# Testing Big Bang Nucleosynthesis

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**Abstract.** Big Bang Nucleosynthesis (BBN), along with the cosmic background radiation and the Hubble expansion, is one of the pillars of the standard, hot, big bang cosmology since the primordial synthesis of the light nuclides (D, $^3$He, $^4$He, $^7$Li) must have occurred during the early evolution of a universe described by this model. The overall consistency between the predicted and observed abundances of the light nuclides, each of which spans a range of some nine orders of magnitude, provides impressive support for the standard models of cosmology and particle physics. Here, the results of recent, statistically consistent tests of BBN are described. This new confrontation between theory and data challenges the standard model. The crises confronting BBN are identified and several possible resolutions are outlined.

## 1. Introduction

The discovery of the Cosmic Background Radiation (CBR) by Penzias & Wilson (1965) transformed forever the study of Cosmology from an exercise in philosophy to the pursuit of science. The presence of the CBR in an expanding Universe favors the hot big bang cosmology. A Universe described by this model was very hot and very dense during early epochs in its evolution. As a consequence, it is a prediction of this "standard" cosmological model that, briefly, the early Universe was a primordial nuclear reactor in which the light nuclides D, $^3$He, $^4$He and $^7$Li were synthesized in astrophysically interesting abundances (for details and references see, e.g., Boesgaard & Steigman 1985; Walker et al. 1991). Thus, along with the CBR and the "Hubble" expansion, Big Bang Nucleosynthesis (BBN) provides one of the three pillars supporting the standard model of cosmology. The standard hot big bang model is, in principle, falsifiable. In contrast to cosmology as theology, this empirical model is not a matter of faith but, rather, demands our eternal vigilance and critical scrutiny. The success of the model is gauged by the degree to which the BBN predictions are consistent with the primordial abundances of the light nuclides inferred from observational data. Over the years BBN has emerged unscathed from the confrontation between theory and observations, providing strong support for the standard, hot big bang cosmological model (e.g., Yang et al. 1984; Boesgaard & Steigman 1985; Walker et al. 1991). This success has, however, not spawned complacency, and the testing continues. In recent years, as the astronomical data has become more precise, hints of a possible crisis have emerged (Copi, Schramm & Turner 1995; Olive &

Steigman 1995; Hata et al. 1995). It is my goal here to describe the impressive success of BBN and to map out the paths leading to the current challenges to the standard model. To better appreciate these challenges and the opportunities they present, an historical analogy may be instructive.

### 1.1. Three Crises For The 19th Century Standard Model

The gravitational theory described by Newton (1686) was outstandingly successful in explaining the motion of the moon and planets. Perhaps one of the most thoroughly tested physical theories in history, Newtonian gravity had become the standard model of 19th Century Physics. Soon, however, some challenges to the standard model emerged. The nature of these challenges and their different resolutions provide some interesting lessons for the emerging crisis in BBN.

(i) Perturbations to the Orbit of Uranus

Deviations in the orbit of Uranus from the predictions of Newtonian gravity (the standard model) led Adams and LeVerrier to predict the existence and location of Neptune. The standard model was used to discover something new, verifying the accuracy of the data (the orbit of Uranus) and providing spectacular support for Newtonian gravity.

(ii) Perturbations to the Orbit of Neptune

Observations of the newly discovered Neptune suggested that its orbit, too, was being perturbed away from the standard model predictions. So began the long search which culminated in the discovery of Pluto. Pluto, however, is not responsible for measurable perturbations to the orbit of Neptune - it is too small. Rather, here we have a case of insufficiently accurate data. The discovery of Pluto was serendipitous; more accurate observations of Neptune's orbit are entirely consistent with the predictions of Newtonian gravity.

(iii) Precession of the Perihelia of Mercury

By the mid-19th century LeVerrier had noted a discrepancy between the predicted and observed precession of the perihelia of Mercury. LeVerrier and others proposed one or more planets (Vulcan) between Mercury and the Sun to resolve this crisis. None were found. Alternately, it was proposed by Newcomb and others that the perturbing mass might be in a ring of dust or asteroids. This, however, would have perturbed the orbits of Mercury and Venus in conflict with observational data. A more radical solution, modifying the inverse square law, was proposed by Newcomb (1895). This, however, is in conflict with the accurately observed lunar orbit.

As is so well known, the resolution of this crisis confronting the 19th century standard model was new physics! Einstein's General Theory of Relativity (1916) predicts a precession in beautiful agreement with that observed.

Three crises, three different resolutions: the standard model preserved and a new discovery (Neptune); the standard model preserved and insufficently accurate data (Neptune/Pluto); the standard model replaced (perihelia of Mercury).

## 2. Consistency of the Standard Model

### 2.1. Predictions

Now let us turn to the standard model of cosmology and the predictions of primordial nucleosynthesis. Employing measured weak interaction rates and nuclear reaction cross sections the primordial abundances of the light nuclides are predicted by BBN as a function of only one adjustable parameter, $\eta$, the universal ratio of nucleons (baryons) to photons ($\eta = N_B/N_\gamma; \eta_{10} = 10^{10}\eta$). The predicted abundances of $^4$He ($Y$ is the $^4$He mass fraction) D and $^7$Li ($y_2 = N_D/N_H, y_7 = N_{Li}/N_H$) are shown for $1 \leq \eta_{10} \leq 10$ in Figure 1 from Hata et al. (1995). For clarity of presentation the predicted abundance of $^3$He, very similar to that of D, is not shown.

The predicted abundances depend on the universal expansion rate, $t^{-1}$, during the epoch of BBN ($\sim 3MeV \gtrsim T_{BBN} \gtrsim 30keV; 0.1 \lesssim t_{BBN} \lesssim 10^3 sec$). For the early Universe $t^{-1} \propto \rho_{TOT}^{1/2}$, where $\rho_{TOT}$ is the total mass-energy density. For the "standard" model (SBBN), $\rho_{TOT}$ is dominated by photons, electron-positron pairs and three flavors of light, left-handed neutrinos ($\nu_e, \nu_\mu, \nu_\tau$).

$$\rho_{TOT}^{SBBN} = \rho_\gamma + \rho_e + 3\rho_\nu^0. \tag{1}$$

In (1), $\rho_\nu^0$ is the contribution from one flavor of light ($m_\nu << T_{BBN}$) neutrinos. To account for a possibly massive $\tau$-neutrino and/or for other, new particles beyond the standard model, it is convenient to modify eq. (1) by introducing $N_\nu$, the "effective" number of equivalent light neutrinos (Steigman, Schramm & Gunn 1977).

$$\rho_{TOT}^{BBN} = \rho_\gamma + \rho_e + N_\nu \rho_\nu^0. \tag{2}$$

For SBBN, $N_\nu = 3$; for $N_\nu \neq 3$ the universal expansion rate at BBN is modified. For $N_\nu \geq 3$, the universe expands more rapidly leaving less time for the conversion of neutrons to protons. Since most neutrons are incorporated in $^4$He, $Y_{BBN}$ increases with $N_\nu$ (and, vice-versa). Therefore, it is convenient to use $N_\nu$ as a second parameter to explore deviations from SBBN and extensions of the standard model of particle physics (Steigman, Schramm & Gunn 1977). The results in Figure 1 are for SBBN ($N_\nu = 3$).

For $1 \leq \eta_{10} \leq 10$, the predicted abundances of the light nuclides span a range of some 9 orders of magnitude from $\sim 10^{-10} - 10^{-9}$ for Li/H, to $\sim 10^{-5} - 10^{-4}$ for D/H and $^3$He/H, to $\sim 0.1$ for $^4$He/H.

### 2.2. Observations

Primordial abundances are, of course, not observed. Rather, they are inferred from astronomical data. Some, such as D and $^3$He, have been mainly observed "here and now" (in the solar system and the interstellar medium (ISM) of our own Galaxy). For these nuclides it is necessary to extrapolate from here and now to "there and then" to derive their universal primordial abundances. $^4$He and $^7$Li are observed (in addition to here and now) in regions where much less chemical processing has occurred (low metallicity, extragalactic HII regions for $^4$He; very metal-poor halo stars for $^7$Li). For these nuclides the extrapolations to primordial abundances are smaller.

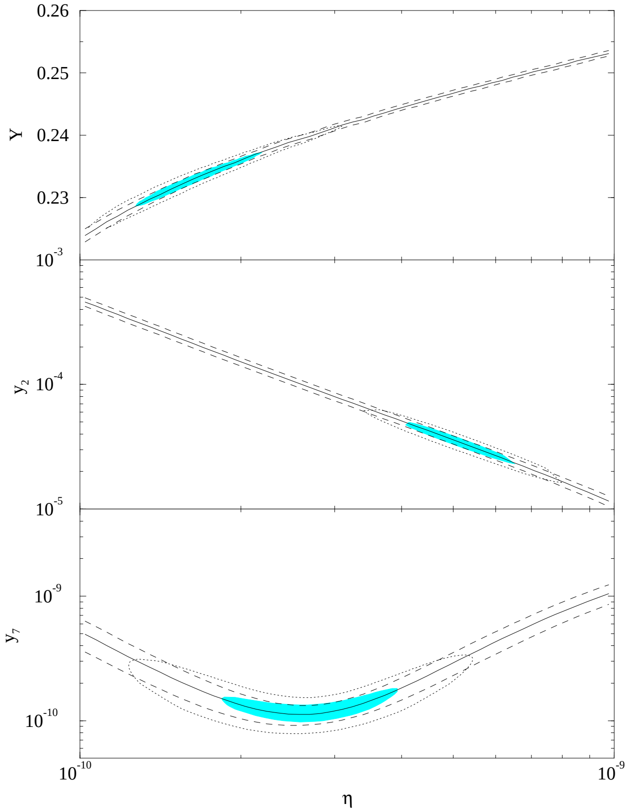

Figure 1. The SBBN predicted abundances (solid lines) of $^4$He (Y is the $^4$He mass fraction), D ($y_2$ = D/H), and $^7$Li ($y_7$ = Li/H) as a function of the nucleon-to-photon ratio $\eta$. The dashed lines are the $1\sigma$ theoretical uncertainties from Monte Carlos. The shaded (dashed) contours are the regions constrained by the observation at the 68% (95%) CL.

In addition to observational uncertainties and those associated with the extrapolations to primordial abundances, systematic effects in deriving abundances from data may contribute to the overall uncertainties. The bad news is that such systematic uncertainties are difficult to constrain. The good news is that the sources of possible systematic errors are different for the different nuclides.

### 2.3. Testing SBBN

The relatively strong and monotonic $y_2$ vs. $\eta$ relation visible in Figure 1 points to D (and, to a lesser extent, $^3$He) as an ideal baryometer. If the primordial abundance of D were known, for example, to $\sim 40\%$, the universal density of baryons would be known to $\sim 25\%$. The large extrapolation from here and now to there and then has inhibited the implementation of this approach. Rather, to avoid this large extrapolation, a more conservative approach has been adopted. Since any D incorporated in a star is burned (to $^3$He) and there are no significant astrophysical sources of post-BBN D, the abundance of D observed anywhere at anytime provides a lower bound to its primordial abundance (e.g., $y_{2P} \geq y_{2\odot}$, $y_{2P} \geq y_{2ISM}$). From Figure 1 it is clear that a lower bound to $y_{2P}$ leads to an upper bound to $\eta$.

It is difficult to avoid the uncertainties of chemical evolution models in using observations of D to infer an upper bound to $y_{2P}$. However, Yang et al. (1984)

noted that since D is burned to $^3$He and some $^3$He survives stellar processing, the primordial abundances of D + $^3$He are strongly correlated with the evolved abundances of D + $^3$He. Burying the stellar and evolution model uncertainties in one parameter, $g_3$, the $^3$He survival fraction, Yang et al. (1984; also, Walker et al. 1991) used solar system data to place an upper bound on primordial D (and/or on D + $^3$He). An upper bound on $y_{2P}$ provides a lower bound on $\eta$ (see Fig. 1).

Due to the "valley" shape in the BBN prediction of Li vs. $\eta$ (see Fig. 1), an upper bound to $y_{7P}$ will provide both lower and upper bounds to $\eta$. The lithium abundance also offers a key test of the standard model since its primordial value must not lie below the minimum predicted ($y_{7BBN} \gtrsim 1 \times 10^{-10}$).

One test of the consistency of SBBN is to use D, $^3$He and $^7$Li to infer lower and upper bounds to $\eta$ ($\eta_{MIN}$, $\eta_{MAX}$) and to check that $\eta_{MIN} < \eta_{MAX}$. If SBBN passes this test, the "$^4$He test" may be applied. The predicted $^4$He mass fraction, $Y_{BBN}$, is a very weak function of $\eta$, increasing from $Y_{BBN} = 0.22$ at $\eta_{10} = 1$ to $Y_{BBN} = 0.25$ at $\eta_{10} = 10$. Thus, it is key to the success of SBBN ($N_\nu = 3$) that for $\eta_{MIN} < \eta < \eta_{MAX}$, $Y_P$ (the inferred primordial abundance) is consistent with $Y_{BBN}(\eta)$ (the predicted abundance).

## 2.4. Consistency

Yang et al. (1984) were among the first to carry out a detailed analysis of the observational data and to implement the tests described above. From D, $^3$He and $^7$Li (with $g_3 \geq 1/4$, see Dearborn, Schramm & Steigman 1986) they found consistency: $3 \lesssim \eta_{10} \lesssim 7$, leading to a predicted range for $^4$He: $0.24 \lesssim Y_{BBN} \lesssim 0.26$. Comparing with the rather sparse data available, they derived $0.23 \lesssim Y_P \lesssim 0.25$ and concluded that SBBN passed the $^4$He test. They did note that SBBN is, in principle, falsifiable and pointed out that if future comparisons should increase $\eta_{MIN}$ and/or decrease $Y_P$, consistency would require $N_\nu < 3$, modifying the standard model.

By 1991 uncertainties in the neutron lifetime (as well as its central value) had been reduced considerably permitting a very accurate prediction of $Y_{BBN}$ vs. $\eta$ (at the $2\sigma$ level, $Y_{BBN}$ is known to $\lesssim \pm 0.001$; see Thomas et al. 1995). At the same time there was extensive new data on lithium (in halo stars) and helium-4 (in extragalactic HII regions). Applying the above tests, Walker et al. (1991) found $2.8 \leq \eta_{10} \leq 4.0$ and $0.236 \leq Y_{BBN} \leq 0.243$. From the HII region data, Walker et al. (1991) derived $Y_P = 0.23 \pm 0.01$ and concluded that SBBN passed the $^4$He test. However, they did emphasize, "that if our lower bound on $\eta$ were increased from $\eta_{10} = 2.8$ to $\eta_{10} = 4.0$, the window on $N_\nu$ would be closed (for $Y_P \lesssim 0.240$)."

## 2.5. Crisis?

Recent applications of the two consistency tests ($\eta_{MIN} < \eta_{MAX}$? $Y_P = Y_{BBN}$?) have provided hints of a possible crisis (Copi, Schramm & Turner 1995; Olive & Steigman 1995; Hata et al. 1995). The two "weak links" are the lower bound on $\eta$ inferred from D and $^3$He observations and the upper bound on $Y_P$ derived from the extragalactic HII region data.

It has long been known that the D + $^3$He analysis of Yang et al. (1984) and Walker et al. (1991) is likely overly conservative. In both analyses the synthesis

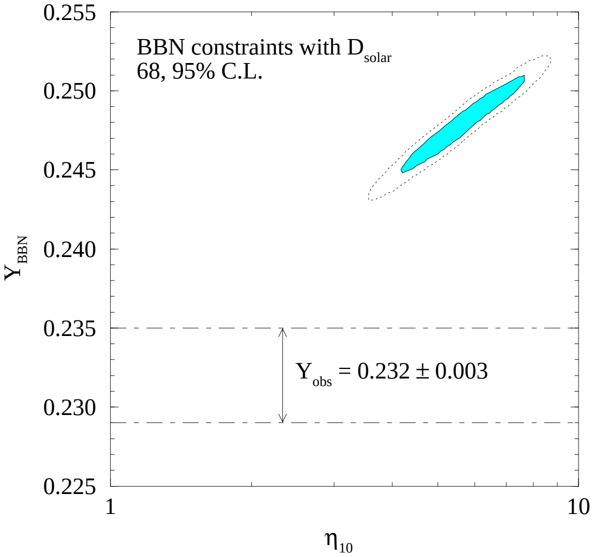

Figure 2. The predicted 68% and 95% CL contours for the $^4$He primordial mass fraction with $\eta$ constrained by D, $^3$He and $^7$Li. Also shown is the $\pm 1\sigma$ range for $Y_P$ inferred from the data.

of new $^3$He in low mass stars (Iben 1967; Rood 1972; Iben & Truran 1978) was neglected. But, Rood, Steigman & Tinsley (1976) had demonstrated that such production might dominate the primordial (D + $^3$He) contribution. Even neglecting this contribution, Steigman & Tosi (1992) had followed the evolution of $^3$He in a variety of chemical evolution models and found more $^3$He survival ($g_3 \gtrsim 1/2$ rather than $g_3 \gtrsim 1/4$) leading to a higher lower bound to $\eta$. More recently, Steigman & Tosi (1995) revisited the "generic" evolution of D and $^3$He and, using updated solar system data (Geiss 1993) inferred (for $g_3 \geq 1/4$) $\eta_{10} \geq 3.1$. In a more sophisticated implementation of the "generic" approach, Hata et al. (1996) found (for $g_3 \geq 1/4$) $\eta_{10} \geq 3.5$.

Although it is still true that $\eta_{MIN} < \eta_{MAX}$, the increasing lower bound to $\eta$ increases the lower bound to $Y_{BBN}$. For $\eta_{10} \geq 3.1$ (3.5), $Y_{BBN} \geq 0.241(0.242)$. An accurate determination of $Y_P$ from observations of $^4$He in low metallicity extragalactic HII regions is required for the $^4$He test. From their analysis of this data, Olive & Steigman (1995) derive $Y_P = 0.232 \pm 0.003$ where 0.003 is the $1\sigma$ statistical uncertainty. Thus, at $2\sigma$, $Y_P^{MAX} < Y_{BBN}^{MIN}$, failing the $^4$He test. It should be noted that $\eta_{MIN}$ and $Y_{BBN}^{MIN}$ have already been pushed to their "$2\sigma$" lower bounds so this discrepancy is at greater than the 95% confidence level. The crisis emerges!

Indeed, Olive & Steigman (1995) used all the data (D, $^3$He, $^4$He, $^7$Li) to infer $N_\nu = 2.17 \pm 0.27$ which deviates from the standard model value ($N_\nu = 3$) by $\sim 3\sigma$. This crisis for SBBN is reflected in Figure 2 where D, $^3$He ($g_3 \geq 1/4$) and $^7$Li have been used to bound $\eta$, leading to predictions of $Y_{BBN}$ at the 68%

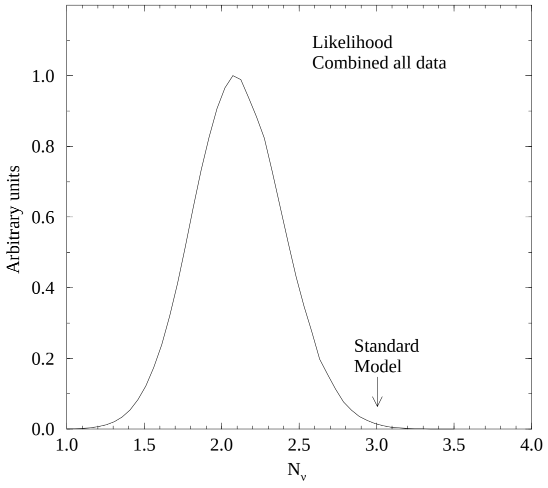

Figure 3. The likelihood function (arbitrary normalization) for the combined fit (D,$^3$He, $^4$He, $^7$Li) of the data and BBN as a function of $N_\nu^{BBN}$. At each value of $N_\nu^{BBN}$ the likelihood is maximized for $\eta$.

and 95% CL. Unless the primordial abundance of $^4$He has been systematically underestimated, the evidence signals a potential crisis for SBBN.

## 3. A Statistical Analysis of BBN

To explore more carefully the consistency of SBBN, my colleagues and I (Hata et al. 1995; Thomas et al. 1995) have undertaken the first comprehensive statistical analysis of the confrontation between theory and observation. We have reexamined the nuclear and weak interactions and their uncertainties and have performed a Monte Carlo analysis of the BBN predictions. Indeed, the curves in Figure 1 reflect the $\pm 1\sigma$ uncertainties in the predictions (for $N_\nu = 3$) of $Y_{BBN}$, $y_{2P}$, $y_{7P}$ vs. $\eta$. From our Monte Carlos we derive $P(A)_{BBN}$, the probability distributions for the predicted BBN abundances (A). We have also reexamined the observational data, accounting for the statistical uncertainties as well as attempting to allow for various systematic uncertainties which may arise in using the data to infer the distribution, $P(A)_{OBS}$, of primordial abundances. These latter uncertainties are not necessarily modelled by gaussian distributions. In contrast to previous approaches which treated each element one at a time, we may use the information on all light nuclides to form a likelihood function (as a function of $\eta$ and $N_\nu$) from $P(A)_{BBN}$ and $P(A)_{OBS}$. The likelihood function, maximized with respect to $\eta$ at each $N_\nu$ is shown in Figure 3 (from Hata et al. 1995). We derive $N_\nu = 2.1 \pm 0.3$, consistent with Olive & Steigman (1995). It is clear that SBBN ($N_\nu = 3$) provides a poor fit to the primordial abundances inferred from the data.

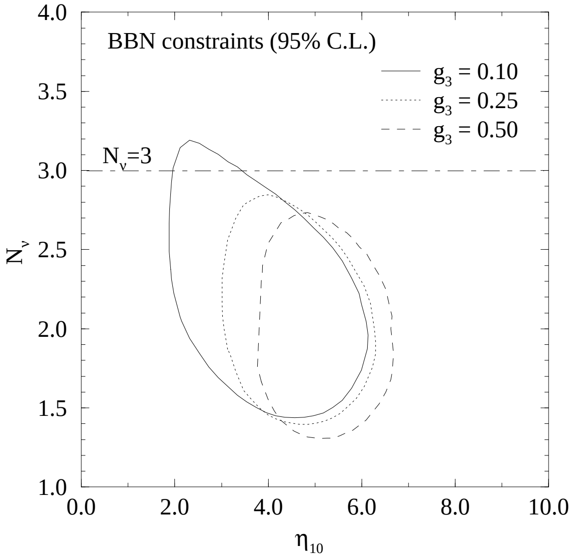

Figure 4. The 95% CL contours in the $N_\nu^{BBN}$ vs. $\eta$ plane for several choices of the $^3$He survival fraction $g_3$.

As with the 19th century standard model, SBBN is challenged. As with the challenges to the 19th century standard model, there are several options for the resolution of this crisis.

### 3.1. Is The Chemical Evolution Extrapolation Wrong?

One source of the challenge to SBBN is the relatively high lower bound to $\eta$ imposed by the relatively low primordial abundances of D and $^3$He inferred from solar system and interstellar observations. These stringent upper bounds to primordial D and $^3$He are suggested by many specific chemical evolution models (Steigman & Tosi 1992) as well as the "generic" model for the evolution of D and $^3$He (Steigman & Tosi 1995; Hata et al. 1996). In the latter case, the crisis worsens with increasing $g_3$ and/or if stellar production of $^3$He is allowed for. The crisis could be ameliorated if $g_3$ is less than the lower bound ($g_3 \geq 1/4$) adopted in the above analyses. In Figure 4 (from Hata et al. 1995), 95% CL contours are shown in the $N_\nu$ vs. $\eta$ plane for several choices of $g_3$. If the "effective" $g_3$ (averaged over stars of all masses and the evolution history of the ISM) is $\sim 0.1$, consistency of SBBN is reestablished.

### 3.2. Is The Primordial $^4$He Abundance Larger?

An alternate source of the challenge to SBBN is the relatively low abundance of primordial $^4$He inferred from the observations of extragalactic HII regions. By allowing only for statistical uncertainties perhaps we've underestimated the true uncertainty in $Y_P$. A larger value for $Y_P$ could reestablish the consistency of $^4$He

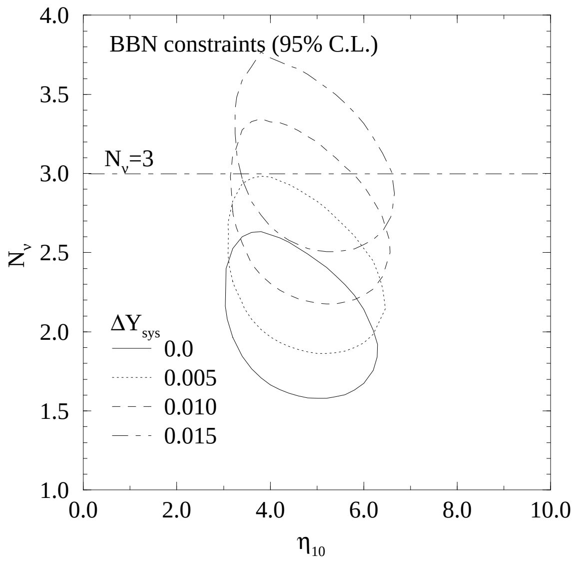

Figure 5. The 95% CL contours in the $N_\nu^{BBN}$ vs. $\eta$ plane for several choices of the systematic error ($\Delta Y_{sys}$) in the $^4$He abundance inferred from HII region data.

with D and $^3$He. Many sources of possible systematic uncertainty in $Y_P$ have been identified and some have been studied (Davidson & Kinman 1985; Pagel et al. 1992; Skillman & Kennicutt 1993; Skillman et al. 1994; Copi, Schramm & Turner 1995; Sasselov & Goldwirth 1995; Olive & Steigman 1995). In Figure 5 (from Hata et al. 1995) are shown 95% CL contours in the $N_\nu$ vs. $\eta$ plane for several choices of $\Delta Y_{sys}$, where $Y_{BBN} = 0.232 \pm 0.003 + \Delta Y_{sys}$. If $Y_P$ is shifted up by $\gtrsim 0.010$, SBBN may be consistent at the 95% CL. It should, however, be emphasized that $\Delta Y_{sys}$ may be negative as well as positive; a negative $\Delta Y_{sys}$ exacerbates the crisis for SBBN.

### 3.3. Is There New Physics?

By employing $N_\nu$ as a second parameter, we have allowed for a class of modifications of the standard model. If, in addition to three flavors of light, left-handed neutrinos ($N_\nu = 3$, SBBN) there are additional light neutrinos or other new particles, $N_\nu > 3$ (Steigman, Schramm & Gunn 1977) and the crisis worsens. However, although $\nu_e$ and $\nu_\mu$ are known to be "light" ($m_\nu < T_{BBN}$), accelerator data on $\nu_\tau$ (ALEPH Collaboration) permits $m_{\nu\tau} \leq 24MeV$. As Kawasaki et al. (1994) have shown, the presence of a massive, unstable tau neutrino with $5 - 10 \lesssim m_{\nu\tau} \leq 24MeV$ and $0.01 \lesssim \tau_{\nu\tau} \lesssim 1\,\mathrm{sec}$. would correspond to an "effective" $N_\nu < 3$. Perhaps the crisis for SBBN is teaching us about extensions of the standard model of particle physics.

## 4. Summary and Conclusions

Primordial nucleosynthesis must have occurred during the early, hot, dense evolution of a Universe described by the hot big bang model. Therefore, BBN offers a test of standard cosmology as well as a probe of particle physics. As with the standard model of 19th century physics, over many years SBBN has provided support for the standard model of cosmology. Indeed, the success of SBBN in predicting the abundances of the light nuclides with only one adjustable parameter $\eta$ ($N_\nu = 3$) restricted to a narrow range ($3 \lesssim \eta_{10} \lesssim 4$) while the abundances range over some 9 orders of magnitude, is impressive indeed.

However, as with the 19th century standard model, some clouds have now emerged on the horizon. Recent analyses (Copi, Schramm & Turner 1995; Olive & Steigman 1995; Hata et al. 1995) point to a crisis unless the data are in error, or the extrapolations of the data are in error, or there is new physics. Some analogies with the crises which confronted the 19th century standard model may be instructive.

(i) Perhaps our extrapolations of the observations of D and $^3$He from here and now to there and then have been naive. Chemical evolution models in which more D is cycled through stars and destroyed (without a concommitant overproduction of $^3$He) would permit a higher primordial abundance of D, allowing a lower $\eta$ and $Y_{BBN}$ consistent with $Y_P$. Thus, as with the discovery of Uranus, the crisis for SBBN may teach us something new about galactic evolution.

(ii) Perhaps our estimates of the primordial abundance of $^4$He are in error because we have overlooked some large systematic error in the abundance determinations. If $Y_P$ is larger than the value inferred form the observational data, $\eta$ may be as large as inferred from D and $^3$He (see Figure 2) and still $Y_{BBN}$ and $Y_P$ may be consistent. Then, as with the discovery of Pluto, our crisis may have been a false alarm from which, nonetheless, we learn something new.

(iii) The most exciting possibility, of course, would be that the data are accurate, the systematic errors small and the extrapolations true. Then, this crisis may point us to new physics beyond the standard models of particle physics or cosmology. The "window" on a massive, unstable $\tau$-neutrino is accessible to current accelerators.

To summarize, then, SBBN ($N_\nu = 3$; $g_3 \geq 1/4$; $\Delta Y_{sys} \leq 0.005$) provides a poor fit to the primordial abundances of the light nuclides inferred from current observational data (Hata et al. 1995). This crisis is not a cause for alarm, but an opportunity to learn something new about astronomy, cosmology, or particle physics.

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