# The origin of galactic disks with exponential z-profiles

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#### ABSTRACT

A new solution is presented for the puzzling, observed universality of the exponential luminosity profiles, perpendicular to the disk plane of spiral and lenticular galaxies. It is shown that such exponential z-profiles result naturally from gaseous protodisks which settle into isothermal equilibrium prior to star formation. Subsequent cooling leads to a gravitational contraction of the gas towards the equatorial plane and to a stellar exponential z-profile if the star formation rate is assumed to be comparable to the cooling rate of the gas. The final stellar scale height depends only on the initial gas temperature and local surface density. This model therefore provides a new method to investigate the early energetic state of galactic protodisks with measured scale heights and surface densities along the disk plane.

Key words: Galaxies:spiral – Galaxies:formation

### 1 INDRODUCTION

Galaxies of similar morphological type exhibit similarities in their structural properties, independent of whether they are isolated in general fields or agglomerated in galaxy clusters. These similarities under different environments may emerge due to some *internal* regularities that have superseeded external, probabilistic disturbances such as interactions and/or mergers.

There is a well-known evidence from optical observations that many edge-on spiral and lenticular galaxies have a universal luminosity profile perpendicular to their disk plane, or a universal z-distribution of stellar mass density if a constant mass-to-luminosity ratio is assumed (Tsikoudi 1977; Burstein 1979; van der Kruit & Searle 1981). It was once claimed that the z-distribution is best fitted by a selfgravitating isothermal model like  $\operatorname{sech}^2(z/2h_z)$  using a scale parameter  $h_z$  (van der Kruit & Searle 1981).

Near-infrared observations of edge-on spiral galaxies however uncovered an excess over the isothermal model at small |z| where the optical photometry is hindered by dust absorption (Wainscoat, Freeman, & Hyland 1989; Aoki et al. 1991; van Dokkum et al. 1994). These infrared observations showed that an exponential distribution  $\exp(-|z|/h_z)$  provides a superior fit to the observed z-profiles. Furthermore it is known from analyses of star counts that the vertical structure of the Galactic disk in the solar neighbourhood is well fitted by an exponential distribution rather than an isothermal model (Bahcall & Soneira 1980; Prichet 1983).

dd/orThe vertical disk structure has been considered as a result of dynamical evolution of the stellar component through<br/>encounters with massive clouds and spiral structures (Vil-<br/>lumsen 1983; Lacey 1984; Carlberg 1987). However, al-<br/>though collisions with clouds might be important in increas-<br/>ing the velocity dispersions of young stars this effect cannot<br/>account for the high velocity dispersion of old stars (Lacey

been found to date.

not exponential. We report that the hydrodynamical equations associated with a simple star formation law possess a remarkable solution which naturally produces an exponential stellar zdistribution in the final stage of gravitational settling of galactic protodisks. This result might provide new insight into the formation of galactic disks and their early evolutionary phases.

1984). In addition, the resulting disks are isothermal and

An exponential z-distribution can be constructed by adding up several stellar disk components with different ver-

tical velocity dispersions, but this is only possible if the con-

tribution from stars with larger velocity dispersion is fine-

tuned to dominate progressively at larger distances from the

disk plane. A mechanism that enables this tuning has not

# 2 THE NUMERICAL MODEL

The key factor which makes galaxy simulations distinct from either hydrodynamical computations is star formation. From theoretical arguments (Hoyle 1953; Low & Lynden-Bell 1976; Silk 1977), the cooling of the gas plays a decisive role in the process of star formation as it provides the required cool environments where density fluctuations in the gas can become gravitationally unstable on stellar mass scales. It is therefore reasonable to assume that stars form at a rate comparable to the local cooling rate. It is this working hypothesis that we use in this paper.

We will restrict ourselves to the gravitational settling of the protodisk in the vertical direction, neglecting radial motions. This approximation is reasonable because of the observational fact that galactic disks are in general rotationally supported. The hydrodynamical equations which describe the evolution of the gas density  $\rho$ , the vertical gas velocity w and the thermal gas energy by unit mass e are then

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(\rho w) = -\frac{\rho}{t_*}$$

$$\frac{\partial w}{\partial t} + w \frac{\partial w}{\partial z} + \frac{2}{3} \frac{\partial}{\rho \partial z}(\rho e) - g_z = 0 \qquad (1)$$

$$\frac{\partial e}{\partial t} + w \frac{\partial e}{\partial z} + \frac{2}{3} e \frac{\partial w}{\partial z} = -\frac{e}{t_c} ,$$

with

$$\partial g_z / \partial z = -4\pi G(\rho + \rho_*) \quad , \tag{2}$$

where  $\rho_*$  is the stellar density and the other notations have their usual meanings. The source terms on the right hand side of equation (1) describe the condensation of gas into stars on a timescale  $t_*$  and the gaseous energy dissipation on a timescale  $t_c = \rho e/\Lambda$  with the cooling rate per unit volume  $\Lambda(\rho, e)$ . We introduce a free parameter  $k = t_*/t_c$ which determines the timescale of star formation  $t_*$  relative to  $t_c$ . Note that k should be of order unity according to our assumption that star formation is related primarily to the cooling activity of the gas.

The cooling rate is a function of the local gas density and temperature and therefore can be approximated as  $\Lambda = \Lambda_0 \rho^a e^b$ . Typical values are a = 2 and, in an ionized hydrogen gas, b = +0.5 or b = -0.5 for free-free cooling or free-bound cooling, respectively. Note that possible heating would compete with the gas cooling and affect a and b. The power-law model also includes the possibility that energy dissipation in the disk is dominated by cloud-cloud collisions. In such a case a = 2 and b = 1.5 (Larson 1969). In order to keep the simulation simple we treat a and b as free parameters and do not specify cooling and heating sources in an explicit way.

The stars will stay most of their time at maximum zdistance, that is approximately at the position where they formed initially. This is at least valid for a majority of stars that make up the disk where the gas is quickly converted into stars and subsequent contraction of the gas no longer has a significant dynamical effect on the stellar orbits (e.g., Yoshii & Saio 1979). As a first approximation we therefore neglect the secular dynamical evolution of stars and assume that their local density is given by

$$\rho_*(z) = \int_0^\infty \frac{\rho(z,t)}{t_*(z,t)} dt \quad .$$
(3)

A commonly perceived theoretical idea on the formation of galaxies (Fall & Efstathiou 1980; Blumenthal et al. 1986; Barnes & Efstathiou 1987) is that their initial state is

more or less the virialized isothermal sphere where baryonic matter relaxes with dark matter. A more recent idea from cosmological simulations (Katz 1992) is that spiral galaxies formed through the hierarchical merging of smaller subunits that were disrupted in the merging process. Whereas their dissipationless stellar and dark matter components now constitute the visible and dark halos of disk galaxies, the gas could dissipate its kinetic energy, spin up and settle into the equatorial plane where it formed an initially extended, gaseous protodisk. As we will demonstrate below, the observed exponential vertical disk profiles indicate that this protodisk was able to achieve a local isothermal equilibrium state prior to the onset of star formation. The initial disk temperature T(r) then is only a function of the disk radius r but independent of z, and the initial surface density  $\Sigma(r)$ determines the initial central density  $\rho(z=0)$  at radius r and its local scale height  $h_z(r)$ :

$$\rho(z) = \frac{\Sigma}{4h_z} \operatorname{sech}^2\left(\frac{z}{2h_z}\right) \quad , \qquad h_z = \frac{k_B T}{2\pi G \Sigma \mu m_H} \quad , \tag{4}$$

where  $\mu$  is the mean molecular weight. We use this as a reasonable initial setup in our calculations.

The hydrodynamical equations are solved on an Eulerian staggered grid, with 100 logarithmically spaced grid points in z and an outer edge placed at a sufficiently large z-distance. The set of differential equations is integrated numerically by means of an explicit, finite difference scheme with operator splitting and monotonic transport. This scheme provides second-order accuracy in space and time (Burkert & Hensler 1987). The simplicity of the above equations allows us to make a comprehensive survey in the parameter space.

# 3 DISCUSSION OF THE NUMERICAL RESULTS

A number of test calculations have shown that the final stellar z-profile depends strongly on the chosen initial distribution of the protodisk gas if one starts from a non-equilibrium state. On the other hand, an exponential stellar disk forms in all cases where we assume the gas to settle into isothermal equilibrium, prior to star formation and gas cooling, as long as the star formation time  $t_*$  is comparable to the cooling time  $t_c$ , that is,  $k \sim 1$ .

In the limit of rapid star formation  $(k \gg 1)$  the initial isothermal distribution of the gas is frozen in the stellar disk which has more or less a flat-top z-profile near the equatorial plane. In the limit of slow star formation  $(k \ll 1)$  the gas condenses into the equatorial plane towards the energy minimum state, giving rise to a power-law z-profile in the stellar disk. Exponential profiles are in between these two extremes, and our calculations show that when k lies inside a preferable range  $0.3 \lesssim k \lesssim 3$ , the final stellar z-profile becomes exponential, independent of the free parameters k,  $\Lambda_0$ , a and b, and also independent of T and  $\Sigma$  of the isothermal protodisk gas. This remarkable result still holds when the isothermal equilibrium state is modified to some extent.

In Figure 1 we show as an example a sequence of models with changing b (upper panel) or changing a (lower panel) while the other parameters are fixed as shown in the panels:  $k = 1, T = 10^{5}$ K,  $\Sigma = 50 M_{\odot} \text{pc}^{-2}$ , and  $\Lambda_{0}$  set to give  $\langle t_c \rangle = 10^9$  yrs at the half-mass z-distance of the protodisk. The stellar z-profile  $\rho_*(z)$  is almost perfectly exponential all the way from  $z \sim 0$  to  $10h_z$  spanning about five orders in density, and the scale length becomes systematically smaller for larger b or larger a. It is evident that the final scale height depends critically on the adopted values of the parameters and in general differs from the initial scale height of the gaseous protodisk.

In Figure 2 we show one of the other sequences of models with decreasing  $\Sigma$  from the left to the right, using a = 2, b =0 and the other parameters fixed as in Figure 1.  $\Sigma$  is chosen to mimic the situation in our own Galaxy with a surface density distribution of  $\Sigma = 50 M_{\odot} \text{pc}^{-2} \times \exp[-(R - 8 \text{kpc})/4 \text{kpc}]$ evaluated at galactocentric radial intervals  $R = 2 \text{kpc} \times i$  with i = 1 to 5. Note that the profiles have been shifted along the horizontal axis. The stellar z-profile  $\rho_*(z)$  is almost perfectly exponential for different choices of  $\Sigma$ , and the stellar disk flares with decreasing surface density, that is, its thickness increases as  $h_z \propto \Sigma^{-1}$ . Such a  $\Sigma$ -dependence of  $h_z$  would be expected if the stellar system remembers the exponential tail of the self-gravitating isothermal disk model which was adopted as an initial condition (see Eq.4). Note however that the final exponential stellar density distribution extends much further inwards, till z = 0. Increasing b while keeping  $\Sigma$  and the other parameters constant decreases the scale height according to  $h_z(b=1) = 0.6 \times h_z(b=0)$  and  $h_z(b=2) = 0.4 \times h_z(b=0).$ 

The disk flaring, when seen edge-on, could largely be masked by projection effects. In Figure 3 we show the edgeon projected z-profiles  $\Sigma_*(z)$  at the same radii as in Figure 2 along the disk plane from the galaxy center. The projection makes the profiles less dependent on R. In such a case the inversion for recovering the original density profiles  $\rho_*(z)$ is generally very unstable. A great accuracy is needed in measuring  $\Sigma_*(z)$  over a wide range of R and z, in order to detect the disk flaring from the observations of edge-on spiral galaxies.

While the resulting stellar z-profiles are always exponential, the scale height  $h_z$  itself depends on the cooling rate  $\Lambda(\rho, e)$  through a and b and also on the initial state of the protodisk gas through T and  $\Sigma$ . In practice, however, the deterministic quantities are only T and  $\Sigma$  because if these are given the cooling source is virtually specified. Then, a and b are are no longer free parameters. In other words, given that  $h_z$  is measured at sufficient accuracy along the disk plane, we could constrain T and  $\Sigma$  as a function of radial distance from the galaxy center. This result can be used in order to explore the global initial structure of the protodisk from the vertical disk structure of an edge-on galaxy observed today.

# 4 CONCLUSIONS

The presented idea that the star formation timescale  $t_*$  is comparable to the cooling timescale  $t_c$  for the origin of exponential stellar z-profiles works when the protodisk has nearly reached an equilibrium prior to star formation and gas cooling.

Under a realistic situation the dynamical timescale  $t_{dyn}$ of the protodisk may initially not be in balance with the cooling timescale  $t_c$ . If  $t_{dyn} < t_c$  in the hot and tenuous gas, the protodisk achieves an equilibrium state and undergoes a quasi-static contraction until the gas density becomes sufficiently large so that  $t_c$  falls below  $t_{dyn}$ . If  $t_{dyn} > t_c$ otherwise, the gas cools rapidly and condenses into cool and dense clouds. In this stage, cloud collisions dissipate kinetic energy and enhance the formation of stars. As demonstrated by Burkert et al. (1992), the increase in energy input through supernova explosions will compensate energy dissipation and cooling, eventually halting the collapse and achieving a quasi-equilibrium state. In either cases, after reaching its equilibrium, the protodisk evolves due to the feedback effects of star formation, independent of its initial formation history and of variations in global physical conditions (e.g., Lin & Murray 1992).

Gas cooling or energy dissipation induces a slow gravitational settling of the protodisk while, at the same time, leading to the formation of stars in the disk. This coupling yields a continuous change of various stellar characteristics (age, metallicity, color, velocity dispersion, etc) as a function of the z-distance from the disk plane. Although our model is not sufficiently detailed to allow an extensive test against such data, the power of producing an exponential stellar z-profile from a vast range of physical conditions is very appealing and should be considered as a strong candidate for understanding the universality of exponential z-profiles.

Burkert et al. (1992) have performed more detailed simulations of the dynamical settling of hot protogalactic gas into the equatorial plane, taking into account star formation and heating and cooling processes in a multiphase interstellar medium. They achieved a good agreement with the observed vertical density structure of the Galactic disk in the solar neighborhood. The complexity of their model leads however to a very large parameter space which cannot be explored in detail due to computational limitations. Among many different processes involved in their physical model, the process of crucial importance is that the rate of star formation is adjusted sooner or later to balance with the local cooling rate by means of the self-regulated star formation mechanism (Cox 1983; Franco & Cox 1983). It is not clear from their models, which process is most important in leading to exponential z-profiles. Our calculations demonstrate that an ideal condition for such a profile is  $t_* \sim t_c$ , independent of the details of heating and cooling.

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## FIGURE CAPTIONS

FIG. 1.—The final stellar z-profile  $\rho_*(z)$  for different values of the power index b (upper panel) or a (lower panel) for the cooling rate defined as  $\Lambda = \Lambda_0 \rho^a e^b$ , with other parameters fixed in the model, as shown in the figure (for details see text). The z-profiles (thick lines) originate from the same initial distribution of the protodisk gas (dotted line).

FIG. 2.—The final stellar z-profile  $\rho_*(z)$  for different values of the initial surface mass density of the protodisk  $\Sigma$  (face-on projected), with other parameters fixed in the model (for details see text). The five z-profiles from the left to the right show a sequence of decreasing  $\Sigma$ , according to the formula appropriate to our own Galaxy:  $\Sigma =$  $50M_{\odot}\mathrm{pc}^{-2} \times \exp[-(R-8\mathrm{kpc})/4\mathrm{kpc}]$  evaluated at galactrocentric radial intervals  $R = 2\mathrm{kpc} \times i$  with i = 1 to 5. Note that the profiles have been shifted along the horizontal axis.

FIG. 3.—The edge-on projected z-profiles  $\Sigma_*(z)$  at the same radii as in Figure 2 along the disk plane from the galaxy center. Note that the profiles have been shifted along the horizontal axis.





