

# MICROLENSING IMPLICATIONS FOR HALO DARK MATTER <sup>1</sup>

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## Abstract

The most accurate way to get information on the mass of the MACHOs (Massive Astrophysical Compact Halo Objects) is to use the method of mass moments. For the microlensing events detected so far by the EROS and the MACHO collaborations in the Large Magellanic Cloud the average mass turns out to be  $0.08M_{\odot}$ . Assuming a spherical standard halo model we find that MACHOs contribute about 20% to the halo dark matter. The eleven events recorded by OGLE, mainly during its first two years of operation, in the galactic bulge lead to an average mass of  $0.29M_{\odot}$ , whereas forty events detected by MACHO during its first year give  $0.16M_{\odot}$ , thus suggesting that the lens objects are faint disk stars.

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## 1. Introduction

It has been pointed out by Paczyński [1] that microlensing allows the detection of MACHOs in the mass range [2]  $10^{-7} < M/M_\odot < 10^{-1}$ . Starting from September 1993 the French collaboration EROS [3] and the American–Australian collaboration MACHO [4] announced the detection of at least six microlensing events discovered by monitoring over several years millions of stars in the Large Magellanic Cloud (LMC). Moreover, the Polish-American collaboration OGLE [5] and the MACHO team [6] found altogether more than  $\sim 100$  microlensing events by monitoring stars located in the galactic bulge. The inferred optical depth for the bulge turns out to be higher than previously thought.

An important issue is the determination of the mass of the MACHOs that acted as gravitational lenses as well as the fraction of halo dark matter in form of MACHOs. The most appropriate way to compute the average mass and other important information is to use the method of mass moments developed by De Rújula et al. [7], which will be briefly presented in section 3.

## 2. Most probable mass for a single event

First, we compute the probability  $P$  that a microlensing event of duration  $T$  and maximum amplification  $A_{max}$  be produced by a MACHO of mass  $\mu$  (in units of  $M_\odot$ ). Let  $d$  be the distance of the MACHO from the line of sight between the observer and a star in the LMC,  $t=0$  the instant of closest approach and  $v_T$  the MACHO velocity in the transverse plane. The magnification  $A$  as a function of time is calculated using simple geometry and is given by

$$A(t) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}, \quad \text{where} \quad u^2 = \frac{d^2 + v_T^2 t^2}{R_E^2}. \quad (1)$$

$R_E$  is the Einstein radius which is  $R_E^2 = \frac{4GMD}{c^2}x(1-x) = r_E^2\mu x(1-x)$  with  $M = \mu M_\odot$  the MACHO mass and  $D$  ( $xD$ ) the distance from the observer to the source (to the MACHO).  $D = 55$  kpc is the distance to the LMC and  $r_E = 3.17 \times 10^9$  km. We use here the definition:  $T = R_E/v_T$ .

We adopt the model of an isothermal spherical halo in which the normalized MACHO number distribution as a function of  $v_T$  is

$$f(v_T)dv_T = \frac{2}{v_H^2}v_T e^{-v_T^2/v_H^2} dv_T, \quad (2)$$

with  $v_H \approx 210$  km/s the velocity dispersion implied by the rotation curve of our galaxy. The MACHO number density distribution per unit mass  $dn/d\mu$  is given by

$$\frac{dn}{d\mu} = H(x) \frac{dn_0}{d\mu} = \frac{a^2 + R_{GC}^2}{a^2 + R_{GC}^2 + D^2x^2 - 2DR_{GC}x\cos\alpha} \frac{dn_0}{d\mu}, \quad (3)$$

with  $dn_0/d\mu$  the local MACHO mass distribution. We have assumed that  $dn/d\mu$  factorizes in functions of  $\mu$  and  $x$  [7]. We take  $a = 5.6$  kpc as the galactic “core” radius (our final results do not depend much on the poorly known value of  $a$ ),  $R_{GC} = 8.5$  kpc our distance from the centre of the galaxy and  $\alpha = 82^\circ$  the angle between the line of sight and the direction of the galactic centre. For an experiment monitoring  $N_\star$  stars during a total observation time  $t_{obs}$  the number of expected microlensing events is given by [7, 8]

$$N_{ev} = \int dN_{ev} = N_\star t_{obs} 2Dr_E \int v_T f(v_T) (\mu x(1-x))^{1/2} H(x) \frac{dn_0}{d\mu} d\mu du_{min} dv_T dx \quad (4)$$

where the integration variable  $u_{min}$  is related to  $A_{max}$ :  $A_{max} = A[u = u_{min}]$ . For a more complete discussion in particular on the integration range see [7].

From eq.(4) with some variable transformation (see [9]) we can define, up to a normalization constant, the probability  $P$  that a microlensing event of duration  $T$  and maximum amplification  $A_{max}$  be produced by a MACHO of mass  $\mu$ , that we see first of all is independent of  $A_{max}$  [9]

$$P(\mu, T) \propto \frac{\mu^2}{T^4} \int_0^1 dx (x(1-x))^2 H(x) \exp\left(-\frac{r_E^2 \mu x(1-x)}{v_H^2 T^2}\right). \quad (5)$$

We also see that  $P(\mu, T) = P(\mu/T^2)$ . The measured values for  $T$  are listed in the Tables 1 and 2, where  $\mu_{MP}$  is the most probable value. We find that the maximum corresponds to  $\mu r_E^2 / v_H^2 T^2 = 13.0$  [9, 10]. The 50% confidence interval embraces for the mass  $\mu$  approximately the range  $1/3\mu_{MP}$  up to  $3\mu_{MP}$ . Similarly one can compute  $P(\mu, T)$  also for the bulge events (see [10]).

**Table 1:** Values of  $\mu_{MP}$  (in  $M_\odot$ ) for the six microlensing events detected in the LMC ( $A_i$  = American-Australian collaboration events ( $i = 1, \dots, 4$ );  $F_1$  and  $F_2$  French collaboration events). For the LMC:  $v_H = 210$  km s $^{-1}$  and  $r_E = 3.17 \times 10^9$  km.

|                                   |       |       |       |       |       |       |
|-----------------------------------|-------|-------|-------|-------|-------|-------|
|                                   | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $F_1$ | $F_2$ |
| $T$ (days)                        | 16.9  | 9     | 14    | 21.5  | 27    | 30    |
| $\tau (\equiv \frac{v_H T}{r_E})$ | 0.097 | 0.052 | 0.08  | 0.123 | 0.154 | 0.172 |
| $\mu_{MP}$                        | 0.12  | 0.03  | 0.08  | 0.20  | 0.31  | 0.38  |

**Table 2:** Values of  $\mu_{MP}$  (in  $M_\odot$ ) as obtained by the corresponding  $P(\mu, T)$  for eleven microlensing events detected by OGLE in the galactic bulge [10]. ( $v_H = 30 \text{ km s}^{-1}$  and  $r_E = 1.25 \times 10^9 \text{ km}$ .)

|            |       |       |       |       |       |       |       |       |       |       |       |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|            | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    |
| $T$        | 25.9  | 45    | 10.7  | 14    | 12.4  | 8.4   | 49.5  | 18.7  | 61.6  | 12    | 20.9  |
| $\tau$     | 0.054 | 0.093 | 0.022 | 0.029 | 0.026 | 0.017 | 0.103 | 0.039 | 0.128 | 0.025 | 0.043 |
| $\mu_{MP}$ | 0.61  | 1.85  | 0.105 | 0.18  | 0.14  | 0.065 | 2.24  | 0.32  | 3.48  | 0.13  | 0.40  |

### 3. Mass moment method

A more systematic way to extract information on the masses is to use the method of mass moments as presented in De Rújula et al. [7]. The mass moments  $\langle \mu^m \rangle$  are defined as

$$\langle \mu^m \rangle = \int d\mu \epsilon_n(\mu) \frac{dn_0}{d\mu} \mu^m . \quad (6)$$

$\langle \mu^m \rangle$  is related to  $\langle \tau^n \rangle = \sum_{events} \tau^n$ , with  $\tau \equiv (v_H/r_E)T$ , as constructed from the observations and which can also be computed as follows

$$\langle \tau^n \rangle = \int dN_{ev} \epsilon_n(\mu) \tau^n = V u_{TH} \Gamma(2-m) \widehat{H}(m) \langle \mu^m \rangle , \quad (7)$$

with  $m \equiv (n+1)/2$  and

$$V \equiv 2N_\star t_{obs} D r_E v_H = 2.4 \times 10^3 \text{ pc}^3 \frac{N_\star t_{obs}}{10^6 \text{ stars/year}} , \quad (8)$$

$$\Gamma(2-m) \equiv \int_0^\infty \left( \frac{v_T}{v_H} \right)^{1-n} f(v_T) dv_T , \quad (9)$$

$$\widehat{H}(m) \equiv \int_0^1 (x(1-x))^m H(x) dx . \quad (10)$$

The efficiency  $\epsilon_n(\mu)$  is determined as follows (see [7])

$$\epsilon_n(\mu) \equiv \frac{\int dN_{ev}^*(\bar{\mu}) \epsilon(T) \tau^n}{\int dN_{ev}^*(\bar{\mu}) \tau^n}, \quad (11)$$

where  $dN_{ev}^*(\bar{\mu})$  is defined as  $dN_{ev}$  in eq.(4) with the MACHO mass distribution concentrated at a fixed mass  $\bar{\mu}$ :  $dn_0/d\mu = n_0 \delta(\mu - \bar{\mu})/\mu$ . In Fig.1 we show the experimental detection efficiency  $\epsilon(T)$  of the MACHO experiment when looking to the LMC [11]. In Fig.2 we plot the corresponding  $\epsilon_0(\mu)$  as calculated from eq.(11). This function indicates how efficient is the experiment to detect a MACHO with a given mass  $M = \mu M_\odot$ .

A mass moment  $\langle \mu^m \rangle$  is thus related to  $\langle \tau^n \rangle$  as given from the measured values of  $T$  in a microlensing experiment by

$$\langle \mu^m \rangle = \frac{\langle \tau^n \rangle}{V u_{TH} \Gamma(2 - m) \hat{H}(m)}. \quad (12)$$

The mean local density of MACHOs (number per cubic parsec) is  $\langle \mu^0 \rangle$ . The average local mass density in MACHOs is  $\langle \mu^1 \rangle$  solar masses per cubic parsec. The mean MACHO mass, which we get from the six events detected so far toward the LMC, is [10]

$$\frac{\langle \mu^1 \rangle}{\langle \mu^0 \rangle} = 0.08 M_\odot. \quad (13)$$

(To obtain this result we used the values of  $\tau$  as reported in Table 1, whereas  $\Gamma(1)\hat{H}(1) = 0.0362$  and  $\Gamma(2)\hat{H}(0) = 0.280$  as plotted in figure 6 of ref. [7]).

The mean MACHO mass, which one gets from the eleven events of OGLE<sup>2</sup> in the galactic bulge is  $\sim 0.29 M_\odot$  [10]. From the 40 events discovered<sup>2</sup> during the first year of operation by the MACHO team [6] we get an average value of  $0.16 M_\odot$ . The lower value inferred from the MACHO data is due to the fact that the efficiency for the short duration events ( $\sim$  some days) is substantially higher for the MACHO experiment than for the OGLE one. The above average values for the mass suggests that the lens are faint disk stars.

The resulting mass depends obviously to some extent on the parameters used to describe the halo (or the galactic centre respectively). In order to

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<sup>2</sup>We considered only the events used by the MACHO team to infer the optical depth without the double lens event.

check this dependence we varied the parameters within their allowed range and found that the average mass changes at most by  $\pm 30\%$ , which shows that the result is rather robust.

Another important quantity is the fraction  $f$  of the local dark mass density (the latter one given by  $\rho_0$ ) detected in the form of MACHOs, which is given by  $f \equiv M_\odot/\rho_0 \sim 126 \text{ pc}^3 \langle \mu^1 \rangle$ . Using the values given by the MACHO collaboration for their first year data [11] (in particular  $u_{TH} = 0.83$  corresponding to  $A > 1.5$  and an effective exposure  $N_* t_{obs}$  of  $\sim 2 \times 10^6$  star-years for the observed range of the event duration  $T$  between 10 - 20 days) we find  $f \sim 0.2$ , which compares quite well with the corresponding value ( $f = 0.19_{-0.10}^{+0.16}$ ) obtained by the MACHO group in a different way.

Once several moments  $\langle \mu^m \rangle$  are known one can get information on the mass distribution  $dn_0/d\mu$ . However, since at present only few events toward the LMC are at disposal the different moments (especially the higher ones) can only be determined approximately. Instead, we can make the ansatz  $dn_0/d\mu = a\mu^{-\alpha}$ . Knowing, for instance,  $\langle \mu^1 \rangle$  and  $\langle \mu^0 \rangle$  (as well as  $\epsilon_1(\mu)$  and  $\epsilon_{-1}(\mu)$  from eq.(11)) we can determine  $a$  and  $\alpha$ . The solution for  $a$  and  $\alpha$  is acceptable only if we get the same values using other moments, such as e.g.  $\langle \mu^{1.5} \rangle$ . Remarkably, we find that  $a \simeq 6.5 \times 10^{-4}$  and  $\alpha \simeq 2$  is a consistent solution. Moreover, from the relation

$$\int_{M_{min}}^{\sim 0.1} \frac{dn_0}{d\mu} \mu d\mu = f \rho_0 \quad (14)$$

with the above values for  $a$ ,  $\alpha$  and  $f \simeq 0.2$  it follows that  $M_{min} \sim 10^{-2} M_\odot$ . Obviously these results have to be considered as preliminary and as an illustration of how one can get useful information with the mass moment method. Once more data are available it will also be possible to determine other important quantities such as the statistical error in eq. (13).

Nevertheless, the results obtained so far are already of interest and it is clear that in a few years it will be possible to draw more firm conclusions.

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## Figure Captions

1.  $\epsilon(T)$  as given by the MACHO collaboration.
2.  $\epsilon_0(\mu)$  as one gets with eq.(11).

This figure "fig1-1.png" is available in "png" format from:

<http://arxiv.org/ps/astro-ph/9601122v1>

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