1. INTRODUCTION

The detection of gravitational waves is one of the Holy Grails of modern physics (Schutz 1989; Giazzoto 1989). Following the pioneering experimental device of J.Weber (see, for example, Shapiro & Teukolsky 1983 for a clear introduction and historical references to Weber's resonant bar detectors) and many other attempts, a series of new-technology interferometric antennas is being planned/constructed to achieve higher sensitivities for the dimensionless wave amplitude h. These coordinated experimental efforts are being complemented by increasingly refined theoretical calculations in order to study the expected waveforms from each potential source.

Although a number of likely sources has been discussed, there is an underlying belief (which has, in fact, strongly influenced the design of the detectors) that non-spherical supernovae and coalescing binaries are among the strongest candidates for objects that the devices should detect. The purpose of this work is to argue quantitatively that wobbling neutron stars, to which comparatively little attention has been paid, may be an equally good bet regarding the probability of detection.

2. DISCUSSION AND CALCULATION

Rotating neutron stars will emit gravitational waves by means of a time-dependent quadrupole moment, generated either by the lack of body symmetry on the equatorial plane or by precession caused by a mis-alignment of the spin and symmetry axes. It is presently quite uncertain how asymmetric a pulsar can be and we shall not refer to the former case (but keep it in mind as a true possibility) in this work. The latter wobbling neutron stars will emit mainly at frequencies close to the rotational one f if the wobble angle θ is small. The resulting amplitude of the waves is calculated as $h = (16\,\pi\,G\,F/c^3\,\Omega^2)^{1/2}$; where the flux is $F = L_{GW}/4\pi\,r^2$ and $\Omega = 2\pi\,f$ is the angular frequency of the pulsar (Zimmermann 1978) yielding

$$h = 1.4 \, \times \, 10^{-18} \, \epsilon \, \theta \, \Big(\frac{I_{zz}}{10^{45}\ g\ cm^2}\Big)\ f_{kHz}^2\ r_{kpc}^{-1} \tag{1}$$

where

$$\epsilon = (I_{zz} \, - \, I_{xx})/I_{zz}$$

θ is the wobble angle, f_{kHz} is the frequency in kHz, r_{kpc} is the distance to the star in kpc, I_{zz} is the moment of inertia with respect to the rotation axis and I_{xx} is any of the moments of inertia orthogonal to it. This equation has been established using the "slow-motion" approximation for the gravitational energy output of the star (Misner, Thorne & Wheeler 1973) but is probably accurate up to a factor $\sim$ 2 in the rapidly rotating regime (Zhong 1985). For a given pulsar, distance and structure, the wobble angle can be used to parametrize the expected amplitude of eq.(1). According to Pines & Shaham (1974) and Zimmermann (1978) an upper physical limit to θ is $\theta_M \sim 10^{-1}$, and values of $\theta \sim 10^{-2} - 10^{-3}$ may be considered as moderate.

The expected sensitivity of the interferometric detectors to the waves (Vogt 1989; Giazzoto 1991) is of the order of $h \sim 10^{-22}$ for short-lived, impulsive bursts ; and $h \sim 10^{-25} - 10^{-26}$ for periodic sources in which integration times of about 10^7 s are possible. According to eq.(1), the emission expected from a pulsar undergoing a wobble motion characterized by θ needs also an accurate estimate of the quantity ϵ.

To study the strenght of the expected emission we have performed, as discussed in de Araújo et al. (1993, hereafter paper I), fully relativistic calculations of the stellar structure base on the approach of Butterworth & Ipser (1976) and Butterworth (1976) (see also Friedman, Ipser & Parker 1986). Our results indicate that the values of the gravitational ellipticity ϵ due to the stellar rotation are typically one or two orders of magnitude higher than those usually *adopted* in the literature, the latter being appropriate for slower radio pulsars (remarkably, comparable results are already implicit, for example, in Friedman, Ipser & Parker 1986). As shown below, this feature leads to outputs of h which may be detectable by the upcoming gravitational antennas generation if those pulsars deformed by rotation can be induced to precess by external/internal torques.

Table 1 gives the stellar parameters calculated at fixed baryon number (that corresponding to $M = 1.4\ M_\odot$ for a static star) for different rotation velocities using the medium-stiff Bethe-Johnson I equation of

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state (Bethe & Johnson 1974) for the neutron matter. While it is not guaranteed that the actual composition of the pulsars can be represented by this choice, it is generally agreed that the chosen equation of state is a reasonable compromise given the present uncertainties on the subject and it will be employed for the sake of definiteness.

In order to relate the putative gravitational wave emission to the expected signal at the detectors it is imperative to estimate the damping timescale for a given wobbling pulsar due to the emission of gravitational waves. The characteristic braking timescale is

$$\tau_{brake} \simeq \frac{5 \, c^5}{128 \, G \, I_{zz} \, \Omega_o^4 \, \epsilon^2} \simeq 2 \left(\frac{\epsilon}{0.1}\right)^{-2} \left(\frac{I_{zz}}{10^{45} \, g \, cm^2}\right)^{-1} \left(\frac{\Omega_o}{5000 \, s^{-1}}\right)^{-4} \, s \qquad (2)$$

where Ω_o is the initial rotational angular velocity. Therefore, even though the sources would have an explicit periodic behaviour, the fact that $\tau_{brake} \ll 10^7 \, s$ qualifies them as "impulsive" or burst one since the duration of the emission is short compared to the experimental observation time (see a full discussion in Thorne 1987). If we require $h \geq 10^{-22}$ (the condition of burst detectability foreseen for LIGO-type interferometers) for a $P = 2ms$ pulsar with a "fiducial" value of $I_{zz} = 10^{45} \, g \, cm^2$ we would need

$$\frac{\theta}{r_{kpc}} \geq 6 \times 10^{-3} \qquad (3)$$

Thus, observation of a "spike" burst with duration limited by τ_{brake} from anywhere in the Galaxy ($r \simeq 20 \, kpc$) would require wobble angles of the order of the more extreme theoretical expectations. However, it is now known that the population of ms pulsars is large (see e.g. Kulkarni & Thorsett 1993) and there have been (unexpectedly) observed in nearby clusters like 47 Tuc (see e.g. Lyne 1992) which is only $\sim 4 \, kpc$ away. Those potential sources would require values of $\theta \simeq 10^{-2}$ to be detected at LIGO-type observatories. We remark that a reliable calculation of ϵ is an important ingredient for such a conclusion. In addition, not only the interesting wobble angles are probably less than extreme, but also that the emission will come out at frequencies which are *not* expected (in principle) to suffer from severe noise problems in contrast with the values required to detect the precession of slower Crab-like sources.

It should be stressed that the actual number of sources in a sphere of radius 20 kpc (roughly the volume of the Galaxy) will depend on the details of the internal dynamics and external torques and can not be reliable evaluated other than in a statistical way. The total number of pulsars in the sample volume may be as high as 10^8 and it has been estimated (Kulkarni, Narayan & Romani 1990) that the subpopulation of ms pulsars in globular clusters only is higher than 10^4. Detailed calculations of external (i.e. encounters with perturbing stars) and internal (i.e. phase transitions) mechanisms capable of exciting moderate wobble motions must be undertaken to address the expected number of sources at a given time. As an example, mechanisms which may conceivably produce small wobble motions in the range $\theta \sim 10^{-4} - 10^{-6}$ have been discussed in Pines & Shaham (1974). Several other excitation mechanisms may be operative as well. If Nature provides even a single source out of the whole galactic population (the case for a precessing Her X-1 and newer data from radio pulsars seem to suggest the possibilty of wobble being an ubiquitous phenomenon) it may be enough greatly to improve our knowledge on gravitational wave phenomena.

Table 1. Stellar parameters of rotating B-J I pulsars

Ω $(rad\ s^{-1})$	e	ϵ	I_{zz} $(10^{45}\ g\ cm^2)$	I_{xx} $(10^{45}\ g\ cm^2)$
3000	0.31	0.047	0.6557	0.6246
4030	0.38	0.089	0.6994	0.6374
5000	0.51	0.14	0.7763	0.6658
6203	0.75	0.25	0.9946	0.7476

Table captions

Table 1. The relevant parameters for gravitational waves emission from wobbling pulsars. Here e is the usual stellar eccentricity and the other quantities are defined in the text. Note that the maximum value of Ω corresponds to the Keplerian value although several instabilities may limit the actual maximum rotation rate to about 0.9 of the former.

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