SURFACE TEMPERATURE OF A MAGNETIZED NEUTRON STAR AND INTERPRETATION OF THE ROSAT DATA. II.

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ABSTRACT

We complete our study of pulsars' non-uniform surface temperature and of its effects on their soft X-ray thermal emission. Our previous work had shown that, due to the effect of gravitational lensing, dipolar fields cannot reproduce the strong pulsations observed in the four nearby pulsars for which surface thermal radiation has been detected: PSR 0833-45 (Vela), PSR 0656+14, PSR 0630+178 (Geminga), and PSR 1055-52. Assuming a standard neutron star mass of 1.4 M_{\odot} , we show here that the inclusion of a quadrupolar component, if it is suitably oriented, is sufficient to increase substantially the pulsed fraction, Pf , up to, or above, the observed values if the stellar radius is 13 km or even 10 km. For models with a radius of 7 km the maximum pulsed fraction obtainable, with (isotropic) blackbody emission, is of the order of 15% for orthogonal rotators (Vela, Geminga and PSR 1055-52) and only 5% for an inclined rotator as PSR 0656+14. Given the observed values, this may indicate that the neutron stars in Geminga and PSR 0656+14 have radii significantly larger than 7 km and, given that very specific quadrupole components are required to increase $P f$, even radii of the order of 10 km may be unlikely in all four cases. However, effects not included in our study may possibly seriously invalidate this temptative conclusion.

We confirm our previous finding that the pulsed fraction always increases with photon energy, below about 1 keV, when blackbody emission is used and we show that it is due to the hardenning of the blackbody spectrum with increasing temperature. The observed decrease of pulsed fraction may thus suggest that the emitted spectrum softens with increasing temperature and that this observed effect must be of atmospheric origin.

Finally, we apply our model to reassess the magnetic field effect on the outer boundary condition used in neutron star cooling models and show that, in contradistinction to several previous claims, it is very small and most probably results in a slight reduction of the heat flow through the envelope.

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1. INTRODUCTION

In a previous paper (Page 1995a; hereafter Paper I) we had presented a simple but realistic model of temperature distribution at the surface of a magnetized neutron star. This model was used to study the effects of temperature inhomogeneity on the neutron star thermal emission and compare them with the observed properties of the four neutron stars for which this emission has been detected by the $ROSAT$ satelite (Ögelman 1995a; Page 1995b): PSR 0833-45 (Vela; Ögelman, Finley, & Zimmermann 1993), PSR $0656+14$ (Finley, Ogelman, & Kiziloğlu 1992), PSR 0630+178 (Geminga; Halpern & Holt 1992) and PSR 1055-52 (Ogelman & Finley 1993). As a first step we did not take into account magnetic effects in the atmosphere which are also very large. We also restricted ourselves to dipolar magnetic fields and the three main results we obtained were:

− Magnetic effects on heat transport in the neutron star envelope do induce very large temperature differences at the surface but when gravitational lensing is taken into account and only dipolar fields are considered the predicted pulsed fractions are smaller than those observed in the soft band where the surface thermal emission is seen. In particular, in the case of 1.4 M_{\odot} neutron stars with small radii ~ 7.0 – 9.0 km, the pulsed fractions $P f$ s are below 1%. With very small radii ≤ 6.5 km, and the same mass, the gravitational beaming can however increase Pf up to 6 -7%. Observed values of Pf range between 10 - 30%. − With dipolar fields the observable light curves are very symmetrical and their shapes do not correspond to the observations. Together with the previous result, this lead us to conclude that the inclusion of dipolar fields only is not sufficiently adequate to model the surface magnetic field of the four observed neutron stars.

− The amplitude of the pulse profile increases with increasing photon energy. This result does not correspond to what is observed in the soft X-ray band in the cases of PSR 0656+14, Geminga and PSR 1055- 52.

We complete here this study and consider also the effects of these surface temperature distributions on the cooling of neutron stars. Since the completion of the work presented in Paper I, complementary results considering the magnetic effects in the atmosphere, where the emerging spectrum is generated, have been presented (Pavlov et al. 1994; Zavlin et al. 1995)

and clearly showed that they can be as important as the magnetic effects in the envelope that we consider here and in Paper I. A complete study must obviously include both aspects of the problem, envelope and atmosphere. However, as long as the exact chemical composition of a pulsar's surface is not known, analysis with blackbody (BB) emission will still remain a mandatory first step and a reference to which atmosphere models will be compared. For this reason it is important to characterize BB emission and determine what can and what cannot be obtained with it. As important as the successes of our model will thus be its failures which can guide us toward the correct atmosphere, or more generally surface, model (Page 1995b; Page, Shibanov, & Zavlin 1995). Nevertheless, several of our results are, we hope, sufficiently general and robust to be valid despite of the limitations of BB emission. We restrict ourselves to study general properties of the model and refrain from detailed analyses of the data which we leave for future work. Our method to generate surface temperature distributions and the observable X-ray fluxes is described in detail in Paper I.

In § 2 we present some complements to our summary of the $ROSAT$ data given in Paper I. In $\S 3$ $\S 3$ we consider the effects of the inclusion of the quadrupolar components of the magnetic field and show that when they are superposed to a dipolar field they provide a sufficiently general configuration for our study. Some basic and straightforward results are presented in § [4](#page-4-0) and [5](#page-4-0). We use these configurations to discuss the reliability of our surface temperature model in § [6](#page-4-0) and also to study the possibility of putting constraints on the neutron star (NS) size through gravitational lensing in § [7.](#page-5-0) We then discuss the energy dependence of the light curves amplitude in $\S 8$ $\S 8$, this section will give a clear indication of the inadequacy of BB emission for understanding detailed features of the observations. The next section, [9,](#page-7-0) adds a second component of emission in order to model the higher energy tail of Geminga. Section [10](#page-7-0) studies the effect of the surface temperature distribution on the boundary condition used for modeling the thermal evolution of neutron stars. Finally, § [11](#page-9-0) presents our conclusions.

2. THE ROSAT DATA

Since the completion of Paper I some information has been presented on the second set of ROSAT observations of PSR $0656+14$ (Ogelman 1995a; Possenti, Mereghetti, & Colpi 1996). The complete set of data clearly shows the presence of a hard tail which is strongly pulsed, as in Geminga and PSR 1055-52. Both the energy dependence of the pulsed fraction and of the pulse phase show some similarity with PSR $1055-52$ (Ogelman 1995a) and also with Geminga (Ogelman 1995b): in all three cases Pf is almost constant up to channels \sim 40 (\sim 30 for Geminga), then decreases and finally increases (very strongly in the case of PSR 1055-52). Moreover, at around the same energy where Pf increases, the phase of the peak changes. The spectral fits with a soft BB for the surface thermal emission and a BB or power law for the hard tail show that the surface thermal emission dominates at channels below channel ∼ 50 in the case of Geminga (e. g., figure [6\)](#page-16-0) and channel ~ 100 in the cases of PSR 0656+14 and 1055-52 (see, e. g., O gelman 1995a). An important point is that in the case of Geminga the change in the peak phase coincides with the spectral shift from the surface thermal emission to the hard tail (see \S [9\)](#page-7-0) and is not surprising. However, in the two other cases this phase shift apparently, and intriguingly, occurs within the band dominated by the surface thermal emission.

3. QUADRUPOLAR FIELDS

3.1. The quadrupolar components and gravity effects

The first natural step beyond dipolar fields is the inclusion of a quadrupolar component which we write as

$$
\mathbf{B}^{Q} = \sum_{i=0,4} Q_{i} \left(\frac{R}{r}\right)^{4} \mathbf{b}_{i}
$$
 (1)

where the five generating fields \mathbf{b}_i are listed in table [1](#page-13-0). Considering the generating quadrupole components separately at the star's surface, one finds that all components reach a maximum strength of 1 and \mathbf{b}_0 has a minimum of $1/\sqrt{5} (=0.45)$ while the other four have a minimum of zero. They have four magnetic poles except for b_0 whose 'south' pole is degenerate and becomes a line covering the whole equator. However, the general quadrupolar field \mathbf{B}^{Q} can have up to six poles, south and north poles always being in an even number. When the quadrupole is added to a dipole we can have again up to six poles, and the number of north (south) poles can be odd. Notice finally that the scale value of the dipolar component we use is the field strength at magnetic pole, i. e., twice the

value commonly considered, so that all field scales we cite always refer to the maximum field strength of the cited component at the stellar surface in flat spacetime.

We also include the effect of gravity on the magnetic field. It is not as important as red-shift and lensing but is nonetheless not completely negligible since it can substantially increase the field strength and is moreover straightforward to implement. It can be written in the form of four multiplicative factors, two for the radial component of the magnetic field and two for its angular components: if B_r , B_τ ($\tau = \theta$ or ϕ) denote the radial and angular components of the magnetic field in the absence of a gravitational field, then:

$$
B_{rg}^D = f_1 \ B_r^D, \qquad \qquad B_{rg}^D = \sqrt{g_{00}} \ g_1 \ B_r^D \qquad (2)
$$

give the dipolar fields in the presence of a gravitational field, and:

$$
B_{rg}^Q = f_2 B_r^Q, \qquad B_{rg}^Q = \sqrt{g_{00}} g_2 B_r^Q \qquad (3)
$$

give the quadrupolar fields in the same situation. The factors in the previous equalities are given by:

$$
f_1 = -\frac{3}{x^3} \left[\ln(1-x) + \frac{1}{2} x(x+2) \right] \tag{4}
$$

$$
g_1 = -2f_1 + \frac{3}{1-x} \tag{5}
$$

$$
f_2 = \frac{10}{3x^4} \left[6\ln(1-x)\frac{(3x-4)}{x} + x^2 + 6x - 24 \right] \tag{6}
$$

$$
g_2 = \frac{10}{x^4} \left[6\ln(1-x)\frac{2-x}{x} + \frac{x^2 - 12x + 12}{1-x} \right] \tag{7}
$$

where $x = R_S/r$, $R_S = \frac{2GM}{c^2}$ being the star's Schwarzschild radius, and $g_{00} = 1 - x$ is the time component of the metric (Muslimov & Tsygan 1987). These corrections of course tend towards 1 at large distances. The values of f_l and g_l at the stellar surface, i. e., at $x = R_S/R$, are plotted in figure [1:](#page-15-0) the effect is stronger on the quadrupole than on the dipole, simply due to the stronger r dependence of the former. If $R \to R_S$ then f_l and g_l tend to ∞ which means that, for a given surface field, the field 'at infinity' vanishes: this is a particular case of the 'no hair' theorem for non rotating blackholes (see, e. g., Misner, Thorne, & Wheeler 1970).

3.2. Statistics of dipole+quadrupole fields

Since the quadrupolar component introduces five more degrees of freedom, a clear general assessment of its possible effects is necessary. To investigate this, we computed nine sets of 1000 models each, in which the quadrupolar components where chosen at random by the computer: three orientations of the dipole and observer, $\alpha = \zeta = 30^{\circ}$, 60° and 90° , were taken to test for geometrical effects (α is the angle between the rotation axis and the dipole and ζ between the rotation axis and the observer's direction) and three values for the stellar radius, $R = 7$ km, 10 km and 13 km, were used to analyze the effect of gravitational lensing. The maximum lensing angles θ_{max} are respectively, 194[°], 132°, and 117° for radii of 7, 10, and 13 km, respectively (figure 3 in Paper I). The dipole strength was fixed at 10^{12} G (value at the magnetic pole in flat space-time; GR effects increase it slightly) and the 1000 quadrupoles were added to the basic dipole. The quadrupole component strengths, Q_i , were restricted to the range 10^{10} G – 5 · 10^{12} G. The star's surface temperature was calculated using an interior temperature of $T_b = 10^8$ K which implies an effective temperture around 10^6 K (the exact value depending on the star size and field configuration). Notice that GR effects depend only on the ratio $R/R_S = Rc²/2GM$ of the star's radius R to its Schwarzschild radius R_S , so that all our results can be extrapolated to other masses and radii (see, e. g., figure [1](#page-15-0) for easy conversion). To save CPU time we only calculated the variation of the observable phase dependent effective temperature, T_e^{Φ} (Paper I, § 5.2), during the star's rotation, i. e., the effective temperature of the portion of the stellar surface visible to the observer at a given phase Φ. Selecting a subset of the 9,000 models so generated we performed complete calculations of the detectable fluxes to calibrate the observable pulsed fraction Pf , in the PSPC's channel range $7 - 50$, in terms of the pulsation of T_e^{Φ} . The results of these calculations are shown in figure [2](#page-15-0) where each model is shown as a black dot and the pure dipole case as a black-white ring. The dramatic effect of lensing is again clearly seen. For later use, figure [2](#page-15-0) plots the ratio of the star's effective temperature T_e (mag.) for the given field configuration to the effective temperature it would have without the magnetic field, T_e (non mag.), at the same interior temperature $T_b = 10^8$ K. A different T_b would change the values of P f but not change significantly the statistics for the number of models giving large $P f$. The observable $P f$ increases

with decreasing temperature for a given surface magnetic field configuration as explained in § [4](#page-4-0).

We want here to correct our definition of the pulsed fraction (Eq. 21 in Paper I) and rather use its standard expression as the fraction of counts above minimum, thus

$$
Pf(i) = \frac{(Cts_{mean} - Cts_{min})}{Cts_{mean}} \tag{8}
$$

where Cts_{mean} and Cts_{min} are, respectively, the mean and minimum count rates detected during the star's rotation in channel $\#$ i. For sinusoidal pulse profiles this is equivalent to our former definition and does not affect the results of Paper I.

The first obvious and natural result of figure [2](#page-15-0) is that most configurations produce little pulsation: most quadrupoles induce several warm regions which flatten the pulse profile even below the value of the pure dipole case. However, many configurations do rise Pf substantially, compared to the purely dipolar case, and to a high value in some special cases. We can thus obtain observable pulsed fractions higher than 30% (at energies below 0.5 keV and at $T_e \sim 10^6$ K) for a star of radius 13 km. For stars of radii 10 km a Pf value comparable to the observed ones can also be obtained. The most interesting case is the 7 km radius star: despite of the enormous gravitational lensing in this star $(\theta_{max} = 194^{\circ})$, we can still find a configuration which gives a P f above 10% at high $T_e \sim 10^6$ K) and above 15% at $T_e \sim 3 \cdot 10^5$ K, as we have checked explicitly.

A last point is worth mentioning: the strong increase in the pulsed fraction possible with the addition of a quadrupolar component is not due to an increase in the area of the cold regions, compared to the purely dipolar case, but rather to a displacement of the magnetic poles and the surrounding warm regions which brings them closer to each other (see, e.g., figure [3\)](#page-15-0). The pulse profiles of Geminga and PSR 1055- 52, which are both considered to be almost orthogonal rotators, show a single wide peak which indicates that the two polar caps and the surrounding warm regions are much less than 180◦ apart. How close the magnetic poles can be pushed toward each other is what will determine the 'success' of the quadrupolar component in increasing the pulsed fraction, whereas quadrupoles which moreover introduce several new warm regions will fail. Figure [2](#page-15-0) illustrates this clearly: the ratio $T_e(\text{mag})/T_e(\text{non mag.})$ is practically the same for the strongly pulsed dipole+quadrupole configurations as for the purely dipolar case, about 0.9, while weakly pulsed configurations which induce several warm regions may have a ratio larger than one. These considerations will be of importance for § [10](#page-7-0).

4. EFFECT OF THE FIELD STRENGTH

Considering the effect of the field strength, in the purely dipolar case we found that the maximum Pf is obtained when the overall surface field reaches a few times 10^{11} G (Paper I): the same result still applies to the dipole + quadrupole case. Below $2-3 \cdot 10^{11}$ G the magnetic effects are getting smaller and above this value, even if the magnetic effects are increasing, the resulting observable Pf varies very little. The reason why the pulsed fraction depends very weakly on the field strength for large enough fields is simple: cold regions contribute so little to the total flux that it does not really matter how cold they are, but only how large they are. We are dealing mostly with warm plates whose extension is determined more by the field orientation than by the field strength as long as the field strength does not drop below a few times 10^{11} G.

5. EFFECT OF THE TEMPERATURE DIS-TRIBUTION ON THE SPECTRUM

The surface temperature distributions induced by dipole + quadrupole fields can have quite complicated structures. However, as discussed in Paper I for dipolar fields, the resulting phase integrated spectra are close to single temperature spectra at the corresponding effective temperature. Figure [4](#page-15-0) shows examples at five temperatures for the dipole+quadrupole field from the set generated for figure [2](#page-15-0) (see $\S 3.2$ $\S 3.2$) which gives the highest pulsed fraction. The surface temperature distribution induced by this field configuration is illustrated in figure [3](#page-15-0). The result is very close to the effect of a simple dipole (figure 6 in Paper I) due to the fact that the chosen quadrupole induces warm regions of approximately the same size as the pure dipole but at different locations (see \S [3.2\)](#page-3-0). When taking into account the PSPC's response (figure [4](#page-15-0) B) the composite spectrum presents a clear excess compared to the single temperature spectrum at channels above channels $50 - 60$. This excess will not affect spectral fits in the case of Geminga since the flux at channels above 60 is dominated by the hard tail. However, in the three other cases where the hard tail only appears above 1 keV, even above 1.2 keV for Vela, this excess will have some impact on the temperature measurement: spectral fits with composite BB spectra will give slightly lower T_e^{∞} 's than single BB spectra and require lower N_H 's. (Since the purely dipole case gives similar results we must correct the corresponding statement made in Paper I about spectral fits and replace it by the preceding one). We will not attempt to quantify this small effect here since we restrict ourselves to general characteristics of our model. This excess in the Wien tail will also affect parameter values for the fits of the hard tail for PSR 0656+14 and 1055-52 while in the case of Geminga our illustrative fit of \S [9](#page-7-0) can be done with the same parameter as used by Halpern & Ruderman (1993) for uniform surface temperature.

6. RELIABILITY OF THE SURFACE TEM-PERATURE MODEL

Our surface temperature model has two ingredients:

1) the value of T_s for a given field strength in the two cases of parallel and orthogonal transport $[T_s(\Theta_B =$ 0°), and $T_s(\Theta_B = 90^\circ)$, respectively, where Θ_B is the angle between the local magnetic field and the radial direction], which are given in figure 1 of Paper I for various field strengths B, and

2) the dependence of T_s on Θ_B for a given field strength B:

$$
T_s(\Theta_B) = \chi(\Theta_B) \times T_s(\Theta_B = 0^\circ)
$$
 (9)

.

where

$$
\chi(\Theta_B) = (\cos^2 \Theta_B + \chi_0^2 \sin^2 \Theta_B)^{1/4} \; ; \; \; \chi_0 \equiv \frac{T_s(\Theta_b = 90^\circ)}{T_s(\Theta_b = 0^\circ)} \tag{10}
$$

As mentioned in Paper I, the dependence given in point (2) is a very good approximation, much better than point (1), and its accuracy is certainly better than the statistical uncertainties present in the ROSAT data. Most of the uncertainty of the model resides in the NS envelope calculations used for point (1). The ' T_b-T_s relationship' $T_s(T_b,\Theta_B,B)$, where T_b is the interior temperature at the bottom of the envelope at density $\rho = 10^{10}$ gm cm⁻³, is known reasonably well at surface temperatures above 3.10^5 K in the case of $\Theta_B = 0^\circ$ and has been calculated by several authors who obtained values in reasonable agreement with each other; the problem is for the case $\Theta_B = 90^\circ$, where there are large uncertainties due to several factors. $T_s(\Theta_B = 90^\circ)$ is much lower than $T_s(\Theta_B = 0^\circ)$,

for a given T_b , and in this case of orthogonal transport the envelope structure is often in the "100% Murphy regime" where everything that can go wrong does go wrong: electron conductivity perpendicular to the field is strongly suppressed and photon transport is the dominant process up to much higher densities than in the case of parallel transport; at these densities and low surface temperatures, the plasma temperature is much higher than the local temperature but, thanks to the magnetic field, extraordinary photons can still exist and transport heat; lattice or ion heat transport can also contribute, a phenomenon that has been observed in laboratory materials under strong magnetic fields (Zyman 1960); the low density region $(\rho < 10^{4-5} \text{g/cm}^3)$ can affect the surface temperature and the magnetic effects on the equation of state become very large. Finally, meridional flow of matter induced by the temperature gradients, rotation, and magnetic field gradients can also smooth the temperature distribution. Unfortunately, none of these effects has been taken into account in the envelope calculation that we use for our model (Schaaf 1990), the only such calculation published to date.

Despite of these limitations we consider our model as quite reliable thanks to the results of \S [3](#page-2-0) which showed that the observable characteristics of thermal emission do not depend sensitively on the field strength for strengths above a few times 10^{11} G (when the emission spectra used are from blackbodies). Maximum magnetic effects are obtained when B is superior to about 10^{12} G, and saturate at higher fields. For a large enough field $\chi_0 \ll 1$ and we can thus write

$$
T_s(\Theta_B) = T_s(\Theta_B = 0^\circ) \times \cos^{1/2} \Theta_B \qquad (11)
$$

to a good approximation as long as Θ_B is not too close to 90°; when Θ_B is close to 90° the region is so cold that it is not seen anyway. The surface fields of the young pulsars we consider can be reasonably assumed to be above this threshold value of $2 - 3 \cdot 10^{11}$ G: values obtained by the standard magnetic dipole radiation braking formula, as reported in table 1 of Paper I, are between $2.2 \cdot 10^{12}$ G (PSR 1055-52) and $8.8 \cdot 10^{12}$ G (PSR 0656+14) without GR effects, and are even larger when GR effects are included (see above). In the case of Geminga the optical data suggest the presence of a synchrotron emission line which implies a magnetic field of at least $3 \cdot 10^{11}$ G (Bignami et al. 1996). So, as long as the exact value of $T_s(\Theta_B = 90^\circ)$ is about 3 times lower than $T_s(\Theta_B = 0^{\circ})$, as it is at

 $3 \cdot 10^{11}$ G in our model, for the field strengths actually present, our interpretation of the ROSAT observations will remain valid.

7. GRAVITATIONAL LENSING AND THE NS SIZE

It was shown in Paper I that when only dipolar fields are used, the pulsed fraction Pf obtained is lower than what has been detected. The addition of a quadrupolar component can substantially increase Pf to values comparable to the observed ones as shown in § [3.2](#page-3-0) (figure [2\)](#page-15-0). However, for this to happen we must choose carefully the strengths of the various \mathbf{b}_i compared to the dipole's strength and orientation: random quadrupolar components generally give temperature distributions which are so complicated that they flatten the light curves.

Vela and PSR 1055-52 have pulsed fractions, in the channel band where surface thermal emission is detected, slightly higher than 10% and are considered to be almost orthogonal rotators: the models of figure [2](#page-15-0) with $\alpha \sim \zeta = 90^\circ$ thus apply to them. Taking into account that the effective temperature of PSR 1055- 52 is lower than 10^6 K the observables Pfs would be slightly higher than values given in figure [2.](#page-15-0) We see that for 1.4 M_{\odot} stars with radii of 13 or 10 km many models can produce Pf 's as high as observed. However only very few, special, field configurations can reproduce the observed Pf at $R = 7$ km, less than 0.1% of all configurations generated randomly for figure [2.](#page-15-0) The high $Pf \sim 20 - 30\%$ of Geminga, also an orthogonal rotator, is however not reproducible with a 7 km radius 1.4 M_{\odot} star (i.e., with $R = 1.7R_S$): the most strongly pulsed dipole+quadrupole model we found gives, at T_e = 5 · 10⁵ K, $Pf \sim 15\%$ in channel band $7 - 50$ as we have checked explicitly, and only a few models at 10 km radius reach 20% modulation. In the case of PSR 0656+14, which has a BB effective temperature of ~ 8 · 10⁵ K and $\alpha \sim \zeta \sim 30^{\circ}$, we can obtain the observed $Pf \sim 14\%$ with a 13 km radius 1.4 M_{\odot} star with many models and at 10 km radius in only a few cases, but with no model at all for a 7 km star: in this latter case the highest Pf we can obtain is about 4%. If we take $\alpha \sim \zeta \sim 8^{\circ}$ (Lyne & Manchester 1988) then Pf is always extremely small: even with a radius of 13 km, for a mass of 1.4 M_{\odot} , the same procedure as used for figure [2](#page-15-0) could not find any model which gives a Pf higher that 4%.

8. ENERGY AND TEMPERATURE DEPEN-DENCE OF THE PULSED FRACTION

One of the most important characteristics of the observed pulsations is the energy dependence of the pulsed fraction Pf . We showed in Paper I that BB emission with dipolar fields always produces an increase of Pf with photon energy. We confirm this here, with a few exceptions, for more general temperature distributions and explain the origin of this feature. BB emission, with the appropriate surface temperature distribution, can explain many of the observed characteristics of surface thermal emission of the four pulsars we are studying. Since there are many theoretical reasons for BB emission to be inadequate it is important to find at least one property not reproducible in our simple model in order to guide us toward the correct atmosphere model(s). The energy dependence of Pf happens to be such a feature: as mentioned in $\S 2$ $\S 2$ in the three cases of PSR 0656+14, Geminga, and PSR 1055-52, Pf decreases at channels around channel 40 – 50. Another strong constraint on atmosphere models is of course that the spectral fit must imply a reasonable pulsar distance (cf. discussion in \S [11\)](#page-9-0).

The reason for the increase of Pf with energy for BB emission was elucidated by Page, Shibanov, & Zavlin (1995): the hardness of the BB spectrum increases with energy. The ratio of the BB fluxes emitted at energy E by two regions of areas A_1 and A_2 , with temperatures T_1 and T_2 respectively, is

$$
\frac{F_{BB}(E,T_1)}{F_{BB}(E,T_2)} = \frac{A_1}{A_2} \times \frac{\exp(E/k_B T_2) - 1}{\exp(E/k_B T_1) - 1} \tag{12}
$$

which is an *increasing* function of E if $T_1 > T_2$. This means that if we have a warm spot at temperature T_1 on a surface with uniform temperature $T_2 < T_1$, then Pf naturally increases with energy. A smooth temperature distribution with only one peak in temperature will obviously give the same result. With several peaks, by experience we can state that Pf always increases (with increasing channel energy) up to channels ~ 60 for $T_e~\sim~5~\cdot~10^5$ K and up to channel ~ 100 for $T_e \sim 10^6$ K: we have tried many models with dipoles, off-centered dipoles, quadrupoles, dipole+quadrupoles, warm plate(s) and found no exception. Only in a few special cases, we have 'observed' a decrease in Pf in the Wien tail which is due to the following (figure [5\)](#page-15-0). The surface presents two warm regions on opposite sides of the star, a very large

one and a smaller one where the temperature reaches a slightly higher value (figure [5](#page-15-0)A), and only one peak is visible, i. e., the peak from the small warmer region does not appear as a peak but is only filling the dip between the successive peaks of the large region (figure [5](#page-15-0)B); by looking at increasing energy the filling of the small peak increases (the small region is warmer than the large one) and Pf thus decreases (figure [5C](#page-15-0)). If the area of the small warmer region is increased then it produces a distinctive peak, i. e., we obtain a double peak light curve, the amplitude of both peaks increases with energy (the amplitude of the small region peak will eventually win over the large region one) and Pf increases. It seems thus that the only mechanism, with BB emission, which can produce a decrease in Pf is this filling of the interpeak by emission from a small warmer region. This is in fact the mechanism that Halpern & Ruderman (1993) invoked to explain the observed decrease of Pf (in channel range $28 - 53$ vs. $7 - 28$) in Geminga but with an essential difference: they proposed that the hard tail component, which is about 100◦ off phase with the thermal component, would fill the dip in the light curve. However the hard tail is much harder than the surface thermal component, i. e., clearly separable from it in the spectrum, and cannot actually produce a decrease in Pf at energies below 0.5 keV as we show in the next section. In the cases where we have 'observed' a decrease in Pf , the responsible component is only slightly warmer than the rest of the star and its emission is practically indistinguishable in the spectrum.

Another feature encountered in Paper I for dipolar fields was a steepening of the increase of Pf in the channel band $50 - 70$ which is due to the response of the PSPC. The carbon present in the detector's window produces a strong absorption edge at 284 eV: the effective area $A(E)$ vanishes just above this energy and starts growing significantly only above 400 eV. Thus, channels below \sim 40 pick up almost exclusively photons with energy below 284 eV while channels above ∼ 50 detect photons almost exclusively above 500 eV: since the 'absolute' pulsed fraction, with BB emission, is growing with energy there is naturally a sharp rise of the observed Pf in the channel band 50 – 70. This feature must hence be present in any model producing a rise of Pf with energy and can be seen, for example, in figures [5C](#page-15-0) and [6C](#page-16-0).

For a given field configuration, the observable Pf at a given energy increases when the overall stellar effective temperature decreases. We stated, wrongly, in Paper I that this is due to the increase of the temperature difference between the warm and cold regions when T_b decreases. However, a much stronger temperature difference increase is induced by the increase of the field strength and it has almost no effect on Pf as stated in $\S 4$. The actual reason for the increase of Pf , at a given energy, with decreasing T_e is the decrease of Pf with photon energy since lowering T_e pushes the Rayleigh-Jeans part of the spectrum (which has a low $P f$) out of the PSPC range. A simple check of this statement has been done by considering 'plate models' with a uniform surface temperature T_0 on which two plates are superposed at temperatures T_+ and $T_$, respectively higher and lower than T_0 : keeping the ratios $T_+ : T_0 : T_-$ constant but lowering T_0 does induce an increase in the observable Pf (at a given energy). If we consider the absolute Pf , i.e., as would be detected by a detector with absolute energy resolution, then the absolute- Pf vs. E curve (see for example figure [5](#page-15-0)C or figure 8 in Paper I) is simply shifted leftwards when T_0 is lowered, at constant $T_+ : T_0 : T_-$ ratios, and, since Pf increases with energy, at a given energy Pf increases when T_0 decreases.

9. SEPARATION OF THE SURFACE THER-MAL EMISSION FROM THE HIGH EN-ERGY TAIL

Our study so far, and in Paper I, has concentrated exclusively on the soft X-ray band assuming implicitly that this thermal component can be separated from the hard tail. We argue here that this separation is justified, and show it explicitly in the case of Geminga. A simple look at the spectral fits (e.g., figure 9A below, or Halpern & Ruderman 1993, for Geminga; Ogelman 1995a for PSRs $0656+14$ and 1055-42) shows that, at energies slightly below the crossover of the soft thermal component with the hard tail, the contribution of the latter becomes rapidly negligible compared to the former. Moreover, the spectral fit of the soft component is not changed significantly if the hard tail is fitted by BB emission or power law emission (e.g., Halpern & Ruderman 1993). One thus does not expect much interference between these two components.

The contribution of the hard tail to the soft band is larger if it is modeled as a power law rather than a BB. To consider the worst case of interference be-

tween the two spectral components we thus model Geminga's X-rays with a composite model including surface thermal emission and a pulsed power law tail, the results being shown in figure [6](#page-16-0). The same surface temperature model complemented with a hard tail as thermal emission from two polar caps has been presented in Page & Sarmiento (1996). The spectrum (figure [6](#page-16-0)A) shows that the hard tail emission is about 30 times weaker than the surface emission at channels below 40 but, due to its strong pulsations, its contribution to the pulse profiles can be about 10% in the soft band $7 - 28$ (figure [6](#page-16-0)B1). Nevertheless this is far from enough to alter the shape of the light curves in both bands $7 - 28$ and $28 - 53$ (figure [6](#page-16-0)B1 & B2). The important consequence of this is the effect on the energy dependence of the pulsed fraction shown in figure [6](#page-16-0)C: the general trend, typical of BB emission, consisting of an increase of Pf with energy (see § [8\)](#page-5-0) is slightly weakened by the addition of the hard tail but not reversed at channels below 100 (figure [6C](#page-16-0)). When modeled as polar cap thermal emission, the hard tail contribution to the soft band is almost completely negligible (Page & Sarmiento 1996). It is thus not possible, within the present framework, to make the hard tail responsible for the observed decrease of the pulsed fraction with energy below 0.5 keV. In conclusion, the observed decrease of Pf with energy, at channels below 53, observed in Geminga is most certainly an intrinsic property of the surface thermal emission which is absolutely irreproducible with BB emission.

The cases of PSR 0656+14 and 1055-52 are more delicate but, from a look at the spectral fits there is no doubt that the observed decrease of Pf at channels below 70 and 50 respectively $(Ogelman 1995a)$ is also a property of the surface thermal emission since the crossover with the hard tail occurs at channels about 100 for these two pulsars. However the shift in the peak phase (\ddot{O}) gelman 1995a) which occurs within the band dominated by the surface thermal emission is another feature totally unexplainable with BB emission. Thus, both the pulsed fraction variation and the peak phase shift must be attributed to the anisotropy induced by the magnetic field in the atmosphere, i.e., anisotropy of the emitted flux, superposed to a non uniform surface temperature distribution. In the case of Geminga the shift in the peak phase coincides with the shift from the surface thermal emission to the hard tail and is not surprising $(Ogelman 1995b)$.

10. SURFACE BOUNDARY CONDITION FOR NEUTRON STAR COOLING

A direct application of our surface temperature model and of the previous analysis concerns the modeling of the neutron star thermal evolution. In such models, the relationship $T_b - T_s$ between the temperature at the bottom of the envelope T_b and at the stellar surface T_s is needed as an outer boundary condition. Taking into account the surface temperature inhomogeneity we must rather speak of a T_b-T_e relationship, where T_e is the effective temperature of the whole surface, and, given different field structures, we can easily generate many such $T_b - T_e$ relationships. Cooling models including magnetic effects in the envelope have been presented by Van Riper (1991) and Haensel & Gnedin (1994) who used a radial field $T_b - T_e$ relationship: this is not a realistic configuration and we will show here that a more careful treatment actually leads to conclusions opposite to the one reached by these authors. For a radial field, T_e is increased compared to the non magnetic case, at a fixed T_b , as shown in figure 1 of Paper I: during the photon cooling era, i. e., neutron star age above $\sim 10^5$ yrs, this results in an increased surface photon emission, for a given internal temperature T_b , and accelerated cooling compared to the non magnetic case. For example, with a radial field of strength 10^{13} G, T_e is increased by about 25%, compared to the non magnetic case, and thus the photon luminosity is increased by a factor of 2.5.

The minimal inclusion of the magnetic field in the $T_b - T_e$ relationship should be done with a dipolar geometry. By integrating Eq. [9](#page-4-0) for a dipolar field one easily obtains

$$
T_e(T_b) = T_s(T_b, \Theta_B = 0) \cdot [1 - 0.47(1 - \chi_0^4)]^{1/4}
$$
 (13)

for the effective temperature T_e as a function of the maximum surface temperature $T_s(\Theta_B = 0)$ and χ_0 (Eq. [10](#page-4-0)): this reduces the magnetic field effects and brings the $T_b - T_e$ relationship close to the non magnetic one as is illustrated in figure 7. Since we netic one as is illustrated in figure [7.](#page-16-0) have shown that the surface field of the four pulsars which show surface thermal emission (and can be compared with cooling models) is not dipolar, we must then include the quadrupolar component in the $T_b - T_e$ relationship. A general idea of the effect can be immediately seen from figure [2](#page-15-0) which plots $T_e(\text{mag.})/T_e(\text{non mag.}).$ At $T_b = 10^8$ K, i. e., $T_e \sim$ 10^6 K, most configurations also induce a *decrease* of

 T_e compared to the non magnetic case. Moreover, $T_e(\text{mag.})/T_e(\text{non mag.})$ decreases for configurations with larger $P f$ and is systematically smaller than one for the cases which can produce the large observed pulsed fractions. We also show in figure [7](#page-16-0) a second $T_b - T_e$ relationship which corresponds to the most strongly pulsed configuration that we have found with a dipole+quadrupole field. The interesting result is that this relationship does not depend significantly on the field configuration: it is very similar to the dipolar case. We have explicitly verified that other field configurations which produce large observable pulsations give almost identical results, as can also be seen from figure [2](#page-15-0) (for $T_b = 10^8$ K). The discussion at the end of § [3.2](#page-3-0) anticipated this result and explained it beforehand. Configurations of figure [2](#page-15-0) which give low P f's can however increase T_e by 10% compared to the non magnetic case: the observed strong pulsations, if really due to large surface temperature inhomogeneities, totally rule out such field configurations and thus such boundary conditions. From this we conclude that the most likely overall effect of the magnetic field, in realistic configurations, is at most a slight decrease of the heat flow in the envelope as compared to the non magnetic case. Notice that the 10^{13} G case is practically identical to the non magnetic case, a result already obtained by Hernquist (1985). Since the strongly pulsed dipole+quadrupole field configurations give $T_b - T_e$ relationships almost identical to the purely dipolar case, one could use the dipolar relationship for magnetic cooling models. The dipolar strengths usually quoted are obtained from the standard magnetic dipolar radiation braking formula and thus only indicative of the actual surface field, and even if that formula were exact, GR effects can increase the surface field as shown in figure [1.](#page-15-0) We thus propose to take as a boundary condition for neutron star cooling models

$$
T_e = a \, g_{s14}^{1/4} \, T_{b8}^b,\tag{14}
$$

where $a = 0.85^{+0.05}_{-0.20}$ and $b \cong 0.55$. The value of $a = 0.85$ is an average of the result of Hernquist (1985) who obtained 0.83 and Gundmundsson, Pethick, & Epstein (1982) who obtained 0.87 and the uncertainties take into account the magnetic field effect: $+0.05$ corresponds to $B \sim 10^{13}$ G and -0.20 to $B \sim 10^{11}$ G. The exponent $b = 0.55$ is from figure [7](#page-16-0) and corresponds also to the value of Gundmundsson et al. (1982).

The cooling calculations of Page (1994), restricted

to the photon cooling era where the $T_b - T_e$ relationship is most important, where explicitly performed with the non magnetic case in anticipation of the present results and his conclusions about the necessity of extensive baryon pairing in the core of the Geminga neutron star are therefore still valid. For illustration, we show in figure [8](#page-16-0) several cooling curves with four boundary conditions from figure [7](#page-16-0) which show explicitly the smallness of the magnetic field effects.

When discussing pulse profiles the reliability of the envelope models is not very important as discussed in § [6](#page-4-0) since most of the effect simply comes from the $\cos^{1/2} \Theta_B$ geometrical factor in equation [11](#page-5-0). When considering the $T_b - T_e$ relationship the reliability of envelope models for $\Theta_B = 90^\circ$ is also not very important since the cold regions are so cold that they contribute very little to the overall effective temperature, [as shown explicitly by the $(1 - \chi_0^4)$ dependence of T_e for dipolar fields in eq. [13](#page-8-0)]. Envelope models for $\Theta_B = 0^{\circ}$ are what really determines the $T_b - T_e$ relationship and more reliable calculations at low temperature are needed for better interpretation of cool neutron star observations which will most certainly be detected with the future generations of X-ray satellites.

11. DISCUSSION AND CONCLUSIONS

11.1. Modeling pulsars' surface temperature distribution

We have completed our general study of thermal emission from magnetized neutron stars, within the framework of blackbody (BB) emission. Our original intent was to determine if the inhomogeneous surface temperature distribution induced by the anisotropy of heat flow in the neutron star envelope is strong enough to produce the pulsed fraction observed by ROSAT in Vela, Geminga, PSR 0656+14, and PSR 1055-52. We have shown explicitly in § [6](#page-4-0) that most of the effect is in the geometrical factor $\cos^{1/2} \Theta_B$ $(Eq. 11)$ $(Eq. 11)$, as presented originally by Greenstein & Hartke (1983), and fortunately the actual value of χ_0 (Eq. [10\)](#page-4-0) is of little importance. Moreover, models of magnetized neutron star envelopes allow us to relate the interior temperature T_b to the maximum surface temperature $T_s(\Theta_B = 0^{\circ})$ quite reliably as long as the latter is higher than about 3 \cdot 10⁵ K. The minimum surface temperature $T_s(\Theta_B = 90^\circ)$ is however poorly determined but its actual value, equivalent to χ_0 , is not really important. In short, the surface temperature distribution of a magnetized neutron star can be modeled well enough for present practical purposes.

Since we do not take into account the effect of the magnetic field on the emitted spectrum but only on the local surface temperature, our results are not sensitive to the actual strength of the field. As long as the magnetic field is stronger than a few times 10^{11} G its effects saturate and we are mostly dealing with warm plates whose size is determined by the field orientation Θ_B . When atmospheric effects are included one can expect a real dependence on the field strength due to the presence of absorption edge(s) and a field dependent anisotropy of the emission. As in the case of dipolar fields (Paper I), the composite BB spectrum produced by the temperature nonuniformity is close to a BB at the star's effective temperature with some excess in the Wien tail (figure [4](#page-15-0)). The quadrupole+dipole configurations which induce strong pulsations show an excess similar to the pure dipolar case since they have warm and cold regions of similar areas but many of the configurations which produce little pulsations give a composite BB spectrum closer to a single temperature BB spectrum since they have many warm regions and smaller cold regions.

11.2. The quadrupolar component

We showed in Paper I that dipolar surface fields do not allow to reproduce the amplitude of the observed pulse profiles; however, we have shown here that the inclusion of a quadrupole component is sufficient to raise the pulsed fraction up to or above the observed values. Gravitational lensing is crucial in this result, and the effect of the quadrupole, in the cases where it does increase the pulsed fraction, is basically to push the two magnetic poles, and the warm regions surrounding them, closer to each other: the areas of the warm and cold regions do not change substantially, compared to the purely dipolar case, but gravitational lensing is then less effective in keeping constantly a warm region in sight. This shift of the magnetic poles location is in fact immediately seen in the cases of Geminga and PSR 1055-52 which are considered to be orthogonal rotators but show only a single wide peak in the soft X-ray band while a dipolar field would produce two peaks (see also discussion in Halpern & Ruderman 1993). If we want to stick to a multipolar description, strong octupole components would make the matter worse, inducing still more warm region and flattening even more the light

curves. Thus, the simple observation of significant pulsations imply that the surface field is dominated by the dipole with a significant quadrupole component, but nevertheless weaker than the dipole, and that higher order components must be weaker. It has been proposed that the presence of the quadrupole is also felt in the anomalous dispersion measure of the high frequency vs. low frequency radio pulses which has been observed in several pulsars (Davies et al. 1984; Kuz'min 1992). Similar observations of PSR 0833- 45, $0656 + 14$, and $1055 - 52$, would allow a direct comparison with the strengths of the quadrupoles needed for the interpretation of the radio and X-ray emission of these pulsars. No confirmation of the presence of a strong quadrupole component could help constraining the size of these neutron stars through gravitational lensing. Another possibility is that the multipole expansion is inappropriate to describe the surface field (i. e., would require the inclusion of many strong high order components) as would be the case if the field has a sunspot like structure: plate tectonics (Ruderman 1991a, b, c) does predict such a structure which may be appropriate for cases like Geminga (Page et al. 1995).

Most dipole+quadrupole configurations produce little observable pulsations and it is surprising that all four observed pulsars have chosen an a priori highly improbable configuration which does increase $P f$. If we visualize the quadrupole as a pair of coplanar dipoles of equal strengths but opposite directions, then the dipole+quadrupoles configurations which are successful in increasing Pf are almost coplanar: there must be a good physical reason for this to happen in all four cases. Possible identification of a new young neutron star in a supernova remnant close to CTB 1 has been recently reported (Hailey & Craig 1996): this object appears to be similar in age and temperature to the Vela pulsar and also shows pulsations at a $10\% \pm 10\%$ level. If the detected Xrays are from the surface thermal emission and Pf is above 10% this would raise to five the number of neutron stars in which a quadrupole with adequate orientation is present. The theory of plate tectonics (Ruderman 1991a, b, c) predicts that the crust of young fast spinning pulsars should break under the stress produced by the internal superfluids and superconductors. Plate motion, towards the equator, will ensue and the mutual attraction of the two magnetic poles may slowly pull them toward each other (as is apparently seen in Geminga and PSR 1055-52) inducing a strong quadrupole component as we need in the present study.

11.3. Constraints on the neutron star size.

We found that, within our present study, the observed pulsed fractions of Geminga and PSR 0656+14 cannot be reproduced if these stars have a radius of 7 km for a mass of 1.4 M_{\odot} , or, more generally, if their radius is of the order of 1.7 \cdot R_S, R_S being the star's Schwarzschild radius. Moreover, a 10 km radius for a 1.4 M_{\odot} star, i.e., $R = 2.4 R_S$, can be 'rejected at the 99% confidence level' in the sense that less than 10 models out of 1000 generated for figure [2](#page-15-0) can reproduce the observed Pf for these same two neutron stars. Similarly, for Vela and PSR 1055-52, $R \sim 1.7 R_S$ can be also be 'rejected at the 99% confidence level' for the same reason but $R = 2.4 R_S$ is acceptable. Our results thus favor rather large radii, above 10 km for a 1.4 M_{\odot} mass, for all four neutron stars. These conclusions must be taken with extreme caution for several reasons: 1) the dipole + quadrupole description of the surface field may not be adequate, 2) anisotropic emission due to the magnetic field may increase significantly the observable pulsed fraction, 3) the emitted flux may be partially absorbed in the magnetosphere, 4) the surface temperature may be controlled by factors other than heat flow from the hot interior. Each of these factors may, however, increase or reduce the observable pulsed fraction. Thus, we consider our results as mostly illustrative but notice that it is encouraging that the observed pulsed fractions fall within the range where we could potentially rule out large classes of neutron star models through gravitational lensing of the surface thermal emission. We finally mention that gravitational lensing of the hot polar cap emission may also provide complementary estimates of the neutron star radius: Yancopoulos, Hamilton, & Helfand (1994) find $R = (2.4 \pm 0.7) R_S$, or 10 ± 3 km at 1.4 M_{\odot} , for PSR 1929+10 but the anisotropy of emission and the lack of information on the location of the second polar cap make this estimate also uncertain.

11.4. Limitations of blackbody emission

We confirmed our previous result that with BB emission the observable pulsed fraction Pf is unavoidably increasing with photon energy, at low energy where the surface thermal emission is detected, and explained that this is due to the hardening of the BB spectrum with increasing temperature. The three pulsars Geminga, 0656+14, and 1055-52 show a decrease of Pf with energy in the soft band, irreproducible with BB emission. The model of Page *et al.* (1995) produced such a decrease of Pf by superposing two different spectra, a non magnetic hydrogen atmosphere at low temperature on most of the stellar surface and a magnetized hydrogen atmosphere on two warm magnetized plates: the magnetic spectrum is much softer than the non magnetic one and Pf decreases strongly with energy. Extrapolating from this result we may speculate that the decrease of Pf with energy is due to a softening of the emitted spectrum with increasing temperature. We also showed in § [9](#page-7-0) that the separation of the soft thermal component and the hard tail is large enough to make the contribution of the latter in the soft band quite small: the observed decrease of Pf is an intrinsic feature of the surface thermal emission and not due to an interference of these two components as proposed by Halpern & Ruderman (1993) in the case of Geminga.

A second clear indication for the inadequacy of BB, or BB-like, emission is directly found in the spectral fits: for a reasonable neutron star size the required distance D must agree with other estimates as, e. g., the distance obtained from the pulsar's dispersion measure or from parallax measurements (Geminga). However, the distance D and stellar radius R^{∞} obtained from spectral fits have to be taken with caution due to the uncertainty in the calibration of the PSPC in the lower channels band; the fit in this band determines the column density N_H which strongly affects the estimate of the emitted flux and the PSPC calibration uncertainty translates into uncertainty on D and R^{∞} (Meyer, Pavlov, & Mészáros 1994). In the case of Vela, BB emission is disastrous since it implies a 3 - 4 km radius neutron star at 500 pc or, alternatively, a distance of ∼ 1500 pc for a standard size neutron star (Ogelman *et al.* 1993). The same is apparently happening in the case of the newly discovered neutron star RXJ 0002+6246 (Hailey & Craig 1996) for which BB spectral fits also require an effective blackbody radius $R^{\infty} \sim 2.5 - 4.5$ km for the estimated distance of 3 kpc. However, magnetized hydrogen atmosphere models do pass the distance test and can fit the Vela spectrum with the standard distance of 500 ± 125 pc (Page, Shibanov, & Zavlin 1996) and a reasonable hydrogen column density. In the case of Geminga, with a parallax measured distance of 157^{+59}_{-34} pc (Caraveo *et al.* 1995), available mag-

netized hydrogen atmosphere models do pretty bad since they need a distance of about 20 pc (Meyer *et al.*) 1994; Page et al. 1995) while BB is acceptable with a required distance between 150 and 400 pc (Halpern & Ruderman 1993). This low distance needed for magnetized hydrogen atmosphere spectral fits may, however, be partly due to the uncertainties in the PSPC calibration at low energies (Meyer et al. 1994). The crucial difference between Vela and Geminga is in the temperature: at 10^6 K (Vela) the hydrogen atmosphere is fully ionized and the models are reliable while in Geminga, at $T \sim 3 - 5 \cdot 10^5$ K, the atmosphere is only partially ionized and the available atmosphere models are not yet very accurate. Atomic thermal motion (Pavlov & Mészáros 1993) and its effects on the bound-bound (Pavlov & Potekhin 1995) and bound-free (Pavlov & Potekhin 1996) opacity will affect significantly the absorption edge present at T_e below 10^6 K, but inclusion of these effects into atmosphere models has not yet been done and the results are hard to predict. The field and temperature dependence of the absorption edge may naturally induce a decrease of Pf with increasing photon energy as observed (Pavlov 1995).

These limitations of BB emission may cast doubts on the validity of our results. As we stated in the introduction we consider that modeling NS thermal emission with BB emission is a mandatory first step and our study does allow us to delimitate more clearly the limitations of this approach. BB is certainly not adequate for reliable estimates of pulsars' surface temperature, as well as size and distance, from spectral fits. However, modeling of the pulse profiles is probably less dependent on the exact atmosphere characteristics since the anisotropy of the emission from the atmosphere is not very strong at low energy and it is unlikely to affect dramatically our conclusions on the effect of gravitational lensing and neutron star sizes. Our results obviously have to be taken only as a first step which hopefully can indicate the direction to be taken for future work(s).

11.5. Neutron star cooling

We have finally applied our model to assess the effect of the crustal magnetic field on the outer boundary condition used in neutron star cooling models. We have shown that the global effect of the magnetic field, compared to the non magnetic case, is very small and most probably consists in a slight reduction of the heat flux in the envelope which results in a lower effective temperature T_e for a given inner temperature T_b . This is in sharp distinction to the case where magnetic effects are naively applied under the assumption of a uniformly magnetized surface, implying an increase of T_e as compared to the non magnetic case.

11.6. Warning about the relevance of the model

Finally, we must repeat the warning expressed in Paper I about the relevance of our study: strong heating of the surface by magnetospheric hard Xrays and/or substantial magnetospheric absorption of the emitted flux, as proposed by Halpern & Ruderman (1993), could invalidate seriously our results (Page 1995b). The possible detection of pulsed γ rays from PSR 0656+14 (Ramanamurthy 1995) would mean that all four pulsars we study are copious γ ray emissors and heating of the neutron star surface by such energetic magnetospheres is not an unreasonable hypothesis. In the original Halpern & Ruderman (1993) model for Geminga the flow of electrons/positrons onto the polar caps produced a luminosity $L_p \sim 2.6 \cdot 10^{32}$ ergs s⁻¹, radiated away as hard X-rays which would then be back scattered onto the neutron star surface and reemitted as soft surface thermal X-rays. The X-ray luminosity of Geminga is about 2 · 10^{31} ergs s⁻¹ for the soft thermal component and $8 \cdot 10^{29}$ ergs s⁻¹ for the hard tail (for a distance of 160 pc as measured by Caraveo et al. 1995) so that even a small proportion of the hard polar X-rays hitting back the surface could explain the observed temperature. The problem of this model was that the *total* predicted X-ray luminosity is more than one order of magnitude higher than observed. The argument has been recently revised and the polar cap luminosity scaled down by Zhu & Ruderman (1996) who now obtain $L_p \sim 8 + 10^{30}$ ergs s⁻¹. This is much closer to what is observed and may be the factor controlling Geminga's surface temperature. However, the soft X-ray flux is about 30 times larger than the hard one so that in this model almost all the polar cap hard X-rays must be scattered back onto the stellar surface. On the other side, neutron star cooling models are also perfectly able to explain the observed temperature (Page 1994). We can only hope that future work(s) will elucidate this dilemma.

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TABLE 1 Spherical components of the five generating quadrupolar fields and location of the magnetic poles.

	b_0	b ₁	b ₂	b_3	\mathbf{D}_4
b_r b_{θ}	$\frac{3}{2}$ cos ² θ - 1 $rac{1}{2}$ sin 2θ	$\sin 2\theta \sin \phi$ $-\frac{2}{3}\cos 2\theta \sin \phi$	$-\sin 2\theta \cos \phi$ $\frac{2}{3}$ cos 2θ cos ϕ	$-\sin^2\theta \sin 2\phi$ $rac{1}{2}$ sin 2θ sin 2ϕ	$\sin^2 \theta \cos 2\phi$ $-\frac{1}{3}\sin 2\theta \cos 2\phi$
b_{ϕ} 'North' poles	$\theta = 0 \& \pi$	$-\frac{2}{2}\cos\theta\cos\phi$ $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ $\left(\frac{3\pi}{4},\frac{3\pi}{2}\right)$ $(\theta, \phi) = \langle$	$-\frac{2}{3}\cos\theta\sin\phi$ $\left(\frac{\pi}{4}, \pi\right)$ $\left(\frac{3\pi}{4}, 0\right)$ $(\theta, \phi) =$	$\frac{2}{3}$ sin θ cos 2ϕ $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$ $\left(\frac{\pi}{2}, \frac{7\pi}{4}\right)$ $(\theta, \phi) = \phi$	$rac{2}{3}$ sin θ sin 2ϕ $(\theta, \phi) = \begin{cases} (\frac{\pi}{2}, 0) \\ (\frac{\pi}{2}, \pi) \end{cases}$
'South' poles	$\theta = \frac{\pi}{2}$	$\left(\frac{\pi}{4}, \frac{3\pi}{2}\right)$ $\left(\frac{3\pi}{4}, \frac{\pi}{2}\right)$ $(\theta, \phi) =$	$\left(\frac{\pi}{4},0\right)$ $\left(\frac{3\pi}{4},\pi\right)$ $(\theta, \phi) =$	$\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$ $\left(\frac{\pi}{2}, \frac{5\pi}{4}\right)$ $(\theta,\phi)=\mathord{\scriptstyle\circ}$	$\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ $(\theta, \phi) = \langle$

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Fig. 1.— Gravitational correction factors for the radial (r) , f_l , and spherical (θ, ϕ) , g_l , parts of the dipole $(l = 1)$ and quadrupole $(l = 2)$ components of the magnetic field at the stellar surface as a function of the ratio of the star's radius R to its Schwarzschild radius R_S or of the actual star's radius at masses of 1.2, 1.4, and 1.6 M_{\odot} .

Fig. 2.— Statistics of observable pulsations with dipole+quadrupole magnetic fields. For each stellar radius and dipole-observer orientation $(\alpha-\zeta)$ with respect to the rotational axis 1000 surface magnetic field and the induced temperature distribution were generated The x axis plots the observable pulsed fraction in the PSPC channel band $7 - 50$ for the given surface temperature distribution and the y axis the ratio of the star's effective temperature for the given magnetic field configuration to the temperature it would have with zero field. The surface temperature are calculated from an interior temperature $T_b = 10^8$ K: all models have an effective temperature around 10^6 K. Lowering T_e does increase Pf : by about 15% at $T_e \sim 3 \cdot 10^5$ K and 30% at 10^5 K. See text, § [3.2](#page-3-0), for more detail.

Fig. 3.— Surface temperature distribution for the dipole + quadrupole magnetic field configuration of figure 2 which gives the highest observable pulsed fraction. The whole star's surface is shown in an area preserving mapping. With an internal temperature of 10^8 K, the effective temperature is $9.33 \cdot 10^5$ K while the maximum and minimum temperatures are, respectively, $1.2 \cdot 10^6$ K and $3.1 \cdot 10^5$ K (1.4 M_{\odot} star with a 10 km radius). Four sets of isotherms are plotted as indicated. Field components: dipolar component of strength 10^{12} G and orientation $\theta = 90^{\circ}$ and $\phi = 0^{\circ}$, and quadrupolar components of strengths Q_0 = 2.48 · 10¹⁰ G, Q_1 = 6.84 · 10¹⁰ G, Q_2 = 5.01 · 10¹⁰ G, Q_3 = 4.80 · 10¹¹ G, and $Q_4 = -6.24 \cdot 10^{10}$ G: the maximum field strength at the surface is 2.18 \cdot 10¹² G and the minimum $1.72 \cdot 10^{11}$ G. The two black dots show the approximate locations of the polar caps, obtained by integrating the last open magnetic field lines from the light cylinder down to the stellar surface while the two small circles at $\theta = 90^{\circ}$ and $\phi = 180^{\circ}$ and 360° show their positions without the quadrupolar component.

Fig. 4.— Effect of the inhomogeneous surface temperature on the spectrum. A: incident flux (i.e. with redshift and lensing) at five temperatures, with zero interstellar absorption. B: the same fluxes with interstellar absorption and passed through the PSPC' response matrix. The temperatures of 5, 7, 9 and 15 $\cdot 10^5$ K correspond to the estimated blackbody temperatures of Geminga, PSR 1055-52, PSR 0656+14 and Vela respectively. We used a 1.4 M_{\odot} , 10 km radius ($R^{\infty} = 13.06$ km) star, at a distance of 500 pc, with the same field configuration as in figure 3. Continuous curves: composite BB spectra, dashed curves: single temperature BB spectra at $T = T_e$.

Fig. 5.— A model producing a decreasing pulsed fraction with blackbody emission. (1.4 M_{\odot} star with a 12 km radius.)

A. Surface temperature distribution. Magnetic field: dipole orthogonal to the rotation axis (in direction $\theta = 90^{\circ}$ and $\phi = 90^{\circ}$) with its center shifted by a half radius in direction of the north magnetic pole. The warm region around the north pole reaches a temperature of $8.15 \cdot 10^5$ K but is much smaller than the warm region around the south pole whose maximum temperature is $7.25 \cdot 10^5$ K.

B. Light curves in three channel bands produced by the temperature distribution of A, as seen through the ROSAT's PSPC by an observer at 150 pc and $N_H = 10^{20}$ cm⁻². Star's rotation axis along $\theta = 0^{\circ}$ and observer at $\theta = 90^\circ$. The dotted lines show light curves where the temperature of the smaller north magnetic pole region has been blocked at $7.25 \cdot 10^5$ K, i. e., the same maximum temperature as the large region around the south magnetic pole: this clearly shows that this warmest region is responsible for the decrease of the Pf .

C. Observable (with the PSPC) and absolute (i. e., as would be seen with a perfect energy resolution detector) Pf . The dotted curves show the corresponding Pf when the temperature of the small warm region has been blocked at $7.25 \cdot 10^5$ K (as in B).

Fig. 6.— Composite model of Geminga's soft Xray emission: surface thermal emission (dashed lines) plus power law emission (dotted lines) and total emission (continuous lines). Surface temperature distribution induced by a dipole+quadrupole magnetic field: dipole of strength 10^{12} G perpendicular to the rotational axis and quadrupole with components Q_0 = -2.28 · 10¹¹ G, Q_1 = +4.95 · 10¹⁰ G, Q_2 = + 3.36 · 10¹⁰ G, Q_3 = + 7.08 · 10¹¹ G, Q_4 = $-4.53 \cdot 10^{11}$ G. The interior temperature is $4.04 \cdot 10^7$ K, giving an effective temperature at infinity $T_e^{\infty} = 4.30 \cdot 10^5$ K for a 1.4 M_{\odot} star with a radius of 12 km. The hard tail is a sum of two components, both power laws with indices 1.47, modulated with the rotational phase Φ by a cos Φ factor. The observer is located within the rotational equatorial plane at a distance of 185 pc and $N_H = 2.1 \cdot 10^{20} \text{ cm}^{-2}$. A. Observable count rates from the surface thermal emission, the power law emission, and the total emission compared with the ROSAT data (from Halpern & Ruderman 1993). B. Pulse profiles in three channel bands, compared with the ROSAT data (from Halpern & Ruderman 1993). C. Pulsed fraction for the two components and the total emission.

Fig. 7.— $T_b - T_e$ relationship: thick lines correspond to purely dipolar fields while the thin lines correspond to one of the maximally pulsed dipole+quadrupole configuration of figure [2.](#page-15-0) The indicated field strengths are the dipole component strength at the magnetic pole (without GR correction: so the actual field value is about 50 - 100% higher, see figure [1\)](#page-15-0).

Fig. 8.— Effect of the crustal magnetic field on the cooling of neutron stars. We show the 'standard' cooling of a 1.4 M_{\odot} neutron star built with the FP equation of state (Friedman & Pandharipande 1981) with and without core neutron ${}^{3}P_{2}$ pairing. The neutron core pairing critical temperature T_c is taken from Hoffberg *et al.* (1970): the high T_c of this calculation shows the maximum effect of pairing. Crust neutrons are always assumed to be paired in ${}^{1}S_{0}$ state with T_c taken from Ainsworth, Wambach & Pines (1989). Core neutron pairing has no effect at the early stage (age below ~ 20 yrs) but (i) slows down the cooling during the neutrino-cooling era (age up to $\sim 10^5 - 10^6$ yrs) by strongly suppressing the neutrino emission, and (ii) fastens the cooling later during the photon-cooling era by suppressing the star's specific heat by about 75% (see, e. g., Page 1994). The two sets of four curves show the effect of the crustal magnetic field on the cooling by its influence on the outer boundary condition. The four $T_b - T_e$ relationships are taken from figure 7. During the neutrino-cooling era the surface temperature follows the evolution of the core temperature: a $T_b - T_e$ relationship which gives a higher T_e , for a given T_b , results naturally in a higher stellar effective temperature. During the following photon-cooling era the effect is inverted since the dominating energy loss is from surface thermal emission with a luminosity $L_{\gamma} = 4\pi R^2 \sigma T_e^4 \propto T_b^{2.2}$ (see equation [14\)](#page-8-0) and a higher T_e for a given T_b results in a larger L_{γ} . Calculations were performed with the cooling code used by Page (1994)

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FIGURE 2

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FIGURE 4

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FIGURE 7

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FIGURE 8