

# Young LMC star clusters as a test for stellar evolutionary models

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**Abstract.** We compare the observed colour-magnitude diagrams (CMD) and the main sequence luminosity functions (LFs) of four Large Magellanic Cloud (LMC) star clusters namely, NGC 1711, NGC 2004, NGC 2164 and NGC 2214 with the synthetic ones derived from the stellar evolutionary models. Four different types of stellar evolutionary models have been used for comparison. The comparison of the features of the observed CMDs with the synthetic ones favour the overshoot models from Bressan et al. (1993). The synthetic integrated luminosity functions from the models can be matched with the observed ones by varying the value of mass function slopes. In order to constrain the models from the comparison of the synthetic ILFs with the observed ones, reliable estimates of mass function slope and binary fraction are desired.

**Key words:** LMC clusters - NGC 1711 - NGC 2004 - NGC 2164 - NGC 2214 - theoretical stellar evolutionary models

## 1. Introduction

Colour magnitude diagrams (CMDs) of young Magellanic Cloud (MC) star clusters are a very good testing ground for the stellar evolutionary models (SEMs) in order to discriminate among the possible evolutionary scenarios of high mass to intermediate mass stars. Since clusters in the MC are rich, they populate almost all the evolutionary phases and also occupy regions of the age and metallicity domain which are not populated in our galaxy. They therefore extend the range of comparison between SEMs and observational data. Furthermore, their study is mandatory for the understanding of star clusters in external galaxies where only integrated properties can be observed.

A detailed comparison of SEMs with the narrow and well defined stellar sequences in the CMDs of MC star clusters could not be done till recently, because most of the earlier CMDs (cf. Sagar & Pandey 1989, Seggewis & Richtler 1989) are based on either photographic or electronographic observations which are, in general, not only restricted to bright stars ( $V \sim 17-18$  mag) but also limited in accuracy. However, the advent of modern detectors and software for doing accurate photometry in crowded regions have recently made such work possible. For example, Chiosi et al. (1989) analyzed the BV CCD data of

the cluster NGC 1866 to disentangle classical stellar evolutionary models from that incorporate convective core overshooting (CCO). They conclude that substantial overshooting ought to occur in stars with mass of about  $5 M_{\odot}$ . Vallenari et al. (1991) also support the above fact on the basis of an analysis of the CCD data of NGC 2164. Bencivenni et al. (1991) while analyzing the similar data for the cluster NGC 2004 conclude that their models based on classical theory can very well explain the number of stars present in the various evolutionary phases. Stothers & Chin (1992) find that in the mass range  $4-15 M_{\odot}$ , very little or no overshooting of the core is needed to reproduce the observed features of the blue populous clusters, NGC 330 and NGC 458 in the Small Magellanic Cloud. This leads to a confusing state regarding the need for CCO in the models for various masses and its amount. The situation gets further complicated by the use of different opacity tables in different SEMs. Various tables of radiative opacity have been published by the Livermore group over the last few years, the two recent ones being from Rogers & Iglesias (1992, hereafter referred to as OPAL1) and from Iglesias et al. (1992, hereafter referred to as OPAL2). These modify the surface parameters of the stars significantly in comparison with the opacities from Los Alamos Opacity Library (LAOL) of Huebner et al. (1977). Stothers & Chin (1991) show that in the structure calculations of SEMs, a large assumed metal abundance can remove the need for assuming significant CCO. At the same time Schaller et al. (1992) and Bressan et al. (1993) show that such an assumption has little effect on the lifetimes and lifetime ratios of the central H and He-burning phases. Here we try to address the above questions by comparing synthetic CMDs and luminosity functions (LFs) obtained from the SEMs given by Castellani et al. (1990), Schaller et al. (1992) and Bressan et al. (1993) with the observed CMDs and LFs of the four LMC star clusters namely, NGC 1711, NGC 2004, NGC 2164 & NGC 2214. The four clusters considered span a wide range in age, thus have a range in turn-off masses. This analysis will be very helpful in discriminating the properties of these different masses as predicted in the models. The remaining sections contain the method adopted for incompleteness correction and field star subtraction, a brief description of the SEMs and the determination of the age of the clusters. The results of comparison between the predicted and the observed CMDs and LFs and conclusions follow.

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## 2. Observational data

The data for the clusters NGC 1711, NGC 2004, NGC 2164 & NGC 2214 are taken from Sagar et al. (1991b) (hereafter referred to as Paper I). The data were obtained at the f/8.5 Cassegrain focus of the 1.54m Danish telescope at the European Southern Observatory, La Silla, Chile using a RCA CCD chip of  $320 \times 512$  pixel in size where one pixel corresponds to 0.47 arcsec. Other details of observations and data reductions are given in Paper I. Since the data for the field region of NGC 2164 in Paper I is not very deep, we have taken those from Vallenari et al. (1991).

Using secondary photoelectric standards covering a range in brightness ( $7.3 \leq V \leq 12.4$ ) and in colour ( $-0.17 \leq (B-V) \leq 1.68$ ), the photometric data have been calibrated with a zero-point accuracy of  $\sim 0.04$  mag in both B and V. The present CCD data agree well with independent photoelectric observations of some stars in the clusters having a large range in brightness ( $11.8 \leq V \leq 18.1$ ) as well as in colour ( $-0.17 \leq (B-V) \leq 2.17$ ) but generally show systematically varying differences with photographic data (cf. Paper I). In the case of NGC 2004 & NGC 2164 a comparison of the present CCD data with independent CCD observations made in Paper I indicate good agreement in the case of NGC 2164 but a significant systematic difference in the case of NGC 2004. Recently CCD observations generally for stars brighter than  $V \sim 17$  have been published by Elson (1991) in the regions of all our programme clusters; by Kubiak (1990) in the region of NGC 1711, by Balona & Jerzykiewicz (1993) for NGC 2004 and by Bhatia & Piotto (1994) for NGC 2214. The last two have compared their data with ours and found a small systematic difference, within the accuracy of zero-points, relative to their measurement. This indicates that these independent CCD measurements agree within zero point errors of the photometry. We plot the photometric differences with Kubiak (1990) and Elson (1991) in Figure 1 and present the statistical results of the comparison in Table 1. Elson (1991) indicates that typical uncertainties in her V measurements are  $\pm 0.10$  mag for the fainter stars and  $\pm 0.05$  for the brighter stars, while those in (B-V) are typically of 0.10 mag. However, the maximum uncertainty in (B-V) could be up to  $\pm 0.50$  mag. According to Kubiak (1990), his data can have a systematic error of  $\sim 0.10$  mag in the zero point of (B-V) and slightly less in V. Considering these facts, we conclude that our data agree fairly well with Elson (1991) but have systematic differences with Kubiak (1990). The differences in (B-V) are minimum ( $\sim -0.04$  mag) for bluest stars but increase systematically to become  $\sim 0.5$  mag for redder stars. A straight line fit through the data points using least squares linear regression gives,

$$\Delta(B - V) = -0.30(\pm 0.014)(B - V) - 0.10(\pm 0.01)$$

with correlation coefficient,  $\gamma = 0.98$ . This may indicate improper use of colour coefficients in the photometric calibration by Kubiak (1990) which yields only (B-V)  $\sim 1.0$  mag even for the reddest stars.

## 3. Observational CMDs and LFs

The CMDs of the clusters analyzed here are presented in Paper I. These CMDs need to be corrected for data incompleteness and field star contamination before they can be compared with SEMs. The first step towards this goal is to define a cluster

region which can be considered as cluster representative, but suffering less from the above defects. The next task is to determine the incompleteness of the data and adopt a suitable method to remove the field stars from the CMDs.

### 3.1. Selection of the cluster region

The idea is to find an annular ring best representing the cluster but suffering less from incompleteness and crowding. The cluster center is derived iteratively by calculating the average x and y positions of stars within 150 pixels from an eye estimated center, until it converges to a constant value. An error of a few arcsec is expected in locating the cluster center. As the crowding is maximum near the center we expect the data to be least complete near the central region. We consider the region where the data completeness factor (CF) is  $< 30\%$  (defined in Sagar & Richtler 1991; hereafter referred as Paper II) as the central region and exclude this circular region from our further analysis. This serves as the inner boundary for the required cluster region. The outer boundary is identified from a plot of stellar density as a function of the distance from the cluster center. The limits of the annular region best representing the cluster are given in Table 2. Since incompleteness varies also within this region, we divide it into two rings, namely, ring 1 and ring 2 as given in Table 2.

### 3.2. Incompleteness correction

The procedure to quantify the stellar completeness factor CF in photometry is described in detail in Paper II. The completeness factor  $\Lambda_i$  at a point ( $V_i, (B-V_i)$ ) in V, (B-V) diagram will be mainly controlled by that CF value of B and V CCD frames where completeness is less, i.e., the value of  $\Lambda_i$  cannot be larger than the smaller value of the pair ( $CF(V_i), CF(B_i)$ ); where  $CF(V_i)$  and  $CF(B_i)$  are the completeness values at the brightness  $V_i$  and  $B_i$  in the V and B CCD frames respectively. Consequently, we have used,

$$CF(V_i, (B - V)_i) = \min(CF(V_i), CF(B_i)), \quad (1)$$

for the data incompleteness correction in our analysis. The completeness values for all regions are taken from Paper II except for the field region of NGC 2164, where they are from Vallenari et al. (1991). The value of  $CF(B_i)$  is generally less than that of  $CF(V_i)$  except in a few cases (see table 4 of Paper II).

For applying the incompleteness correction to get a complete CMD, the CMDs for the rings 1 and 2 are constructed separately. Then the CMD of each ring is divided into a number of boxes having a width of 0.2 mag in V and 0.1 mag in (B-V). The  $\Lambda_i$  for the mean values of  $B_i$  and  $V_i$  for  $i^{th}$  box is found using the relation (1). The actual number of stars in the  $i^{th}$  box is then calculated as  $N_{ci} = (N_{oi})/\Lambda_i$ , where  $N_{oi}$  is the observed number of stars in the  $i^{th}$  box. Now, the extra number of stars in the  $i^{th}$  box,  $\Delta N_i$ , given by,  $\Delta N_i = N_{ci} - N_{oi}$ , is then randomly distributed inside the box. In this way the stellar incompleteness is corrected for both the rings and then the CMD of the rings are combined to obtain a complete CMD for the chosen cluster region. The same technique is applied to the field region without dividing it into different rings since the incompleteness does not depend on position here.

### 3.3. Correction for field star contamination

We have used the zapping technique described by Mateo & Hodge (1986) to remove the field stars from the cluster CMD after accounting for the difference in the areas of field and cluster regions considered. In this method, for each star in the CMD of the field region, the nearest star in the CMD of the cluster region is identified and removed. The maximum box size for such an identification is varied and finally fixed at  $V \sim 1$  mag and  $B - V \sim 0.5$  mag. This box size is not related to the one used for the completeness correction. Lupton et al. (1989) have measured the radial velocities of some stars in NGC 2164 and NGC 2214 and identified a few of them either as galactic foreground stars or LMC field stars. We have therefore removed them from our further analysis. In this way, we get a cluster CMD ready to be compared with the theoretical SEMs.

### 4. Observed CMDs

For all the programme clusters, Figure 2 shows the observed CMDs corrected for photometric incompleteness and field star contamination. For comparing the number of stars present in different stellar evolutionary phases, the CMDs are divided into three parts consisting of MS stars, bright red giants (BRGs) and faint red giants (FRGs). The BRGs are the evolved stars of the cluster and the FRGs are the intermediate age core helium burning stars of the LMC field. Even after the field star subtraction using zapping technique we do find some FRGs, which we reject assuming them as the left out field stars. Here we discuss the various features of the final cluster CMDs and the main sequence LFs.

- (i) NGC 1711 has a well populated MS ( $\sim 590$  stars) up to  $V \sim 16$  and some stars are seen scattered up to 14.2 magnitude. A total of 10 BRGs are distributed between 0.0 and 2.0 mag in  $(B-V)$ . The red clump of BRGs are at a  $(B-V)$  value of 1.6.
- (ii) NGC 2004 is the youngest of the clusters considered here, having a well populated MS of  $\sim 520$  stars continuous up to  $V \sim 13.5$  mag. There are six BRGs confined to a very small range in brightness,  $14.5 \leq V \leq 15.25$  but with somewhat larger range in their colours,  $1.3 \leq (B-V) \leq 2.0$ .
- (iii) The brightest MS star in NGC 2164 has  $V \sim 14.5$  mag and  $(B-V) \sim -0.1$ . Then there are no MS stars up to  $V \sim 15$  mag. Below that the cluster MS is well populated with  $\sim 530$  stars. The 16 BRGs are seen scattered between  $-0.1$  and 1.4 mag in  $(B-V)$ .
- (iv) Recently, the cluster NGC 2214 has drawn a lot of attention regarding its peculiar nature. This cluster is understood to have two components (cf. Bhatia & MacGillivray 1988), but the debate is on the ages of these two components as to whether they are equal or different. Sagar et al. (1991a) not only assign two different ages for the two components but also indicate that the older population is more centrally concentrated ( $R < 50$  pixels) while the younger one shows a more extended distribution. However, recently, Lee (1992) and Bhatia & Piotto (1994) have questioned the reality of the presence of the older population. Since we are not considering the central portion in our analysis here, we are omitting the controversial older population. The cluster MS containing  $\sim 550$  stars is well populated up to  $V \sim 16$  mag. The  $V$  magnitude difference between the brightest and the next brightest MS star is  $\sim 0.8$  mag. The 14 BRGs are populated between  $-0.1$  and 1.3 mag in  $(B-V)$ .

### 5. Luminosity functions of the MS stars

The differential luminosity functions (DLFs) for the MS stars are calculated from the CMDs discussed in last section using a bin width of 0.2 mag in  $V$ . The resulting DLFs for the clusters under study are shown in Figure 3, where the effects of correction of photometric incompleteness as well as of the field star contamination are also indicated. Despite the incompleteness correction, the luminosity functions seem to be complete only for stars brighter than  $V \sim 19.5$  mag. We have fitted a straight line to the  $\log N$  vs  $V$  plots using the least squares regression. The value of the slopes are  $0.16 \pm 0.03$ ,  $0.19 \pm 0.04$ ,  $0.15 \pm 0.03$  and  $0.16 \pm 0.02$  for the clusters NGC 1711, 2004, 2164 & 2214 respectively. This indicates that the slopes of their mass functions are similar which is in agreement with the results given in the Paper II.

### 6. Brief discussion of evolutionary models

We use four SEMs in this analysis, two classical models and two with CCO. Both CCO models use the new opacity values. The  $Z$  value considered is 0.02 for all the models.

The new opacities are significantly higher than the LAOL around two temperatures, i.e. a few  $10^5$  K and about  $1 \cdot 10^6$  K. For solar metallicities, the difference in opacity amounts to a factor of three and 20% respectively (cf. Bressan et al. 1993). The OPAL2 supersedes the previous OPAL1 because of the inclusion of the spin-orbit interaction in the treatment of Fe atomic data and the adoption of the recent measurements of the solar photospheric Fe abundance by Grevesse (1991) and Hannaford et al. (1992).

The differing input physics and computational techniques from model to model makes a direct comparison very difficult. The necessary homogeneity is provided by the Bressan et al. (1993) models as they include both classical and CCO models with all the other inputs remaining the same. The details of the models are given below:

- a Model 1 given by Castellani et al. (1990) is a classical model. It covers a mass range of  $0.8$  to  $20 M_{\odot}$ . This model is used to explain successfully the LF of NGC 2004 by Bencivenni et al. (1991) and NGC 1866 by Brocato et al. (1990). They have used old opacity tables (LAOL) for  $\log T_{\text{eff}} > 4$  and at low temperatures opacities are taken from Cox & Tabor (1976). The evolutionary sequences are computed from zero-age main-sequence (ZAMS) to the thermally pulsing asymptotic giant branch (TPAGB) phase.
- b Model 2 given by Schaller et al. (1992), cover a mass range of  $0.8 M_{\odot}$  to  $120 M_{\odot}$ . They have used new opacities (OPAL1) supplemented by radiative opacities by Kurucz (1991) at temperatures below 6000 K. As a result of the increased value of opacities, the convection parameters  $l/H_p$  and the overshooting distance change with respect to their previous model given by Maeder & Meynet (1989), where  $l/H_p$  is the ratio of mixing length to the pressure scale height. The value of  $l/H_p$  is  $1.6 \pm 0.1$ . The ratio of overshooting distance to the pressure scale height,  $d_{\text{over}}/H_p$  is 0.2 in the mass range 1.25 to  $25 M_{\odot}$ . Mass loss is included for all the stellar masses throughout the HR diagram.
- c Model 3 given by Bressan et al. (1993) present SEMs in the mass range of  $0.6$  to  $120 M_{\odot}$ . The evolutionary tracks extend from ZAMS to TPAGB phase for low and intermediate mass stars, and to the central carbon-ignition for

higher masses. The opacities at temperatures between 6000 K and  $10^8$  K are taken from OPAL2. The opacities at lower and higher temperatures are taken from LAOL and Cox & Stewart (1970a,b) respectively. The value of  $l/H_p$  is taken as 1.63. The value of  $d_{over}/H_p$  is 0.25 for the mass range 1.0 to  $1.5 M_\odot$  and equals 0.5 for masses above. This value of 0.5 corresponds to the overshoot distance favoured by Schaller et al. (1992). They have also included the effects of envelope overshooting at the bottom of the convective envelopes, by adopting a value of 0.7 for the envelope overshooting distance. The mass loss is included only for stars more massive than  $12 M_\odot$ .

- d Model 4 given by Bressan et al. (1993) is same as model 3 without incorporating the core overshooting but with all other inputs being the same. This set is available within the mass range 2.5-20  $M_\odot$ .

## 7. Determination of age of the clusters

To determine the age of the clusters, we fit the isochrones obtained from the SEMs mentioned above to the  $(M_V, (B-V)_O)$  CMDs of the clusters obtained from their  $V, (B-V)$  CMDs. We adopt a value of 18.6 for the distance modulus to the LMC based on the recent studies on cepheid variables in LMC (Viswanathan 1985, Caldwell & Coulson 1985, Walker 1987, Welch et al 1987, 1991 ). The reddening values used by us are given in the Table 2. For NGC 1711, the  $E(B-V)$  value is taken from Mateo (1988) and for others from Cassatella et al. (1987). We assume the extinction in V as  $3.1 \times E(B-V)$ . For fitting the theoretical isochrones in the  $M_V, (B-V)_O$  diagram, we converted them from the  $\log L/L_\odot$  vs  $\log T_{eff}$  plane to the  $M_V, (B-V)_O$  plane using colour-temperature relations and bolometric corrections from Kurucz (1979) and Vandenberg (1983) complemented with values given by Johnson (1966) for the temperatures below 4000°K. The isochrones fitted to the CMDs of the clusters under study are shown in Fig. 4. The ages and the turn-off masses obtained from different models are given in Table 3 which indicate that they depend strongly on the SEMs used. Isochrones with CCO make the object older but the turn-off mass lighter while those without make the cluster younger but the turn-off mass heavier. The ages and the turn-off masses of the clusters derived from the two classical models agree very well with each other. This may indicate that use of new opacities in SEMs has not changed the hydrogen burning time scales significantly. Amongst the CCO models, model 3 makes the object oldest, even though the overshoot amount is the same as in model 2. However, both the CCO models yield the same turn-off masses. We also notice that the red giant stars are generally best fitted by the CCO models. The isochrones produced from the classical models could not reach the observed red end of the giant branch. Bencivenni et al. (1991) using the model 1 found an age of 8 Myr for the cluster NGC 2004, but using the same model we find an age of 12Myr. We find that an age of 8 Myr predicts the BRGs roughly 1 mag brighter than the observed ones, instead a 12 Myr isochrone fits the red giants better though it falls short by  $\sim 0.25$  in  $(B-V)$  mag.

## 8. Comparison of observed CMDs with synthetic CMDs

In order to compare the theoretical stellar evolutionary models with present observations, we produced synthetic CMDs from

the SEMs discussed in section 6 using the method described in the Appendix. A comparison of the observed distribution of stars in the CMD of a cluster with the that of synthetic CMD produced from different SEMs tell us about the reliability of the physical assumptions involved in theoretical calculations. For this we compare the features as well as the integrated luminosity function (ILF) of the observed CMDs with the synthetic ones.

### 8.1. Comparison of features

Here, we compare the features of the synthetic CMDs with the observed ones. The synthetic CMDs for the clusters which are best matching with the observation are shown in Fig. 5. The absolute magnitude of the MS tip, the absolute magnitude of the faintest BRG, the  $(B-V)$  value of the reddest BRG and the number of blue supergiants predicted by the models and the values obtained from the observed CMDs are tabulated in Table 4. Models 2 and 4 predict blue supergiants in all the clusters which is contrary to the observation, while models 1 and 3 produce almost none in their synthetic CMDs. In all the clusters the evolved part of the CMD is more closely reproduced by Model 3. It is interesting to note that even though the amount of CCO is the same in models 2 and 3, the number of blue super giants and the evolved parts of the CMDs are better matched with model 3. This may be due to the differing input physics and computational techniques involved in the two models.

In the cluster NGC 2004, the brighter end of the MS is not populated up to the observed value by any of the synthetic CMDs. Therefore we tried to populate the MS up to the observed tip by adopting an age spread. This gives rise to a spread out BRG population, both in  $M_V$  and  $(B-V)_O$ , which is contrary to what is observed. In order to populate the MS up to observed bright end, an age 6 Myr less than that obtained from BRGs is needed, in all the models. This may indicate that the stars which populate the tip of the MS are 6 Myr younger than the BRGs. Bencivenni et al. (1991) has assigned an age of 8 Myr to this cluster using model 1 and still find a group of 10 stars lying above the observed bright end of the MS. They attribute such an evidence to the occurrence of binary accreting systems.

### 8.2. Comparison of observed ILFs with simulated ILFs

Chiosi et al. (1989) show that the ILF of main sequence (MS) stars normalized to the number of evolved stars can be used to differentiate among different evolutionary scenarios, since it is just the ratio of core H to He burning lifetimes which is very much affected by the mixing scheme used. Following Chiosi et al. (1989), we derive the ILF normalized to the number of evolved stars for these clusters from the observed data as well as from synthetic CMDs constructed using different SEMs and compare them in Fig. 6. for the Salpeter value of mass function slope ( $\alpha=1.35$ ) and an assumed binary fraction of 30%. The observed ILF depends on the number of evolved stars present in the sample. In order to minimize the effect of incompleteness and crowding in the cluster sample, it is restricted to the outer annular region. It is assumed that such a sample has the same proportion of stars in all mass ranges. This assumption is valid except in the case of NGC 1711 where slight mass segregation among the MS stars has been found (cf. Subramaniam et al.

(1993). Also stochastic nature in the star formation if present can also affect the number of observed evolved stars.

Since model 4 is available only for masses  $\geq 2 M_{\odot}$  it is not possible to compare the ILFs from this model with the observed ones, instead we compare the ratio of the core He to H burning lifetimes. An inspection of Fig 6. indicates that different models produces best fits for different clusters. In order to see the effects of uncertainties in the value of  $x$  and binary fraction (BF) we have varied one of these two parameters keeping the other constant. The results of such an exercise is summarized below:

- (i) For a given model, the value of BF was fixed at 30% and the value of  $x$  was changed to get the best fit between observed and synthetic ILFs. The values of  $x$  for the best fits are tabulated in Table 5. This table indicates a general trend that the model 1 needs steeper whereas model 3 needs flatter mass function slope. The value most deviating from the Salpeter value is for NGC 1711 in the case of model 1. Hence, it can be seen that with values of  $x$  not very different from Salpeter's value, the various models are able to fit the observed ILFs.
- (ii) For a given model, the value of  $x$  is fixed at 1.35 (Salpeter's value) and the BF is varied to get the best fit. The values of BF are varied between 0 to 50% and the results are tabulated in Table 6. In some cases, this change in BF could not bring the synthetic ILFs closer to the observed one. Such cases are given as blanks in the table. It can be seen that to obtain the best fit some models require rather extreme values of BF.

Thus it can be seen that the comparison of ILFs does not show a clear view as to which model is to be preferred. Since the ILFs reflect the ratio of the H to He-burning lifetimes, the values of He to H-burning lifetime ratios for the turn-off masses of the clusters as given by the models are tabulated in Table 7. For a cluster, these values are very different, while the turn-off masses from the models are not very different (see table 3). One striking point being, both the classical models give very similar values for the ratio. Hence, it is difficult to find out the value of the ratio which is fitting the observation, mainly because of the uncertainty in the value of the mass function slope. Similar conclusion has been arrived at by Chiosi et al. (1994) while analyzing the SMC cluster NGC 330.

## 9. Conclusions

We have compared the CMDs and LFs derived from a homogeneous set of data comprising of the LMC star clusters NGC 1711, NGC 2004, NGC 2164 and NGC 2214 having turn-off masses in the range 6-15  $M_{\odot}$  with the synthetic ones produced using the SEMs from Castellani et al. (1990), Schaller et al. (1992) and Bressan et al. (1993). The main results of the present investigation are:

1. Classical models make the clusters younger but the turn-off masses slightly heavier in comparison to the models incorporating convective core overshooting (CCO).
2. Stellar evolutionary models with CCO given by Bressan et al. (1993) reproduce the observed featured of CMDs best amongst the models used in the analysis.
3. The synthetic ILF derived from a model strongly depends on the value of the mass function slope, and lightly on the binary fraction whereas the observed ILFs are affected by

the uncertainty in the number of evolved stars. Hence the comparison of the synthetic with the observed ILFs does not favour any model specifically. In order to constrain the models from the comparison of the synthetic ILFs with the observed ones, reliable estimates of mass function slope, binary fraction are desired.

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## A. Appendix

The computer code used to construct synthetic colour magnitude diagrams using SEMs is described below. This method reveals the time scales involved in various evolutionary phases thereby allowing to compare the number of stars predicted by an evolutionary model with the number of stars observed in the respective phases. We have included the effects due to contact as well as optical binaries and the photometric errors. The details of the computer code can be summarized as follows.

1. Monte-Carlo method is used to distribute stars randomly in a mass interval for a given mass function slope. This randomness also accounts for the stochastic nature of the mass function. The expression for the initial mass function is given by,

$$dN = AM^{-(1+x)}dM \quad (A1)$$

where  $dN$  is the number of stars in the mass interval  $dM$  and  $x$  is the mass function slope, the Salpeter (1955) value being, 1.35. The constant of proportionality is fixed by the number of post-main sequence stars. Different values of  $x$  has been used to find the best fit of synthetic ILFs with the observed ILFs.

2. The cluster age obtained by fitting isochrones as described earlier is assigned to the masses thus obtained, if the star formation is assumed to take place instantaneously. It is also possible to give an age spread to the masses, if we assume a spread in the formation time of the cluster members. This spread can either be a gaussian with peak at the cluster age or a step function.
3. The photometric errors present in the observations will produce a scatter in the CMD of the cluster. It is essential to include the photometric errors in the synthetic

CMD. These errors are taken from Paper I. The method adopted for including these errors is adopted from Robertson (1974). A gaussian distribution has been assumed for the errors with the standard deviation  $\Delta B$  and  $\Delta V$  given by,  $\Delta B = a(B - b)^2$  and  $\Delta V = c(V - d)^2$ . The coefficients  $a$ ,  $b$ ,  $c$  and  $d$  are chosen to fit the observations where the errors increase with magnitude.

4. Since the width of the MS and the scatter at the tip of the MS are produced by the presence of the binaries, inclusion of binaries is very essential in the synthetic CMD. In the galactic open clusters the binary stars amount up to 30% of the cluster population (Mermilliod & Mayor (1989)). Apart from the presence of the actual binaries, contribution from the optical binaries, which is instrumental is also substantial because of the high stellar density. Since it is not clear how much of binaries the LMC clusters contain, we have assumed a value of 30% for the overall binary population. The procedure works as follows: to each star of mass  $M_1$  we associate a probability of being a member of a binary system with a companion of mass  $M_2$  such that the mass ratio  $M_1/M_2$  is confined within a given range. We have chosen the range of mass ratio as 0.75 - 1.25. These two stars are then replaced by a single star whose flux in various pass bands is given by the sum of the flux of the component stars.

In general the main sequence is populated randomly according to a fixed MF slope until the required number of evolved stars are obtained. The lower limit of the mass is determined by the observational limit. In general we have adopted the Salpeter value for the MF slope. The age is found by fitting the isochrones as explained earlier. The age spread is not given in general, i.e., an instantaneous star formation is assumed. The binary stars are included and the photometric errors are added to the final  $M_V$  and  $(B - V)_0$  values. The LFs are found in an interval of 0.2 mag from the MS.

## Figure Captions:

Fig. 1. The photometric differences with Kubiak (1990) and Elson (1991) are plotted against our data for the four clusters. The difference  $\Delta$  is in the sense literature minus our CCD values. (a) and (b) give the differences for V and (B-V) with respect to V whereas (e) gives (B-V) differences with respect to (B-V) for the clusters NGC 2004, NGC 2164 and NGC 2214, which are indicated by different symbols. (c), (d) and (f) are same as above but for the cluster NGC 1711 and two symbols indicate two sets of data.

Fig. 2. The cluster CMDs after completeness correction and field star subtraction are shown here. The regions occupied by MS stars, BRGs and FRGs are also shown.

Fig. 3. The differential luminosity functions (DLFs) of the four clusters are plotted in solid lines. The DLFs from the raw data are indicated in dashed lines, whereas the DLFs from the data after completeness correction (before field star subtraction) is shown in dotted lines.

Fig. 4. The determination of the age by isochrone fitting for the four clusters are shown here. The continuous line represent model 1, dotted line model 2, short dashed line model 3 and long dashed line model 4.

Fig. 5. The synthetic CMDs those match the observation for  $x=1.35$  and binary fraction of 30% for the four clusters are shown here. The models from which the CMDs are made are also shown.

Fig. 6. The ILFs for the four clusters are plotted here. The continuous line indicate the observed ILFs. The dotted line represent the ILFs from model 1, short dashed line from model 2 and long dashed line from model 3. The error bars indicate the statistical error at the magnitudes shown for the observed ILF. The synthetic ILFs are constructed for  $x=1.35$  and a binary fraction of 30%.

**Table 1.** Statistical results of the photometric differences  $\Delta$  in the sense literature minus our CCD values. V and (B-V) are from our photometry. The mean and standard deviation ( $\sigma$ ) are based on N stars. A few points discrepant by more than  $3\sigma$  have been excluded from the analysis.

A. Comparison with Kubiak (1991) CCD data in NGC 1711

V (mag)	$\Delta V$ in mag			$\Delta(B-V)$ in mag			(B-V) (mag)	$\Delta(B-V)$ in mag		
	Mean	$\sigma$	N	Mean	$\sigma$	N		Mean	$\sigma$	N
13.0 - 14.0	-0.10	0.03	5	-0.16	0.07	4	-0.22 - -0.10	0.04	0.04	7
14.0 - 14.8	-0.20	0.13	4	-0.13	0.10	4	-0.10 - 0.10	-0.10	0.02	5
14.8 - 15.2	0.08	0.02	8	-0.52	0.07	8	0.10 - 1.00	-0.22	0.07	4
15.2 - 16.1	-0.26	0.24	8	-0.07	0.00	8	1.00 - 1.70	0.52	0.00	9

B. Comparison with Elson (1991) CCD data in NGC 1711

V (mag)	$\Delta V$ in mag			$\Delta(B-V)$ in mag			(B-V) (mag)	$\Delta(B-V)$ in mag		
	Mean	$\sigma$	N	Mean	$\sigma$	N		Mean	$\sigma$	N
13.5 - 15.0	-0.03	0.09	10	-0.16	0.03	10	-0.20 - 0.20	-0.19	0.09	69
15.0 - 16.0	-0.06	0.05	10	-0.17	0.13	10	0.20 - 1.20	-0.07	0.62	7
16.0 - 16.5	-0.11	0.07	9	-0.18	0.08	9	1.20 - 1.60	-0.14	0.30	12
16.5 - 17.0	0.00	0.15	22	-0.18	0.16	19				
17.0 - 17.5	-0.05	0.15	18	-0.09	0.12	15				
17.5 - 18.2	-0.06	0.19	21	-0.17	0.09	22				

C. Comparison with Elson (1991) CCD data in NGC 2004

V (mag)	$\Delta V$ in mag			$\Delta(B-V)$ in mag			(B-V) (mag)	$\Delta(B-V)$ in mag		
	Mean	$\sigma$	N	Mean	$\sigma$	N		Mean	$\sigma$	N
13.0 - 15.0	-0.08	0.09	8	-0.01	0.17	9	-0.20 - 0.20	-0.02	0.09	52
15.0 - 16.0	-0.12	0.16	12	0.00	0.07	13	0.20 - 1.70	0.15	0.16	10
16.0 - 17.0	-0.03	0.24	18	-0.01	0.10	20				
17.0 - 17.5	-0.18	0.16	9	-0.03	0.16	10				
17.5 - 18.6	0.01	0.17	9	-0.02	0.08	10				

D. Comparison with Elson (1991) CCD data in NGC 2164

V (mag)	$\Delta V$ in mag			$\Delta(B-V)$ in mag			(B-V) (mag)	$\Delta(B-V)$ in mag		
	Mean	$\sigma$	N	Mean	$\sigma$	N		Mean	$\sigma$	N
13.5 - 15.0	-0.03	0.06	7	-0.12	0.05	7	-0.20 - 0.20	-0.10	0.06	21
15.0 - 16.0	-0.10	0.08	10	-0.11	0.08	9	0.20 - 1.60	-0.09	0.14	15
16.0 - 17.0	-0.06	0.15	12	-0.08	0.14	12				
17.0 - 17.6	0.01	0.18	8	-0.08	0.08	8				

E. Comparison with Elson (1991) CCD data in NGC 2214

V (mag)	$\Delta V$ in mag			$\Delta(B-V)$ in mag			(B-V) (mag)	$\Delta(B-V)$ in mag		
	Mean	$\sigma$	N	Mean	$\sigma$	N		Mean	$\sigma$	N
13.0 - 16.0	-0.08	0.06	7	0.03	0.16	8	-0.20 - 0.20	-0.09	0.12	12
16.0 - 17.0	0.04	0.12	8	-0.05	0.13	6	0.20 - 1.70	0.05	0.16	9
17.0 - 18.0	0.02	0.18	8	-0.08	0.15	7				



**Table 5.** The values of  $x$  which produces the best fit between synthetic and observed ILFs for a binary fraction of 30% for different models.

cluster	x value for models		
	1	2	3
NGC 1711	1.90	1.60	1.35
NGC 2004	1.65	1.35	1.25
NGC 2164	1.35	1.13	1.05
NGC 2214	1.35	1.25	1.10

**Table 6.** The value of binary fraction which produces the best fit between the observed and synthetic ILFs for the Salpeter mass function slope ( $x=1.35$ ) for different models.

Cluster	value of the BF for models		
	1	2	3
NGC 1711	–	50%	30%
NGC 2004	50%	30%	0%
NGC 2164	30%	20%	–
NGC 2214	30%	20%	5%

**Table 2.** Coordinates of cluster centre ( $X_c$ ,  $Y_c$ ), annuli of selected rings in pixels and  $E(B-V)$  values in mag for the clusters are given.

Cluster	$X_c$	$Y_c$	Selected annuli		$E(B-V)$
			Ring 1	Ring 2	
NGC 1711	140	242	$50 \leq R < 115$	$115 \leq R < 200$	0.09
NGC 2004	150	407	$45 \leq R < 115$	$115 \leq R < 200$	0.09
NGC 2164	132	237	$45 \leq R < 110$	$110 \leq R < 180$	0.10
NGC 2214	171	231	$45 \leq R < 110$	$110 \leq R < 200$	0.07

**Table 3.** Age and turn-off mass of the clusters determined from the models.

Cluster	Ages in Myr from models				Turn-off masses in $M_\odot$ from models			
	1	2	3	4	1	2	3	4
NGC 1711	22	28	35	23	9.7	8.3	8.3	9.4
NGC 2004	12	14	18	12	14.3	12.3	11.4	13.7
NGC 2164	35	50	60	38	7.5	6.4	6.4	7.2
NGC 2214	37	50	60	40	7.3	6.4	6.4	7.0

**Table 4.** The observed features of the cluster CMDs are compared with the ones predicted by the models from the synthetic CMDs. In the table, (a), (b), (c) and (d) denote the  $M_V$  of the MS tip,  $M_V$  of the faintest BRG, number of blue supergiants and the  $(B-V)$  value of the reddest BRG respectively. The last column gives the error in the values obtained from models.

cluster	quantity	observed	Values from models				error
			1	2	3	4	
NGC 1711	(a)	-4.5	-4.0	-3.6	-4.0	-3.9	$\pm 0.2$
	(b)	-3.8	-4.5	-4.5	-4.4	-4.3	$\pm 0.1$
	(c)	1	2	4	0	3	
	(d)	1.55	1.3	1.7	1.65	1.34	$\pm 0.2$
NGC 2004	(a)	-5.5	-4.0	-4.75	-4.6	-4.2	$\pm 0.1$
	(b)	-4.45	-5.8	-5.6	-5.35	-5.0	$\pm 0.1$
	(c)	0	0	2	0	4	
	(d)	1.9	1.65	1.63	1.45	1.40	$\pm 0.2$
NGC 2164	(a)	-3.3	-3.0	-3.3	-3.4	-3.6	$\pm 0.3$
	(b)	-3.0	-3.8	-3.2	-3.1	-3.4	$\pm 0.1$
	(c)	1	0	6	0	5	
	(d)	1.38	1.23	1.47	1.46	1.24	$\pm 0.1$
NGC 2214	(a)	-3.6	-3.3	-3.3	-3.4	-3.1	$\pm 0.3$
	(b)	-3.2	-3.5	-3.5	-3.1	-3.3	$\pm 0.1$
	(c)	0	2	4	0	4	
	(d)	1.3	1.0	1.48	1.2	1.25	$\pm 0.1$

**Table 7.** The values for the ratio of the helium to hydrogen burning lifetime is listed for different masses. The masses are in solar units.

Mass	$\tau_{He}/\tau_H$			
	Model 1	Model 2	Model 3	Model 4
7.0	0.169	0.109	0.071	0.146
9.0	0.144	0.099	0.063	0.111
12.0		0.098	0.062	0.087
15.0		0.096	0.065	0.093