

**Mixed dark matter models with a non-thermal
hot component: fluctuation evolution.**

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Abstract – We calculate the linear evolution for a class of mixed dark matter models, where the hot component derives from the decay of a heavier particle (CntHDM models). These models differ from ordinary mixed models based on massive neutrinos (CHDM models), for which the hot component has a phase space distribution which derives from a thermal one. In CntHDM models the density of the hot component and the derelativisation redshift of its quanta are independent parameters. In this work we provide the spectra for a number of CntHDM models, and compare them with CDM and CHDM spectra. If PQ symmetry and SUSY simultaneously hold, the lightest standard neutralino can be expected to decay into axino and photon. We briefly summarise the features of this particle model which gives rise to the cosmological framework discussed here, as a fairly generic consequence. Other frameworks which lead to similar models are also briefly discussed.

key words: Cosmology – galaxies : origin.

1. Introduction.

Mixed dark matter (MDM) models (Valdarnini & Bonometto 1985, Bonometto & Valdarnini 1985, Achilli et al. 1985, Holtzman 1989, see also Fang, Li, & Xiang 1984) allow a better fit of large scale data than most other models (Schaefer, Shafi, & Stecker 1989; van Dalen & Schaefer 1992; Schaefer & Shafi 1992; Taylor & Rowan–Robinson 1992; Davis, Summers, & Schlegel 1992, Holtzman & Primack 1993; Pogosyan & Starobinsky 1993; Liddle & Lyth 1993, Klypin et al. 1993, Klypin, Nolthenius & Primack 1993, Bonometto et al. 1994). Recent treatments of MDM were mostly based on a mixture of CDM, HDM and baryons with $\Omega_c/\Omega_h/\Omega_b = 0.6/0.3/0.1$ (Ω : density parameters), although different ratios were also considered. If HDM is made of fermions of mass m_h , with g_f spin states, which decoupled when $T \gg m_h c^2$, it is $\Omega_h \simeq 0.3(m_h g_f / 14 \text{ eV})$. The standard candidate as HDM component is the τ -neutrino, assumed to have a mass $m_h = 7 \text{ eV}$.

Hereafter this version of MDM will be called CHDM. Within its context Ω_h and the derelativization redshift for the hot component $z_{de} \simeq m_h c^2 / 3 T_{o,\nu}$ are connected through m_h ($T_{o,\nu}$ is the present neutrino temperature). In turn, z_{de} fixes the mass scale $M_{D,h}$ above which fluctuations in the hot component are not erased by free streaming after their entry in the horizon.

Instead of being composed of massive neutrinos, the hot component could have arisen from the decay of heavier particles. In this case, the link among m_h , g_f , Ω_h , z_{de} , and $M_{D,h}$ is not so cogent. There are still precise constraints due, e.g., to primeval nucleosynthesis, but the parameter space is spanned by two independent continuous parameters (e.g., Ω_h and z_{de}), instead of a continuous and a discrete parameter (e.g., Ω_h and g_f). The distribution of hot particles in momentum space is then quite different from the one originating from a thermal distribution. In this note we shall discuss such alternative to CHDM (hereafter denominated CntHDM).

The decay of mother particles takes place when fluctuations over the relevant scales are still well outside the horizon. Daughter particles are then relativistic and an analytical treatment of the evolution of fluctuations during these stages can be given. The critical events to set up the shape of the final spectrum are the entry of the fluctuations in the horizon and the derelativization of the particles forming its hot component. These two events can take place in different order, according to the scale considered and to the model parameters. At a redshift $\sim 10^3$, primeval plasma (re)combines. In purely baryonic models recombination is a critical step, as it sets up a minimal mass scale for the survival of baryonic fluctuations and provides information on very–small–scale CBR fluctuations. In models dominated by DM, these stages are not so essential, as baryons will later fall in potential wells created by CDM and/or HDM, while the relevant scales for CBR fluctuation observations however enter the horizon after recombination. The presence of radiation and baryons is critical for fluctuations entering the horizon before equivalence. The growing of the CDM component is then almost stopped by Meszaros effect, i.e. by the fact that baryon and radiation can only oscillate as sound waves while, until equivalence, they are the dominant

substance in the Universe. After equivalence, as is known, DM fluctuation growing can restart.

Furthermore, the baryonic component has a non negligible influence on the detailed final spectrum. Our models take also into account a massless neutrino background and its fluctuations (which are rapidly dissipated at the entry in the horizon). Three, two or one massless neutrinos were considered, according to the model parameters, as is discussed below.

The main output of this paper are a number of spectra for CntHDM models, computed in detail through all their linear evolution, down to $z = 0$. The models considered are given in Table 1 of sec. 2. As is also outlined in sec. 4, the study of each model still requires a fairly large amount of CPU, and further numerical analysis is still in progress.

A particle model, recently discussed by Bonometto, Gabbiani & Masiero (1994, BGM), originates CntHDM models as a fairly generic consequence. We shall briefly outline some features of this particle model in sec. 2. The class of cosmological models described here is however relevant, also independently of that specific particle physics. In sec. 3 we will study the analytical part of the evolution of density fluctuations. In sec. 4 the numerical treatment will be described. Sec. 5 is devoted to a discussion of the results.

2. A particle model.

In this section we debate a specific particle model which gives rise to a cosmological framework of the kind discussed in the next sections. Although this model is completely consistent, the cosmological results have a validity which extends well beyond this peculiar particle physics. In recent years cosmologists have become familiar with supersymmetry (SUSY) and Peccei–Quinn (PQ) symmetry. Both of them provide CDM candidates. The lightest mass eigenstate of neutral fermion SUSY partners, called *neutralino* (χ), has a mass $m_\chi \gtrsim 30$ GeV and must therefore decouple when T is significantly below m_χ (Lee & Weinberg 1977, Ellis et al., 1984). The goldstone boson associated to PQ symmetry, arising at $T < V_{PQ}$ (scale of PQ symmetry breakdown), is called axion (a). If both SUSY and PQ symmetry are implemented, also SUSY partners of a 's exist; the fermion partner of a is called axino (\tilde{a}). In BGM it is shown that we can expect two \tilde{a} background to exist. Besides thermal \tilde{a} 's, decoupling slightly below V_{PQ} , probably diluted by entropy inflow(s) at later phase transition(s), non–thermal \tilde{a} 's originate from χ decay. The former component is cold, the latter one is a non–thermal hot component. It is possible that all DM consists of \tilde{a} 's, which can account for both components of MDM.

The decay $\chi \rightarrow \tilde{a} + \gamma$ takes place at $z_{\chi, Dy} \sim 10^9$ and \tilde{a} 's have initially momentum $P = m_\chi c/2$. They derelativize at $z_{de} \simeq z_{\chi, Dy} 2m_{\tilde{a}}/m_\chi$. The outcoming density parameter $\Omega_{\tilde{a}} = \Omega_h$ is linked to the axino mass ($m_{\tilde{a}}$) and χ number density at the eve of their decay (n_χ). These quantities are linked by a number of particle relations, discussed in BGM and summarized in fig. 1. In such figure m_{sf} is the scale of the soft SUSY breaking (*sfermion* scale).

Different curves refer to possible choices of m_χ and m_t (top quark mass), as is detailed in its caption.

Table 1: Parameters for CntHDM models

model	1	2	3	4	5	6	7	8	9	10
$\Omega_{\tilde{a}}$	0.30	0.30	0.30	0.15	0.20	0.15	0.20	0.20	0.15	0.20
$z_{de}/10^4$	1.5	6	3	1.5	0.7	0.7	0.9	1	0.4	0.4
massless ν 's	2	3	3	3	2	2	2	1	2	1

For each value of m_{sf} , fig. 1 provides the values allowed for the axino mass $m_{\tilde{a}}$ (decreasing curves) and for the quantity $z_{de}V_{PQ}/\Omega_{\tilde{a}}$, where the particle parameter V_{PQ} , expected to range between $5 \times 10^9 \text{ GeV}$ and 10^{12} GeV , is combined with z_{de} (\tilde{a} derelativization redshift) and $\Omega_{\tilde{a}}$ (hot DM density parameter).

Fig. 1 shows that the quantity $z_{de}V_{PQ}/\Omega_{\tilde{a}}$ can be expected to range between 10^{13} GeV and 10^{16} GeV , for reasonable values of m_{sf} . In turn, it is possible to find various combinations of particle parameters which give $\Omega_{\tilde{a}}$ in the range 0.1–1, and z_{de} in the range 10^3 – 10^5 .

Further constraints are set by the number of massless neutrinos still present during primeval nucleosynthesis. Let g_{ν} be the number of massless ν spin states present during the nucleosynthesis (ν 's with mass much below nucleosynthesis temperatures are to be considered massless). Standard stellar abundance data for light elements allow up to $g_{\nu} \simeq 7$. Accordingly, if $g_{\nu} = 6$ (3 standard neutrinos) we are allowed $g^+ = 1$ extra spin states. It can be $g^+ = 3$ or $g^+ = 5$, if one or two neutrino(s) are supposed to be quite heavy and to have already decayed before nucleosynthesis. Neutrinos with a mass $m_{\nu} = 7 \text{ eV}$ ($= 8 \times 10^4 \text{ K}$) derelativize at a redshift $z \simeq 1.5 \times 10^4$ and yield $\Omega_{\nu} = 0.3$. A particle allowed half of their energy density at nucleosynthesis and derelativizing at z_{de} will then yield $\Omega_h \simeq z_{de}/10^5$. If the number of extra neutrino spin states allowed during nucleosynthesis is g^+ , the limit for Ω_h reads:

$$\Omega_h \lesssim g^+ z_{de}/10^5. \quad (2.1)$$

The number g_{ν} of effective massless neutrinos is used in the equations ruling the expansion of the Universe through the quantity $w_{\nu} = (7/4)(4/11)^{\frac{4}{3}}g_{\nu}$, that will be used below.

For all our models we took $\Omega_{total} = 1$, $\Omega_b = 0.05$, and $H = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Values of Ω_h and z_{de} ranging in the intervals 0.15–0.30 and 4×10^3 – 6×10^4 were considered. Models with low z_{de} show interesting features and the parameter space was explored in that region in more detail. In Table 1 we report the values of Ω_h , z_{de} , and the number of massless neutrinos, for those models whose spectra are worked out here.

3. Density fluctuation evolution: analytical aspects.

Let $f(x_{\alpha}, P_{\alpha}, t)$ be the ntH (non-thermal hot) \tilde{a} distribution in phase space (x_{α} , $P_{\alpha} = P n_{\alpha}$: particle

coordinates and momentum). We must also consider the source of \tilde{a} 's due to χ -decay: $S(x_\alpha, P_\alpha, t)$. Up to first order,

$$f = f_o(P, t)[1 + \tilde{\epsilon}(x_\alpha, P, t)] , \quad S = S_o(P, t)[1 + \sigma_s(x_\alpha, P, t)] \quad (3.1)$$

(Notice that $\tilde{\epsilon}$ and σ_s do not depend on n_α). The analysis is based on the equation

$$S = \partial f / \partial t + (\partial f / \partial x_\alpha) \dot{x}_\alpha + (\partial f / \partial P) \dot{P} \quad (3.2)$$

taking into account that $\dot{x}_\alpha = cn_\alpha(P/P_o)(T_o/T)$, $\dot{P} = P[(\dot{T}/T) + y/4]$. Here $P_o c$ is energy, T is radiation temperature, y yields perturbation self-gravity and will be given in eq.(3.7) below. At the 0-th order, eq. (3.2) yields

$$S_o = \partial f_o / \partial t + P(\dot{T}/T)(\partial f_o / \partial P) , \quad (3.3)$$

while the source term, due to χ -decay, is

$$S_o = (h^3/4\pi)n_{\chi, dg}(T/T_{\chi, Dy})^3 \exp(-t/t_{\chi, Dy}) (t_{\chi, Dy} P^2)^{-1} \delta(m_{\chi, Dy} c/2 - P) \quad (3.4)$$

($t_{\chi, Dy}$: χ mean life, $n_{\chi, dg}$: χ 's number density at the decoupling). With such source terms the solution of eq. (3.3) reads

$$f_o = (h^3/4\pi)n_{\chi, dg} \exp(-Q)(2Q/P^3) \theta(m_{\chi, Dy} c/2 - P) \quad (3.5)$$

with

$$Q = (2P/m_\chi c)^2 (t/t_{\chi, Dy}) \quad (3.6)$$

(notice that $Q \delta(m_\chi c/2 - P) = t/t_{\chi, Dy}$); χ decays occur in radiation dominated era, when $t/t_{\chi, Dy} = (T_{\chi, Dy}/T)^2$.

We shall treat the first order equation for the case of a plane wave in x_3 direction. In such direction the component of P is μP . Let also $h_{\alpha\beta} = \eta_{\alpha\beta} - g_{\alpha\beta}$ express the deviation of the metric tensor $g_{\alpha\beta}$ from purely spatially flat $\eta_{\alpha\beta}$ components ($\alpha, \beta = 1..3$; t being the universal time, only g_{oo} does not vanish, besides of $g_{\alpha\beta}$). Then

$$y = (1 - \mu^2)3\dot{h}_t - (1 - 3\mu^2)\dot{h}_{33} = 2P_o(\mu)\dot{h}_t + 2P_2(\mu)\dot{h}_3 , \quad (3.7)$$

where

$$3h_t = \sum_{\alpha=1}^3 h_{\alpha\alpha} , \quad h_3 = h_t - h_{33} . \quad (3.8)$$

Let us then take $\tilde{\epsilon} = \epsilon(k, P, \mu, t) \exp(ikx_3 T/T_o)$, in order to treat each length scale separately (ρ_c : present critical density; P_l : Legendre Polynomials).

The first order equation splits in two parts. Let be $K = kcT/T_o$. For $P < m_\chi c/2$,

$$\dot{\epsilon} + i\epsilon\mu KP/P_o = (1 + 2Q)y/4 , \quad (3.9)$$

while, for $P = m_\chi c/2$, it is $\epsilon = \sigma_s + yt/2$. The latter equation holds at χ decay and sets initial conditions to eq. (3.9). Such equation is to be solved for any μ and P ; however P is related to Q (see 3.6) and the distribution f_o decreases exponentially with Q (see 3.5). This will allow to select a finite set of Q values. As far as μ is concerned, we perform the expansion

$$\epsilon(k, P, \mu, t) = \sum_{l=0}^{\infty} (-i)^l \sigma_l(k, P, t) P_l(\mu) . \quad (3.10)$$

Then, taking eq. (3.7) into account, eq. (3.9) yields the system

$$\begin{aligned} \dot{\sigma}_o &= -(1/3)K(P/P_o)\sigma_1 + (1/2 + Q)\dot{h}_t , \quad \dot{\sigma}_1 = K(P/P_o)(\sigma_o - 2\sigma_2/5) \\ \dot{\sigma}_2 &= K(P/P_o)(2\sigma_1/3 - 3\sigma_3/7) + (1/2 + Q)\dot{h}_3 , \quad \dot{\sigma}_l = K(P/P_o)(l_- \sigma_{l-1} - l_+ \sigma_{l+1}) \quad (l > 2) \end{aligned} \quad (3.11)$$

[here $l_- = l/(2l - 1)$, $l_+ = (l + 1)/(2l + 3)$].

Radiation is to be treated very much alike. The critical difference is that photons are massless and therefore all P behave in the same way (furthermore, of course, the P distribution is different). Integrating the photon distribution over P and expanding the outcoming $\delta(k, \mu, t) = \sum_{l=0}^{\infty} (-i)^l \delta_l(k, t) P_l(\mu)$, we obtain a closed system analogous to (3.11), which reads

$$\begin{aligned} \dot{\delta}_o &= -(1/3)K\delta_1 + 2\dot{h}_t , \quad \dot{\delta}_1 = K(\delta_o - 2\delta_2/5) - (\delta_1 - 4w)\nu \\ \dot{\delta}_2 &= K(2\delta_1/3 - 3\delta_3/7) + 2\dot{h}_3 - \delta_2 9\nu/10 , \quad \dot{\delta}_l = K(l_- \delta_{l-1} - l_+ \delta_{l+1}) - \delta_l \nu \quad (l > 2). \end{aligned} \quad (3.12)$$

Apart of the replacement $1/2 + Q \rightarrow 2$, the main difference are the terms accounting for matter–radiation collisions: $\nu = n_e \sigma_T c$ is the inverse γ collision time (n_e : electron number density, σ_T : Thomson cross–section) and w is the velocity field in baryons. Eqts. (3.11) and (3.12) are to be implemented by the equations for baryon and CDM density fluctuations (δ_b , δ_c), metric perturbations (\dot{h}_t) and t dependence of T . Such equations read:

$$\begin{aligned} (\dot{T}/T)^2 &= \Gamma_3[\Omega_b + (1 + w_n u)T/T_e + \Omega_h e_h + \Omega_c] , \\ \dot{\delta}_b &= -Kw + 3\dot{h}_t/2 , \quad \dot{w} = w\dot{T}/T + (\delta_1 - 4w)(\rho_r/\rho_b)\nu/3 , \\ \ddot{h}_t &= 2\dot{h}_t\dot{T}/T + \Gamma_3[\delta_b\Omega_b + 2(\delta_o + \delta_{o,\nu})T/T_e + e_o\Omega_h + \delta_c\Omega_c] , \\ \dot{h}_3/2 &= -\dot{h}_t + (\Gamma_2/kc)[w\Omega_b + (\delta_1 + \delta_{1,\nu})T/3T_e + e_1\Omega_h] . \end{aligned} \quad (3.13)$$

Here ρ_r/ρ_b is the radiation/baryon density ratio, $\Omega_c T_e = 10^4 \Omega_c T_o$ is the temperature when CDM and radiation have equal densities, $\Gamma_n = (8\pi/3)G\rho_c(T/T_o)^n$, and

$$e_h = \int_o^\chi dQ e^{-Q} \sqrt{1 + Q \frac{T^2}{T_{de}^2}} , \quad e_o = \int_o^\chi dQ e^{-Q} \frac{Q \frac{T^2}{T_{de}^2 + \frac{1}{2}}}{\sqrt{Q + \frac{T^2}{T_{de}^2}}} \sigma_o , \quad e_1 = \frac{T}{2T_{de}} \int_o^\chi dQ e^{-Q} Q^{1/2} \sigma_1 . \quad (3.14)$$

Here $\chi = (T_{\chi, Dy}/T)^2$ and, in most cases, it cannot be numerically distinguished from ∞ . T_{de} is the typical derelativization temperature of axinos.

We took into account also massless neutrinos. Their fluctuations are rapidly damped as soon as they enter the horizon. In principle, to follow such damping in detail, the same number of equations as for radiation are to be used. Some approximations, introduced when residual neutrino fluctuations are below the precision limit of the whole algorithm, have been set up in order to limit the number of harmonic components. They will not be discussed here. In eq.(3.13), $\delta_{o,\nu}$ and $\delta_{1,\nu}$ indicate the first two harmonics for neutrinos, multiplied by a factor which depends on the number of massless neutrinos in the model.

4. Density fluctuation evolution: numerical aspects.

We performed a set of numerical integrations of the integro–differential system of equation (3.11), (3.12), (3.13), plus massless neutrinos. The integral part of the system was treated by taking 10 values of P associated with those values of Q needed to perform the integrations in eq. (3.14) with the Gauss–Laguerre projection procedure. The number of P_l in eqs. (3.11)–(3.12) was self regulated to obtain a precision of $1:10^5$. The algorithm allowed up to a maximum of 499 harmonics, which were never reached. The integration routine is a distant descendant of the one used by Bonometto, Caldarà, & Lucchin (1983), although the treatment of collisionless particles with ntH spectrum required substantial changes.

Altogether the differential equations to integrate numerically are up to 5505. For each model 9 length–scales were considered (1 Mpc, 5 Mpc, 10 Mpc, 20 Mpc, 50 Mpc, 100 Mpc, 200 Mpc, 500 Mpc, 1000 Mpc), approximately spanning the mass–scale range from $10^{12}M_{\odot}$ to $10^{21}M_{\odot}$. Each model requires approximately 37 hours of CPU at a 4000/90 vaxstation. Small length–scale cases are obviously taking much more time than great length–scale ones.

For the sake of example, in fig. 2 we report the detailed behaviour of density fluctuations for the various components, for the model 5 and the 100 Mpc case. Some irregularities are present in the plot of matter and radiation sound–waves, due to a number of prints not fully adequate to follow the oscillations in detail.

In fig. 3 and 4 we plot the outcoming spectra at $z = 0$, arising from primordial Zel’dovich spectra, for the models 2 and 6. In the same plots the spectra for a pure CDM and a standard CHDM model are also shown. The CDM spectrum has also been worked out with our algorithm by setting $\Omega_h = 10^{-3}$. The discrepancy between such spectrum and the one given by Holtzman (1989), for the same cosmological parameters, is typically $\lesssim 0.5\%$, and nowhere exceeds 2%.

In order to argue on the physical interpretation of the outcoming spectra, a direct comparison with CDM and CHDM is more suitable than the detailed spectral shape. Henceforth we preferred to plot the differences $\Delta \log[P(k)] \equiv \log[P(k)] - \log[P(k)]_{CDM,CHDM}$, for the 10 cases we evaluated, rather $P(k)$ themselves. In fig. 4, 5 and 6 such $\Delta \log[P(k)]$ are shown for the models 1–5, 5–7 and 8–10, respectively.

Table 2: Transfer function coefficients for CntHDM models

model	c_1	c_2	c_3	c_4
1	-0.12991D+01	0.11015D+02	0.34827D+02	0.40777D+03
2	-0.45687D+01	0.60267D+02	-0.13429D+03	0.37425D+03
3	-0.24013D+01	0.38752D+02	-0.81353D+02	0.44185D+03
4	-0.17309D+01	0.17613D+02	0.69582D+02	0.13268D+03
5	-0.11754D+01	0.49944D+01	0.13379D+03	0.16949D+03
6	-0.20006D+01	0.16444D+02	0.10363D+03	0.11294D+03
7	-0.13807D+01	0.10134D+02	0.10300D+03	0.18045D+03
8	-0.12392D+01	0.10929D+02	0.86743D+02	0.16254D+03
9	-0.22635D+01	0.20171D+02	0.10958D+03	0.11857D+03
10	-0.19229D+01	0.11527D+02	0.14706D+03	0.15760D+03

5. Discussion.

The main outputs of this work are reported in the figures and in Table 2, where we provide the coefficients for a numerical interpolation of the transfer functions

$$\mathcal{T}(k) = \sqrt{P(k)/k} = \mathcal{N} / \left(1 + \sum_{j=1}^4 c_j [k/(\text{Mpc}^{-1})]^{j/2} \right) \quad (5.1)$$

through the coefficients c_j (the normalization coefficient \mathcal{N} will not be given here).

In fig. 5, fig. 6 and fig. 7 we compare the spectra of the models studied here and those of CDM and standard CHDM. Most features reported on these figures can be qualitatively understood taking into account the corresponding values of Ω_h and z_{de} .

Model 1 (in fig. 5, solid curve). The behaviour of this model is not far from standard CHDM. Its transfer function nowhere exceeds CHDM by more than a factor 1.4. No attempt was made to fine tune Ω_h and z_{de} , to obtain a still nearer spectrum, but a better fit is possible.

Models 2 and 3 (in fig. 5: dashed curve and dotted curve). They are characterized by a greater z_{de} and the same Ω_h , in respect to CHDM. Their transfer functions are then greater than CHDM over all those scales which enter the horizon when the hot component is already derelativized. At large k , the curves fall down towards CHDM, as is to be expected.

Model 4 (in fig. 5: short-dashed dotted curve). This model differs from model 1 for a smaller Ω_h (z_{de} is the same). It is therefore intermediate between CDM and CHDM. The spectrum bends at the same scale as for CHDM and the transfer function ratio is then decreasing fairly slowly, in respect to CDM.

Models 5–6–7 (in fig.6). Their behaviours show that a decrease of z_{de} can widely compensate an increase of Ω_h . The spectrum falls more rapidly than model 4 and begins to flatten at a smaller k . These models were considered to explore the parameter space with intermediate values for z_{de} .

Models 8–9–10 (in fig.7). While model 8 still reproduces a situation similar to CHDM, just with smaller density for the hot component, the other two models (with low z_{de}) show a consistent decrease of the spectrum if compared to CDM for small k 's and a stabilization of their behaviours at $k/Mpc^{-1} \simeq 1$. This is likely to cause a significant improvement in respect to CHDM for the scales over which objects can form at high z .

The linear evolution of primeval fluctuations can be used to evaluate a number of observational quantities that will not be fully enumerated here. Each one of them can be obtained by performing suitable integrations over particular regions of the spectrum. After COBE evaluation of $\delta T/T$, it can be convenient to compare different models by requiring them to give the same result over the COBE angular scale. This is approximately obtained here by requiring that all spectra coincide at 500 Mpc. A more detailed integration of the numerical outputs for radiation can be performed to improve $\delta T/T$ results; the radiative component is evaluated by our algorithm down to $z \sim 500$, and we shall use all relevant harmonics to give the expected $\delta T/T$, over a wide range of scales, in a forthcoming work.

Among the advantages of CHDM in respect to CDM is the behaviour of the spectrum over the scales yielding bulk velocities. However, the faster decrease of CHDM also leads to predicting a rather unadequate amount of high-redshift objects. This can be a problem for high redshift galaxies and QSO's (see, e.g., Pogosyan e Starobinsky 1994). More recently, it has been pointed out that the problem can be even more severe for objects associated to damped Ly_α clouds (Mo and Miralda-Escude', 1994, Subramanian and Padmanabhan, 1994, Kauffmann and Charlot, 1994, Ma and Bertschinger, 1994, see however also Klypin et al., 1994).

In this connection we wish to draw the attention on the characteristics of the models 5–6 and 9–10. Such choice of parameters give a decrease of $P(k)$ (in respect to CDM) down to intermediate scales, while the difference between CDM and such models tends then to stabilize, instead of further decreasing as in standard CHDM. More detailed computations are needed to verify up to which degree this improving of outcoming spectra can ease the fit with observational data.

There can be scarce doubts that, thanks to the presence of an extra parameter, CnTHDM models are more flexible than CHDM ones. However such extra parameter is connected with precise data of particle physics, which can be tested through experiments which are either already feasible or can be expected to be performed in a fairly nearby future. This does not only concern the axino model discussed in sec.2, but also other models with decaying particles. Among them, models in which there can be decays of one neutrino flavour into lighter neutrinos can be of particular relevance. Although these decays have a more complex kinematics, the cosmological behaviour due to the outcoming spectra can be very similar to the

ones discussed here.

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FIGURE CAPTIONS

Fig. 1 – Particle and cosmological parameter for the \tilde{a} model. The four lines refer to different choices of m_χ and m_{top} . The solid lines are for $m_\chi = 30$ GeV and $m_{top} = 120$ GeV. The dotted lines are for $m_\chi = 30$ GeV and $m_{top} = 180$ GeV. The short dashed lines are for $m_\chi = 60$ GeV and $m_{top} = 120$ GeV. The long dashed lines are for $m_\chi = 60$ GeV and $m_{top} = 180$ GeV. The area between solid and long dashed lines is therefore however allowed for a combination of m_χ and m_{top} , in their allowed ranges.

Fig. 2 – Examples of time evolution of the different components (CDM, \tilde{a} 's, massless ν 's, baryons, radiation); $t_{rif} = 1.44 \times 10^{11}$ s. The ordinate scale is arbitrary.

Fig. 3 – Spectrum at $z=0$ for the model 2 (see text). Dotted lines refer to a CDM and a CHDM model (the latter has $\Omega_c/\Omega_h/\Omega_b = 0.6/0.3/0.1$).

Fig. 4 – The same as fig. 3 for the model 6 (see text).

Fig. 5 – The difference $\Delta \log[P(k)]$ between the models 1 to 4 and CDM (lower curves) or CHDM (upper curves) is given in function of k . The solid curves refer to model 1. The dotted curves refer to model 2. The dashed curves refer to model 3. The short-dashed dotted curves refer to model 4.

Fig. 6 – The same as fig. 5. Here the models 5 to 7 are considered. The solid curves refer to model 5. The dotted curves refer to model 6. The dashed curves refer to model 7.

Fig. 7 – The same as fig. 5. Here the models 8 to 10 are considered. The solid curves refer to model 8. The dotted curves refer to model 9. The dashed curves refer to model 10.

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