

Non-minimal coupling of the scalar field and inflation

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Abstract

We study the prescriptions for the coupling constant of a scalar field to the Ricci curvature of spacetime in specific gravity and scalar field theories. The results are applied to the most popular inflationary scenarios of the universe; their theoretical consistency and certain observational constraints are discussed.

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1 Introduction

The concept of inflation has dominated the cosmology of the early universe for the last fifteen years. Despite the success of the inflationary paradigm in resolving the problems of the standard big-bang model and in providing a mechanism for the formation of structures in the universe, there is no universally accepted model for inflation: rather, many different inflationary scenarios have been proposed. Moreover, it has not been possible to unambiguously identify the inflaton with any known field from a particle physics theory. A comparison of the inflationary models with observations has been made possible in recent years by the discovery of anisotropies in the cosmic microwave background [1]. A difficulty that is often encountered in comparing theory and observations is that a specific inflationary scenario typically contains several free parameters, and a *ad hoc* choice of their values may render the scenario viable, sometimes at the price of fine-tuning the parameters or the initial conditions of the model (see e.g. Refs. [2, 3, 4]). In the present paper, we study the possible prescriptions for one of the parameters appearing in many inflationary scenarios, namely the coupling constant ξ of the inflaton with the Ricci curvature of spacetime. To fix the ideas, let us consider the Lagrangian density for Einstein gravity and a non-minimally coupled scalar field as the only form of matter:

$$\mathcal{L} = \left[\frac{R}{16\pi G} - \frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi - V(\phi) - \frac{m^2}{2} \phi^2 - \frac{\xi}{2} R \phi^2 \right] \sqrt{-g}, \quad (1.1)$$

where R denotes the Ricci curvature of spacetime, g is the determinant of the metric $g_{\mu\nu}$, ∇_μ is the covariant derivative operator, m and $V(\phi)$ are, respectively, the mass and the potential of the scalar field ϕ . ϕ obeys the Klein-Gordon equation

$$\square \phi - \xi R \phi - m^2 \phi - \frac{dV}{d\phi} = 0. \quad (1.2)$$

The term $-\xi R \phi^2/2$ in the Lagrangian density (1.1) describes the non-minimal coupling of the field ϕ to the curvature [5]. It is well-known [6, 2, 4] that the viability of inflationary models is deeply affected by the value of the parameter ξ . Although a popular choice is setting $\xi = 0$ (minimal coupling) in order to simplify the calculations, this prescription for ξ is often unacceptable. In quantum field theory in curved spacetimes it is argued that a non-minimal coupling is to be expected when the spacetime curvature is large. Non-minimal couplings are generated by quantum corrections even if they are not present in the classical action [7]. The coupling is actually required if the scalar field theory is to be renormalizable in a classical gravitational background [8, 9]. When

the problem of the correct value of ξ is not ignored, the prevailing point of view in the literature on inflation is that the coupling constant ξ is a free parameter, and that the values of ξ that are acceptable are those that, *a posteriori*, make a specific inflationary scenario viable. In this paper, we show that this point of view is unacceptable in many cases, and that often there exist definite prescriptions for the coupling constant. The value of ξ depends on the nature of the inflaton ϕ and on the theory of gravity under consideration. With the value of ξ known *a priori*, specific scenarios are analyzed and their theoretical consistency is discussed, before comparing their predictions with the available observations.

The plan of the paper is as follows: in Sec. 2 we illustrate the various prescriptions for the value of ξ in different theories, and we study their applicability to inflation. Emphasis is given to metric theories of gravity, in particular general relativity and theories formulated in the Einstein conformal frame. In Sec. 3 we examine the consequences of these prescriptions for the most popular inflationary scenarios proposed so far. Section 4 contains considerations on the effects of non-minimal coupling in power-law inflation and observational constraints on a specific model. In Sec. 5 we provide further constraints on chaotic and new inflation. Section 6 contains the conclusions.

2 Prescriptions for the coupling constant ξ

The coupling constant ξ is often regarded as a free parameter in inflationary scenarios. This view arises from the fact that there is no universal prescription for the value of ξ . Indeed, some prescriptions for ξ do exist in specific theories, although they are not widely known, and they depend on the nature of the scalar field ϕ and on the theory of gravity. In this section, we will review the prescriptions for the coupling constant, before applying them to cosmology in Sec. 3.

2.1 Quantum theories of the scalar field ϕ

The available prescriptions for the coupling constant ξ differ depending on whether the scalar is a fundamental field, or is associated with a composite particle. In Ref. [10] it was argued that, if ϕ is a Goldstone boson in a theory with a spontaneously broken global symmetry, then $\xi = 0$. It has been pointed out that if the scalar field ϕ is associated to a composite particle, the value of ξ should be fixed by the known dynamics of its constituents [11]. In particular, in Ref. [11], the Nambu–Jona–Lasinio model was analyzed and, in the large N approximation, the value $\xi = 1/6$ was found for this specific

model. Reuter [12] considered the $O(N)$ -symmetric model with a quartic self interaction, in which the constituents of the ϕ boson are scalars themselves. The resulting ξ depends on the coupling constants of the elementary scalars [12]. Other arguments restrict the range of allowed values of ξ ; Hosotani [13] examined the back reaction of gravity on the stability of the scalar field ϕ assuming the Lagrangian of Einstein gravity with a general coupling $\xi R\phi^2/2$ and a potential

$$V(\phi) = V_0 + \frac{m^2}{2} \phi^2 + \frac{\eta}{3!} \phi^3 + \frac{\lambda}{4!} \phi^4 . \quad (2.1)$$

He found that, for cubic self-interactions, $\xi = 0$ is the only value allowed. For Higgs scalar fields in the standard model [14], it must be $\xi \leq 0$ or $\xi \geq 1/6$. However, the results of Ref. [13] are based on the use of canonical gravity and the conclusions may change if an alternative theory is adopted for the background gravity.

To our knowledge, no other prescriptions for the coupling constant ξ are available from quantum theories of the field ϕ . It is likely that every theory which provides a candidate for the inflaton will provide a specific value, or range of values, for ξ . To make things worse, in a quantum theory ξ is subject to renormalization, like masses or other coupling constants [12, 15]. It appears, therefore, that the prospects for an unambiguous determination of ξ are not promising. However, this would be a pessimistic conclusion, because inflation is essentially a *classical*, low energy, phenomenon. It has been argued that “the tensor contribution to the cosmic microwave background quadrupole implies that the vacuum energy that drives inflation is not a quantum-gravitational phenomenon” [16]. To be more specific, the potential energy density of the scalar field 50 e-folds before the end of inflation is subject to the constraint $V_{50} \leq 6 \cdot 10^{-11} m_{pl}^4$ [16]. Hence, gravity is classical during inflation. In many scenarios, the inflaton ϕ is a *gravitational* field, (e.g. the field of Brans–Dicke theory), and hence it is classical. What if the inflaton is non-gravitational in origin? The problem whether a classical treatment of the inflaton is appropriated has been studied in a number of papers ([17, 18, 19] and references therein). Under certain conditions, the distribution of the field is peaked around classical trajectories and the evolution of the scalar field can be considered as classical. This justifies the use of *classical* equations to describe inflation, and it is not inconsistent with the fact that quantum fluctuations of ϕ around its classical value provide seeds for density perturbations [18, 19]. Therefore, the problem of the determination of the correct value of the coupling constant ξ may be restricted to the consideration of classical theories of gravity *and* of the inflaton ϕ .

2.2 Classical theories of ϕ and metric theories of gravity

According to the previous discussion, we will assume that gravity is described by a classical theory based on a spacetime manifold, and that the inflaton field ϕ is classical. There exists a prescription for the coupling constant ξ of a scalar field with the Ricci curvature, and for the coupling constants with other curvature scalars which can, in principle, be considered. The generalization of the flat space Klein–Gordon equation to a curved spacetime includes couplings with the Ricci curvature, as well as couplings with the other scalars constructed from the curvature tensor:

$$\square\phi - m^2\phi - \left(\xi R + \alpha_1 R^2 + \alpha_2 R^{\alpha\beta} R_{\alpha\beta} + \alpha_3 R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} + \dots\right)\phi - \frac{dV}{d\phi} = 0. \quad (2.2)$$

In Ref. [20] it was proved that, under the assumptions

- *i*) the scalar field ϕ satisfies Eq. (2.2);
- *ii*) the field ϕ satisfies the Einstein equivalence principle (hereafter EEP—see Ref. [22] for a formulation), i.e. the propagation of ϕ resembles locally the propagation in flat space;

the coupling constants are forced to assume the values

$$\xi = 1/6 \quad , \quad \alpha_1 = \alpha_2 = \alpha_3 = \dots = 0. \quad (2.3)$$

This result arises from the study of wave propagation and tails of radiation in a curved spacetime, and was derived by requiring that the structure of tails of radiation become closer and closer to that occurring in flat spacetime when the curved manifold is progressively approximated by its tangent space (i.e. by imposing the EEP on the field ϕ). Although the requirement of Eq. (2.3) reproduces the usual case of conformal coupling in four spacetime dimensions, the derivation of this result is completely independent of conformal transformations, the conformal structure of spacetime, the particular spacetime metric and the field equations for the metric tensor in the particular theory [20]. The conclusions of Ref. [20] were confirmed in Ref. [21].

If the assumption *i*) is satisfied but *ii*) is not there is, in principle, the disturbing possibility that massive scalar particles propagate on the light cone in a space in which the Ricci curvature is different from zero [20].

A question arises naturally: can we impose the EEP on the scalar field (inflaton) ϕ in a particular theory of gravity (inflationary scenario) ? The answer depends on the gravitational theory under consideration. If the nature of the field ϕ is gravitational (e.g.

the scalar field of Brans–Dicke theory), the statement that its physics resembles locally the physics in flat spacetime goes beyond the EEP, which regards only *non-gravitational* physics [22]. Metric theories of gravity [22] (including general relativity—hereafter GR) satisfy the EEP. Therefore, if the correct theory of gravity during inflation is GR, or any metric theory in which the inflaton field ϕ is non-gravitational, then the EEP holds and the coupling constant assumes the value $\xi = 1/6$. GR is widely used in the construction of inflationary scenarios, but it is not the only theory used for this purpose. Almost all the existing scenarios of inflation employ a metric theory of gravity: however, the inflaton field ϕ can have gravitational origin, like in scalar–tensor theories (of which Brans–Dicke theory is the simplest example). The prescription $\xi = 1/6$ clearly does not apply to the latter case for the above mentioned reason [23], and also for a second reason: the field ϕ in these theories satisfies an equation more complicated than Eq. (2.2) [22]. However, if the field ϕ satisfies an equation of the kind (2.2) and is massive ($m \neq 0$ is assumed in many inflationary scenarios), any value of ξ different from $1/6$ leaves the possibility that the massive ϕ propagates along the light cones when $R \neq 0$ [20]. This argument supports the choice $\xi = 1/6$ for *any* massive field satisfying Eq. (2.2). However, in the following, we will regard the prescription (2.3) as valid only for GR and all the metric theories of gravity in which the inflaton field ϕ is non-gravitational.

2.3 Theories formulated in the Einstein frame

A wide class of theories of gravity can be grouped into this category. They have the common feature that the final formulation of the theory is made in the “Einstein frame” conformally related to the “Jordan frame”, in which the theory was formulated at the start (for definitions and terminology, we refer the reader to [24] and references therein). This class of theories includes Kaluza–Klein, R^2 , supergravity and string–inspired theories, and many generalized scalar–tensor theories. The conformal transformation to the Einstein frame has also been used as a mathematical technique to transform a non-minimally coupled scalar field to the (computationally much easier) case of a minimally coupled field. In the literature, there is plenty of ambiguity on which conformal frame should be regarded as physical. For some theories, it has been proved that the “original” formulation in the Jordan frame is physically unacceptable because the kinetic energy of the scalar field is negative–definite, and a unique conformal transformation to the Einstein frame is singled out. In these cases, the formulation in the Einstein frame is the only acceptable possibility. The necessity (and uniqueness) of the conformal transformation has been established for Brans–Dicke [25] and Kaluza–Klein theories [26, 27], and has been generalized to a wider class of theories [24]. The theory in the Einstein

frame is, in general, very different from the Jordan frame formulation. The conformal transformation to the Einstein frame (and the associated redefinition of the scalar field – see [27, 24]) has the consequence that the “new” scalar field in the Einstein frame is minimally coupled to the curvature, $\xi = 0$, irrespective of the value of the coupling constant in the Jordan frame. This prediction applies to all theories formulated in the Einstein frame, which have been used extensively to construct inflationary cosmologies.

We remark that, according to Ref. [20], the minimally coupled scalar field of a theory formulated in the Einstein frame violates the EEP. Therefore, strictly speaking, the theory is not Einstein gravity, in which the EEP (and also the strong equivalence principle) are satisfied. This fact conflicts with the current use of the term “Einstein gravity” in many papers. The violation of the EEP in a theory formulated in the Einstein frame is not surprising, since also the weak equivalence principle is violated in these theories. In fact, if a form of matter (let us say a field ψ , to fix the ideas) other than the inflaton is included in the Jordan frame Lagrangian, then the stress–energy tensor of ψ in the Einstein frame is non–minimally coupled to the inflaton. This causes the presence of a fifth force violating the equivalence principle [27] and a time dependence of the coupling constants of physics, which is actually regarded as an important low–energy manifestation of string (and other) theories. When the weak equivalence principle is violated by the conformally transformed field ψ , the violation of the EEP (a stronger version of the equivalence principle) by the inflaton does not appear to be surprising. It is to be noted that if the scalar field decays and disappears from the universe during the radiation era, or at an early time during the matter–dominated era [28] (or even earlier [29]), the violation of the equivalence principle leaves no trace in present day experiments performed in the Solar System.

3 Consequences of the prescriptions of ξ for inflation

In this section, we apply the predictions of Sec. 2 to cosmology. We examine the inflationary scenarios most studied in the literature and we answer the two following questions: 1) is any of the prescriptions examined in Sec. 2 for the value of ξ applicable ? and 2) if the answer to question 1) is affirmative, what are the consequences for the specific inflationary scenario ?

It is well–known that the viability of a particular inflationary model can depend strongly on the value of the coupling parameter ξ . The following arguments have been used to argue against or in favour of specific scenarios:

- the existence of inflationary solutions;

- the amount of inflation necessary to solve the problems of the standard big-bang model;
- the fine-tuning of initial conditions for the inflaton.

These conditions regard the unperturbed model of the universe. A fourth argument to be taken into account is the evolution of density perturbations generated during inflation.

Many results on the viability of inflationary scenarios with a non-minimally coupled scalar field are already available in the literature. In these papers, the choice of the value of ξ was motivated *a posteriori* by the viability of the inflationary scenario, according to the prevailing point of view that sees ξ as a free parameter. Our point of view is radically different from previous works: While ξ was a free parameter for the previous authors, we have the prescription $\xi = 1/6$ for the metric theories of gravity in which the inflaton is non-gravitational. We review the results available in the literature from our new point of view. The scenarios analyzed in the following are the most well-studied, but do not constitute a complete list of the models proposed in the literature.

3.1 New inflation

The new inflationary scenario [30, 31] currently is not regarded as a successful one because of the extreme fine-tuning of parameters in the effective potential required to reproduce the observable universe [32, 33, 34]. However, the study of new inflation provides insight in the way non-minimal coupling affects a slow-roll inflationary scenario. The background gravity for new inflation is assumed to be GR and the (unperturbed) inflaton field ϕ is treated as classical, and is non-gravitational. The prescription $\xi = 1/6$ applies. Abbott [6] considered this scenario with the Coleman-Weinberg potential

$$V(\phi) = B\phi^4 \left[\ln \left(\frac{\phi^2}{\sigma^2} \right) - \frac{1}{2} \right] + \frac{B\sigma^4}{2}, \quad (3.1)$$

where B is constant and $\sigma = 10^{15}$ GeV, and realized that, if $\xi > 0$, the term $\xi R\phi^2/2$ in the Lagrangian density acts like an extra term in the scalar field potential, and creates a barrier that prevents the GUT phase transition from being completed. During the slow-roll of the inflaton on the flat section of the potential, the universe behaves very much like de Sitter space ($R = \text{constant}$), and the term $\xi R\phi^2/2$ behaves like a mass term for the scalar field [35], destroying the flatness of the potential. This happens, in particular, for $\xi = 1/6$. It is to be concluded that this version of the new inflationary scenario is not theoretically consistent, regardless of the fine-tuning problems.

Flat potentials different from (3.1) can also achieve new inflation. For example, the potentials

$$V(\phi) = V_0 - \alpha\phi^2 - \beta\phi^3 + \lambda\phi^4, \quad (3.2)$$

$$W(\phi) = W_0 - \alpha\phi^4 + \beta\phi^6, \quad (3.3)$$

(where V_0 , W_0 , α , β and λ are constants) and have been employed [37, 16]. The effect of non-minimal coupling appears to be the same in this potential as in the Coleman–Weinberg potential. In general, *the argument of Ref. [6] applies to all scenarios with a slow-rollover inflationary potential: the flatness of the potential is destroyed by the non-minimal coupling of the inflaton.* What about non flat potentials (used, e.g., in chaotic or power law inflation) ? In principle there is the possibility that the term $\xi R\phi^2/2$ in the Lagrangian density balances a suitable potential $V(\phi)$ in such a way that a section of the resulting “effective potential” is almost flat, thus giving again slow-roll of the inflaton field. The energy density and pressure of a non-minimally coupled scalar field are given by

$$\rho = \left(1 - 8\pi G\xi\phi^2\right)^{-1} \left[\frac{(\dot{\phi})^2}{2} + V(\phi) + 6\xi H\phi\dot{\phi} \right], \quad (3.4)$$

$$P = \left(1 - 8\pi G\xi\phi^2\right)^{-1} \left[\left(\frac{1}{2} - 2\xi\right)\dot{\phi}^2 - V(\phi) - 2\xi\phi\ddot{\phi} - 4\xi H\phi\dot{\phi} \right]. \quad (3.5)$$

It is possible to consider a suitable potential such that the equation of state approaches $P = -\rho$ and thus achieve inflation in the presence of a non-minimal coupling. A concrete example was given in Ref. [38] in the context of GR with conformal coupling ($\xi = 1/6$), by assuming the equation of state $P = (\gamma - 1)\rho$ and deriving numerically (for small values of the constant γ) the necessary potential. This potential is very different from the corresponding potential derived analytically for $\xi = 0$ for the same values of γ in Ref. [39]. Unfortunately, when the “effective potential” $V + \xi R\phi^2/2$ has a flat section on which the inflaton rolls slowly, the complication of the Friedmann and the Klein–Gordon equations prevents us from developing an elegant slow-roll formalism in terms of slow-roll parameters like the one available for a minimally coupled scalar field [37]. The introduction of an effective potential mimicking the effects of non-minimal coupling does not appear to be possible, as will be shown in Sec. 4.

3.2 Power-law inflation

For a minimally coupled scalar field, power-law inflation [40, 41, 42, 43] arises from an exponential potential

$$V(\phi) = V_0 \exp\left(\pm\sqrt{\frac{16\pi}{p}} \frac{\phi}{m_{pl}}\right), \quad (3.6)$$

where $p > 1$ is constant and the scale factor has the time dependence

$$a(t) = a_0 t^p. \quad (3.7)$$

It was recognized in Ref. [41] that the potential (3.6) is motivated in the context of Kaluza–Klein cosmologies. Actually, exponential potentials arise in string theories, supergravity and, in general, in any theory which is obtained by means of a conformal transformation to the Einstein frame. As discussed in Sec. 2.3, the prescription $\xi = 0$ is the only possibility in this case. The expression “power-law inflation” generically denotes a scenario in which the scale factor has the time-dependence (3.7), rather than a realization of inflation in a particular theory of particle physics. Most of the times, the particular theory in which power-law inflation is considered is not specified in the literature. We conclude that the power-law inflationary scenarios based on a theory formulated in the Einstein frame are theoretically consistent only if $\xi = 0$. Examples are given by the class of models [44] (representative of Kaluza–Klein cosmologies and other theories) of Ref. [46], and by extended inflation reformulated in the Einstein frame [47].

3.3 R^2 inflation

Higher derivative theories of gravity have the peculiar feature that inflation is generated by the R^2 term in the Lagrangian density for gravity, and a scalar field is not needed [48, 49], thus bypassing the problem of the value of ξ . However, a scalar field is sometimes included in the scenario to “help” inflation (see e.g. Ref. [50]). Since gravity is not Einstein gravity, a prescription for the coupling constant ξ is not available. To give an idea, we consider the proposed form of the Lagrangian density:

$$L = \frac{m_{pl}^2}{16\pi} \left(R + \frac{R^2}{6M^2} \right) + L_{non-gravitational}; \quad (3.8)$$

the justification for this Lagrangian density comes from supergravity [51]. There is no point in imposing the EEP in the context of supergravity: in fact it is known that already

the weak equivalence principle is violated at least in $N = 2$ and $N = 8$ supergravity [52] even in the low energy, weak field limit, with consequences testable by current experiments (which are actually used to constrain these theories [53]). In any case, it appears that both isotropic and anisotropic cosmologies have inflationary solutions as attractors, irrespective of the value of ξ [54].

By means of a conformal transformation to the Einstein frame, R^2 inflation can be recast as “standard” gravity with a minimally coupled scalar field [55]. This version of the theory is theoretically consistent.

3.4 Extended and hyperextended inflation

Extended inflation in its original formulation [56] made use of Brans–Dicke theory; the original scenario was soon abandoned due to the “big bubble problem”. Extended inflation can be recast as power–law inflation after a conformal transformation to the Einstein frame, with p in Eq. (3.7) given by $p = \frac{\omega}{2} + \frac{3}{4}$ (where ω is the Brans–Dicke parameter) [47]. In this formulation, the scenario is theoretically consistent.

A version of extended inflation in which the inflaton χ is different from the Brans–Dicke field ϕ and is coupled non–minimally to the spacetime curvature ($\xi_\chi \neq 0$) has been proposed [57]. The field χ is non–gravitational and Brans–Dicke theory is a metric theory of gravity, hence the prescription $\xi_\chi = 1/6$ applies. However, there are two other parameters (χ_0, ω), which make difficult to draw conclusions on the viability of this scenario, and a conformal transformation to the Einstein frame may be necessary [24].

Hyperextended inflation [58, 59, 60, 61] is based on scalar–tensor theories that generalize Brans–Dicke theory. The inflaton is a gravitational scalar field which is not subject to the prescription $\xi = 1/6$. ϕ is directly coupled to the Ricci curvature via a term $Rf(\phi)$, where $f(\phi)$ is an arbitrary function of ϕ , and the equation satisfied by ϕ is different from Eq. (1.2).

3.5 Induced gravity inflation

Induced gravity inflation [62] is also based on a scalar–tensor theory and the inflaton has gravitational origin. A non–minimal coupling $\xi \neq 0$ has been used [63] in conjunction with the Coleman–Weinberg potential (3.1). Chaotic inflation has been achieved in the context of induced gravity [64]. No prescription for ξ is available in these cases. Induced gravity inflation has also been reformulated in the Einstein frame [65]; this scenario with $\xi = 0$ is theoretically consistent, as explained in Sec. 2.3.

3.6 Chaotic inflation

Several results on chaotic inflation with a non-minimal coupling are available in the literature. The chaotic inflationary scenario originally introduced by Linde [66] employs GR and the Einstein equations are generally used in papers on the subject (e.g. [67]).

Futamase and Maeda [2] considered this scenario with a massive or massless scalar field, with the potential

$$V(\phi) = \lambda\phi^4 \tag{3.9}$$

(the same potential originally introduced by Linde [66]) and a non-minimal coupling of the inflaton, $\xi \neq 0$. They found that if $\xi \geq 10^{-3}$, chaotic inflation requires fine-tuning in the initial conditions for the scalar field. They concluded that “the chaotic scenario in the non-minimally coupled model does not work unless the coupling constant ξ is negative or sufficiently small ($\xi \leq 10^{-3}$) ... Thus, in order to know whether the chaotic inflationary scenario does really work, one has to investigate first whether the inflaton couples minimally or nonminimally with the spacetime curvature. If it turns out that the inflaton couples nonminimally classically or possibly through quantum corrections, one has to investigate how strong the coupling is” [2]. It is clear from our discussion of Sec. 2.2 that the scenario considered by Futamase and Maeda is theoretically consistent only if $\xi = 1/6 \gg 10^{-3}$ and hence fine-tuning in the initial conditions for the scalar field cannot be avoided. For the particular value $\xi = 1/6$, Futamase and Maeda gave an additional proof that chaotic inflation cannot be realized [2]. The non-existence of inflationary solutions for the potential (3.9) with a conformally coupled scalar field and Einstein gravity was also pointed out in Ref. [68].

Chaotic inflation with the potential (3.9) for a non-minimally coupled scalar field was considered in Ref. [3]. The purpose of that paper was to reduce the fine-tuning of the parameter λ in the potential imposed by observations of the cosmic microwave background: $\lambda \leq 10^{-12}$. A non-minimal coupling of the inflaton achieves this goal, but the price to be paid is a fine-tuning in the value of the coupling constant: it has to be $|\xi| \simeq 10^4$ [3]. However, the prescription $\xi = 1/6$ to be applied to the model rules out this possibility.

Chaotic inflation with the potential

$$V(\phi) = \mu^2 \left(\frac{\phi^2}{2} + \frac{\lambda}{2n} \phi^{2n} \right) \tag{3.10}$$

($\mu^2, \lambda > 0$) and $\xi \neq 0$ was studied in Ref. [4] with a dynamical systems approach. Consistently with Ref. [2], the authors found that, for $\xi = 1/6$, no trajectory in the phase space exists which corresponds to inflation [4].

Chaotic inflation with the Ginzburg–Landau potential

$$V(\phi) = \frac{\lambda}{8} (\phi^2 - v^2)^2 \quad (3.11)$$

and the Einstein Lagrangian for the pure gravity part of the action was considered in Ref. [69]. In the case $\xi = 1/6$, the authors of Ref. [69] deduced that there are no inflationary solutions. However, their analysis was performed in the regime $\phi^2 \gg v^2$, in which the potential reduces to the case of the quartic self–interaction (3.9).

Chaotic inflation can be achieved in the context of induced gravity [64]. No prescription for ξ can be given in this case.

3.7 Natural inflation

In the natural inflationary scenario [70], the inflaton is a massless pseudo Nambu–Goldstone boson with the potential

$$V(\phi) = \Lambda^4 \left[1 + \cos \left(\frac{\phi}{f} \right) \right] , \quad (3.12)$$

which exhibits two energy scales: $f \sim m_{pl}$ and $\Lambda \sim 10^{-5} f$ is the scale of spontaneous symmetry breaking. This scenario is motivated by superstring theories [16], and therefore there seems to be little indication on what prescription for ξ is correct, apart from the fact that GR is used in this scenario. This would imply that inflation occurs in the low energy limit, in which the prescription $\xi = 1/6$ applies. Since the analysis of the potential is difficult, two regimes are considered [16]: *i*) $f \leq m_{pl}$, $V \simeq 2\Lambda^4$, which is extremely fine–tuned [37]; *ii*) $f \gg m_{pl}$, $V(\psi) = m^2 \psi^2 / 2$ (where $\psi = \phi - \sigma$, $\sigma = \text{constant}$), which is equivalent to the chaotic inflationary scenario already considered.

3.8 Double field inflation

Inflation with two (or more) scalar field has been considered [71, 72, 73, 74]; in Ref. [72] the potential is

$$V(\phi, \psi) = \frac{\lambda}{4} (\psi^2 - M^2)^2 + \frac{m^2}{2} \phi^2 + \frac{\lambda'}{2} \phi^2 \psi^2 . \quad (3.13)$$

A particular realization for ψ is a Peccei–Quinn field. Like in many other papers, the theory considered is *classical*, but it is supposed to simulate the full quantum theory of the inflaton(s) by choosing a suitable potential. Such a hybrid theory may not be

inconsistent for some values of the coupling constant(s) of the scalar field(s) with the Ricci curvature. For example, in Ref. [74], Brans–Dicke theory is used, and the inflaton is an extra (other than the Brans–Dicke) scalar field non–minimally coupled to the curvature. The coupling constant is $\xi < 0$, $|\xi| \ll 1$. This scenario is inconsistent: in fact Brans–Dicke theory is a metric theory of gravity and any scalar field other than the Brans–Dicke field is non–gravitational: therefore the EEP and the prescription $\xi = 1/6$ apply to the inflaton in the scenario of Ref. [74].

In the scenario of Ref. [75] the Lagrangian for Einstein gravity and two minimally coupled scalar fields are used. The theory is supposed to be “a toy model for the scalar field sector of the string–derived supergravity theory” [75]; in supergravity there is no point in imposing the EEP (in fact the weak equivalence principle is already violated in at least some realizations of the theory [52]), which would guarantee the conformal coupling. We can only say that, in GR, the minimal coupling for the two scalar fields of Ref. [75] is not acceptable, since they must be conformally coupled according to Ref. [20]. The same conclusion applies to the soft inflation of Ref. [76].

3.9 Anisotropic cosmologies

The occurrence of inflation has been studied also in anisotropic spaces for a non–minimally coupled inflaton field. Starobinski [77] showed that, for $\xi = 1/6$ (and therefore in GR), the anisotropic shear diverges as the inflaton ϕ approaches the critical value $\phi_c = (3/4\pi)^{1/2} m_{pl}$. This result was recovered in Ref. [78], in which it was also shown that the divergence of the anisotropic shear also occurs if $\xi > 0$ and for almost all initial conditions $\phi_0 > \phi_c$ (which do not reproduce the present universe). In general, the addition of anisotropy rules out the possibility of chaotic inflation for $\xi > 10^{-2}$ [78].

4 Power–law inflation with the potential $V(\phi) = \lambda\phi^n$

In Ref. [2], the case of a potential

$$V(\phi) = \lambda\phi^n \quad , \quad n > 6 \tag{4.1}$$

and $\xi \neq 0$ was considered: power–law inflation (3.7) was obtained, with

$$p = 2 \frac{1 + (n - 10)\xi}{(n - 4)(n - 6)|\xi|} . \tag{4.2}$$

By substituting Eq. (3.7) in Eq. (1.2), one obtains

$$\ddot{\phi} + \frac{3p}{t}\dot{\phi} + \frac{dV}{d\phi} + \frac{6\xi p(2p-1)}{t^2}\phi = 0, \quad (4.3)$$

which has the solution

$$\phi = \bar{\phi} t^\alpha \quad (4.4)$$

where

$$\alpha = \frac{2}{2-n} \quad (4.5)$$

($\alpha < 0$). It is to be noted that, in this particular case, it is possible to write

$$\frac{\xi}{2}R\phi^2 = \beta V(\phi), \quad (4.6)$$

where

$$\beta = 3\xi p(2p-1)\lambda^{-1}\bar{\phi}^{2-n}. \quad (4.7)$$

Usually, power-law inflation is associated to an exponential potential for a minimally coupled field. The possibility of obtaining power-law inflation with a power-law potential is due to the non-minimal coupling of the inflaton to the Ricci curvature, and shows the effect of non-minimal coupling on the physics of the scalar field and the dynamics of the universe. From Eq. (4.6), it may appear that one could substitute the physical system under consideration with an “equivalent” Friedmann universe dominated by a minimally coupled scalar field with the effective potential

$$V_{eff}(\phi) = V(\phi) + \frac{\xi}{2}R\phi^2 = \Lambda\phi^n, \quad (4.8)$$

where $\Lambda = 1 + \beta$. However, Eqs. (3.7), (4.2), (4.4) and (4.5) do not constitute a solution of the coupled Friedmann–Klein–Gordon equations for $\xi = 0$. To see this, it is sufficient to consider the Friedmann equations for the “equivalent universe”

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}\rho, \quad (4.9)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3P); \quad (4.10)$$

these, together with Eqs. (3.4), (3.5) for $\xi = 0$ and (4.5), (4.8) give $p \propto t^{4/(2-n)}$, which contradicts the constancy of p . Therefore, the introduction of the effective potential

(4.8) is not useful even when Eq. (4.6) is satisfied. The conformal technique used in Ref. [79] appears more promising.

In order to achieve inflation, it must be $p > 1$. In the space of parameters (n, ξ) , the inequality $p > 1$ is satisfied only in the regions:

$$n > 6 \quad , \quad 0 < \xi < \frac{2}{n^2 - 12n + 44} \quad , \quad (4.11)$$

$$6 < n < 4 + 2\sqrt{3} \simeq 7.464 \quad , \quad \xi < 0 \quad , \quad (4.12)$$

$$n = 4 + 2\sqrt{3} \quad , \quad \xi < \frac{1}{4(3 - \sqrt{3})} \simeq 0.197 \quad , \quad (4.13)$$

$$n > 4 + 2\sqrt{3} \quad , \quad \frac{-2}{n^2 - 8n + 4} < \xi < 0 \quad . \quad (4.14)$$

The range of values $6 \leq n \leq 10$ is interesting for superstring theories [80]; only a very narrow range of values of ξ is allowed for high n . However, it must be kept in mind that fine-tuning arguments rule out the scenario for $\xi > 0$ [2].

5 Observational constraints on the coupling parameter ξ

As discussed in the previous section, many inflationary scenarios are not viable for certain ranges of values of ξ . Other scenarios are viable, with the parameter ξ spanning a range of values, which can be constrained by the available observations of cosmic microwave background anisotropies.

Kaiser [79] considered chaotic inflation with the potential $V(\phi) = \lambda\phi^4$ and a non-minimally coupled scalar field, and computed the spectral index of density perturbations as a function of ξ [81]:

$$n_s = 1 - \frac{32\xi}{1 + 16\alpha\xi} \quad , \quad (5.1)$$

where α is the number of e-folds of the scale factor before the end of inflation. In the following, we will use the value $\alpha = 60$ adopted in Ref. [79]. This model is different from the one given by Eqs. (3.7) and (4.2). The statistical analysis of data from the *COBE* experiment detecting anisotropies in the cosmic microwave background gives $n_s = 1.1 \pm 0.5$ [1], and the combined statistical analysis of the *COBE* and Tenerife observations yields the 1σ limit $n \geq 0.9$ [82]. We adopt the limits

$$0.9 \leq n_s \leq 1.6 \quad (5.2)$$

which, using Eq. (5.1) yields the constraints on ξ :

$$\xi \leq -1.56 \cdot 10^{-3} \quad , \quad \xi \geq -9.87 \cdot 10^{-4} . \quad (5.3)$$

The GR prediction $\xi = 1/6$ implies $n_s = 0.967$. However, values of ξ greater than $\sim 10^{-3}$ lead to fine-tuning problems [2, 68, 3, 4], as explained in Sec. 3.

Chaotic inflation with a non-minimally coupled scalar field and the Ginzburg–Landau potential (3.11) was also considered in Ref. [79]. The spectral index of density perturbations was computed in the two regimes: *a*) $\phi_{end}^2 \gg v^2$; *b*) $\phi_{end}^2 \simeq v^2$, respectively, where ϕ_{end} is the value of the scalar field at the end of inflation. Case *a*) is reduced to the case, already considered, of a quartic potential and of Eq. (5.1). Case *b*) yields [79]

$$n_s(\xi, \delta) = 1 - \frac{16\xi(1 + \delta^2)}{8\alpha\xi(1 + \delta^2) - \delta^2} . \quad (5.4)$$

From Eqs. (70) and (71) of Ref. [79], one derives

$$\delta^2(\xi, v) = -\frac{8\pi G\xi v^2}{1 + 8\pi G\xi v^2} , \quad (5.5)$$

where $G = m_{pl}^{-2}$ is the present value of Newton’s constant. Although n_s was given in Ref. [79] for a range of values of δ and ξ , it turns out that n_s depends only on the square of the parameter v and not from ξ [83]. Using Eq. (5.5), one obtains

$$n_s(v) = 1 - \frac{2}{\alpha + \pi (v/m_{pl})^2} . \quad (5.6)$$

The limits (5.2) are satisfied for all values of v , hence Eq. (5.6) does not constrain the parameter v .

The last scenario considered in Ref. [79] is the case of a non-minimally coupled scalar field, the Ginzburg–Landau potential (3.11), $\phi_{end}^2 \simeq v^2$ and new-inflationary initial conditions, which give

$$n_s = 1 + 8\xi \frac{1 + \delta^2}{\delta^2} . \quad (5.7)$$

Again, the use of Eq. (5.5) reveals that n_s is independent of ξ and is a function of v^2 only:

$$n_s(v) = 1 - \frac{1}{\pi (v/m_{pl})^2} . \quad (5.8)$$

The limits (5.2) provide a constraint on the parameter v of the Ginzburg–Landau potential:

$$|v| \geq \left(\frac{10}{\pi}\right)^{1/2} m_{pl} \simeq 1.78 m_{pl} . \quad (5.9)$$

We are not aware of other viable scenarios in which the spectral index n_s has been computed as a function of the coupling constant ξ . In the last two scenarios considered in this section, the independence of n_s of ξ rules out the possibility of determining this parameter with the data currently available.

6 Discussion and conclusions

The problem of the correct value of the coupling parameter ξ in any given inflationary scenario containing scalar fields cannot be neglected, if the scenario is to be theoretically consistent. We have analyzed the inflationary scenarios which are most studied in the literature: some of them are theoretically consistent, while some others are not, and in other cases the value (or range of values) of ξ is unknown. A clear prescription for the value of ξ emerges in GR, and it has been shown that GR is an attractor for scalar–tensor theories [28, 61, 29]. If scalar–tensor theories approach GR during the matter–dominated epoch of the universe, as suggested in Ref. [28], these arguments are irrelevant for inflationary scenarios. It has also been proposed that the “GR as an attractor” behavior occurs *during* inflation [61, 29]; in this case the coupling parameter ξ assumes the value $1/6$ before the end of inflation. The relevance of this phenomenon depends on the time during inflation at which the scalar–tensor theory approaches GR, and is worth studying in the future.

It is also to be remarked that, if GR is the correct theory of gravity during inflation, or if the inflaton field is conformally coupled to the Ricci curvature in some other theory of gravity, the universe has a peculiar feature: the cosmological tail problem [84] for the ϕ –field (i.e. the backscattering off the background curvature of spacetime) is trivially resolved in some cases: due to the conformal flatness of the Friedmann universe and to the conformal invariance of the Klein–Gordon equation, a massless scalar field with potential $V = 0$ or $V = \lambda\phi^4$ (chaotic inflation) propagates without tails.

It should also be kept in mind that, in the inflationary scenarios not based on GR, there is, in principle, the possibility (not explored so far) that the inflaton couples non–minimally to scalars constructed from the Riemann tensor and different from R . These couplings are not allowed in GR, or in any metric theory of gravity in which a non–gravitational inflaton satisfies Eq. (1.2).

Finally, we remark that it is believed that particles associated to the inflaton field may survive as dark matter in boson stars. In this case, the correct value of the coupling constant ξ in a specific inflationary scenario must also be used in the study of the structure and stability of boson stars, both of which depend on ξ . Another possible application of the prescriptions of Sec. 2 is the field of classical and quantum wormholes.

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