

Matter/Microwave Correlations in an Open Universe

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Abstract

In an intriguing recent paper, Crittenden and Turok proposed cross-correlating the cosmic microwave background (CMB) with tracers of the matter density to probe the existence of a cosmological constant. Here I emphasize that a similar cross-correlation arises in an open Universe and, depending on the redshift distribution of the tracer population and the matter density, may be comparable to or stronger than that in a flat cosmological-constant Universe with the same matter density. The two cases can be distinguished through cross-correlation with tracer populations with different redshift distributions.

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In an intriguing recent paper, Crittenden and Turok proposed cross-correlating the cosmic microwave background (CMB) with tracers of the matter density to probe the existence of a cosmological constant Λ [1]. In a flat matter-dominated Universe, CMB anisotropies are produced at (or near) the surface of last scatter (the Sachs-Wolfe (SW) effect). If $\Lambda \neq 0$, additional anisotropies are produced as photons pass through potential wells along the line of sight (the integrated Sachs-Wolfe (ISW) effect).¹ Here I re-emphasize that a similar effect exists in an open Universe and point out that, depending on the redshift distribution of the tracer population, this cross-correlation can be comparable to or stronger than that in an open Universe with the same matter density Ω_0 .

A cross-correlation between the ISW signal and a tracer of the mass density at low redshift must be picked out from a noisy background due to the SW effect. The contribution to the signal-to-noise ratio from the ℓ th multipole moment therefore depends on the ratio $C_\ell^{\text{ISW}}/C_\ell^{\text{tot}}$ [1], where C_ℓ^{ISW} and C_ℓ^{SW} are the ISW and SW contributions to the ℓ th multipole moment. Although the precise value of this ratio may depend slightly on the large-scale power spectrum, it can be approximated roughly by [3,4]

$$\frac{C_\ell^{\text{ISW}}}{C_\ell^{\text{SW}}} \simeq \frac{g(\Omega_0)}{\ell} \equiv \frac{36\pi}{\ell} \int_0^\infty \left(\frac{dF}{d\eta} \right)^2 (\eta_0 - \eta) d\eta, \quad (1)$$

where $F(\eta) \propto (H/a) \int (da/a)(Ha/a_0)^{-3}$ (normalized to $F(0) = 1$) is the growth factor for gravitational-potential perturbations as a function of conformal time η . Here, a and H are the scale factor and Hubble parameter of the Universe.

Figure 1 illustrates that $g(\Omega_0)$ is much larger in an open Universe than in a flat Λ Universe with the same matter density. A more accurate treatment of the ISW effect in an open Universe confirms qualitatively the results presented here [3]. For example, for $\Omega_0 = 0.3$, the quadrupole is due almost entirely to the ISW effect (e.g., see Fig. 7 in Ref. [3]), whereas in a flat Λ Universe, it contributes only a fraction of the SW term [4].

Realistically, however, the signal will be due to the ISW contribution from redshifts probed by the tracer population, and the noise will be due to the SW effect and the ISW contribution from larger redshifts. If one has a survey which traces the mass distribution out to a redshift z_s , then to a first approximation, Eq. (1) should be replaced by

$$\frac{C_\ell^{\text{late-ISW}}}{C_\ell^{\text{SW}} + C_\ell^{\text{early-ISW}}} \simeq \frac{\int_0^{z_s} (dg/dz) dz}{\ell + \int_{z_s}^{z_{ls}} (dg/dz) dz}, \quad (2)$$

where dg/dz the differential contribution to $g(\Omega_0)$ as a function of redshift z shown in Fig. 2, and $z_{ls} \simeq 1100$ is the redshift of the surface of last scatter. For example, suppose we cross-correlate the COBE map of the CMB with the x-ray background, which probes redshifts up to $z_s \simeq 2$. Then for $\Omega_0 = 0.5$, Eq. 2 falls in the range 0.11–0.39 for an open Universe and

¹The ISW term discussed here arises in linear perturbation theory and produces large-angle CMB anisotropies. Although the two terms are often used interchangeably, it should be distinguished from the Rees-Sciama effect [2], in which anisotropies are produced on much smaller angular scales from nonlinear gravitational collapse of galaxies or clusters of galaxies.

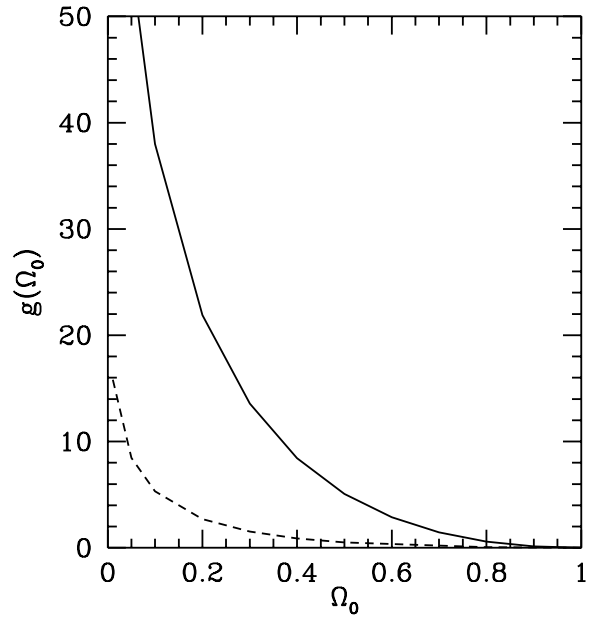


FIG. 1. The function $g(\Omega_0)$ in an open Universe (solid curve) and in a flat Λ Universe (dashed curve).

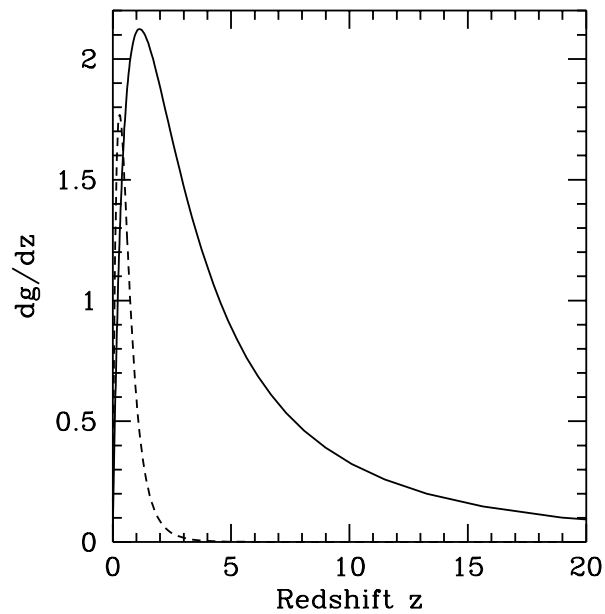


FIG. 2. The function dg/dz for $\Omega_0 = 0.3$ in an open Universe (solid curve) and in a flat Λ Universe (dashed curve).

0.03–0.19 for a cosmological-constant Universe for values of $\ell \simeq 2–15$ probed by COBE. This simple estimate suggests that the cross-correlation of the CMB with the x-ray background in an open Universe with $\Omega_0 = 0.5$ should at least be comparable to that in a cosmological-constant Universe with the same matter density. For larger values of Ω_0 , the signal-to-noise becomes larger in an open Universe relative to its value in a cosmological-constant Universe. If a tracer population which extends to a redshift $z_s \simeq 5$ can be found, the signal-to-noise in a cosmological-constant Universe remains unaffected, but it will increase by more than a factor of two in an open Universe.

Of course, more detailed numerical calculations, which take into account realistic redshift distributions as well as the angular resolutions and sky coverages of the CMB and tracer surveys, will be needed for comparison with data. Still, the estimates provided here combined with the results of Ref. [1] suggest that this cross-correlation may provide a useful probe of Ω_0 in an open Universe. Although these calculations were performed assuming primordial adiabatic perturbations, a similar cross-correlation should arise in models with primordial isocurvature perturbations. More work on large-angle anisotropies in topological-defect models must be done to determine whether this test will be effective in these scenarios.

Finally, the flat and open models should be distinguishable if two (or more) tracer populations with differing redshift distributions can be used. This might also be accomplished by varying the flux cutoff of a single tracer population.

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