfering possiblities of having anisotropies due to inflation (and other sources) in addition to those due to strings.

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