

A Reanalysis of Small Scale Velocity Dispersion in the CfA1 Survey

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ABSTRACT

The velocity dispersion of galaxies on scales of $r \sim 1h^{-1}$ Mpc, $\sigma_{12}(r)$, may be estimated from the anisotropy of the galaxy-galaxy correlation function in redshift space. We present a reanalysis of the CfA1 survey, correct an error in the original analysis of Davis and Peebles (1983), and find that $\sigma_{12}(r)$ is extremely sensitive to the details of how corrections for infall into the Virgo cluster are applied. We conclude that a robust value of σ_{12} cannot be obtained from this survey. We also discuss results from other redshift surveys, including the effect of removing clusters.

Subject headings: cosmology - large scale structure of the universe, galaxies - clustering

1. Introduction

Davis and Peebles (1983, hereafter DP83) calculated the velocity dispersion of galaxies, $\sigma_{12}(r)$, on scales of $r \sim 1 - 5h^{-1}Mpc$ for the CfA1 redshift survey, a survey containing 1840 redshifts covering 1.83 steradians in the North galactic hemisphere (Huchra et al. 1983). Their result, $\sigma_{12}(1) = 340 \pm 40$ km/s on scales of $1h^{-1}$ Mpc, became the standard by which N-body simulations were judged for perhaps ten years, and a primary argument against the Cold Dark Matter scenario for structure formation with the assumption that galaxies trace the mass fluctuations in an unbiased way, which yields much higher velocities on this scale (e.g., Davis et al. 1985, Gelb & Bertschinger 1994). The same calculation was done on the Southern Sky Redshift Survey (SSRS1, da Costa et al. 1991), with results of $\sigma_{12}(1) \sim 300$ km/s (Davis 1988), in apparent agreement with the CfA1 result. It is only recently that there have been attempts to reproduce the results of DP83 (Mo, Jing, & Borner 1993,

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Zurek et al. 1994), and to perform this analysis on new, larger redshift surveys (Fisher et al. 1994a, 1994b, Marzke et al. 1995, Guzzo et al. 1995). It is now apparent that there is a large variation in σ_{12} between different surveys (see Table 1). In addition, different workers (Mo et al. 1993, Zurek et al. 1994) obtain very different results (from DP83 and from each other) when they analyze the CfA1 survey. In this paper we clarify some details of the original calculation of DP83 which were not spelled out in the original paper, and present a reanalysis which shows why the results are so unstable. We also reproduce the earlier results for SSRS1 (Davis 1988), and investigate how removing clusters affects σ_{12} .

2. Method

In this section we briefly describe the method used to extract the pairwise velocity dispersion σ_{12} from the redshift-space correlation function $\xi(r_p, \pi)$. Readers should refer to DP83 and Fisher et al. (1994a, b) for more details.

The correlation function in redshift space, $\xi(r_p, \pi)$, is estimated by counting the number of pairs in a bin in r_p (separation perpendicular to the line of sight) and π (separation parallel to the line of sight). It is normalized by constructing a catalog of Poisson distributed points with the same selection function and angular limits as the data, and counting pairs between the data and the Poisson catalog.

$$1 + \xi(r_p, \pi) = \frac{n_R}{n_D} \cdot \frac{DD(r_p, \pi)}{DR(r_p, \pi)} \quad (1)$$

where DD is the number of pairs between data and data, and DR is number of pairs between the data catalog and a Poisson catalog. The quantities n_R and n_D are the minimum variance weighted densities (see Davis & Huchra 1982) of the Poisson and data catalogs, respectively.

Let $F(\mathbf{w} | \mathbf{r})$ be the distribution function of velocity differences \mathbf{w} for pairs of galaxies with vector separation \mathbf{r} , and $f(w_3 | r)$ the velocity distribution function averaged over the directions perpendicular to the line of sight. The first moment of $F(\mathbf{w} | \mathbf{r})$, $\overline{v_{12}}(r)$, is the mean streaming velocity relative to the Hubble flow, and from isotropy it must be a function only of the magnitude of \mathbf{r} . Correspondingly, the first moment of $f(w_3 | r)$ is $\langle w_3 \rangle = y \overline{v_{12}}(r)/r$ where y is the component of \mathbf{r} along the line of sight.

A reasonable form for the distribution function $f(w_3 | r)$, parameterized by its moments, is

$$f(w_3 | r) \propto \exp \left(\nu \left[\frac{w_3(r) - \langle w_3(r) \rangle}{\sigma_{12}(r)} \right]^n \right) \quad (2)$$

It has been found empirically from studying observations and N-body simulations (Peebles 1976, Fisher et al. 1994b, Zurek et al. 1994, Marzke et al. 1995) that on small scales an exponential form (n=1) fits the data better than a Gaussian (n=2) or any higher power of the argument. Adopting this form for $f(w_3 | r)$, and using $r^2 = r_p^2 + y^2$ and $w_3 = \pi - y$ we have:

$$1 + \xi(r_p, \pi) = \frac{H_0}{\sqrt{2}} \int \frac{dy}{\sigma_{12}(r)} [1 + \xi(r)] \exp \left[\frac{-\sqrt{2}}{\sigma_{12}(r)} \left| \pi - H_0 y \left[1 - \frac{\overline{v_{12}}(r)}{H_0 r} \right] \right| \right] \quad (3)$$

An approximation based on self-similar solutions of the BBGKY hierarchy suggests a form for $\overline{v_{12}}(r)$ (Davis & Peebles 1977):

$$\overline{v_{12}}(r) = \frac{FH_0 r}{[1 + (r/r_0)^2]} \quad (4)$$

where F is an adjustable parameter of the model. The assumption of stable clustering (that the collapse of the cluster is exactly balanced by the Hubble flow) leads to $F = 1$. DP83 investigated different values of F but the usually quoted result is for $F=1$. The velocity dispersion $\sigma_{12}(r)$ is obtained by fitting the model of equation (3) to $\xi(r_p, \pi)$ as estimated from equation (1).

3. Reanalysis of CfA1

In the CfA1 survey, the Virgo cluster dominates the foreground of the sample. At the time when the analysis of DP83 was done, an accepted approach to correcting for the infall of our galaxy and other galaxies into Virgo was to assume a spherical infall model. In this model the infall velocity of any galaxy was taken to be inversely proportional to the galaxy’s distance from Virgo and the model was scaled so that the infall of the Local Group had the then-favored value of 440 km/s (currently favored values are 200-300 km/s). Therefore in DP83 the redshift of each galaxy was corrected both for the infall of the local group, and for its own infall into Virgo. Each redshift was also corrected for galactic rotation using a rotation velocity of 220 km/s. We will subsequently refer to the correction to each galaxy’s distance due to the infall of the local group towards Virgo, plus the galactic rotation correction, as the “dipole” correction, and the correction due to each galaxy’s own infall into Virgo (plus galactic rotation and the dipole correction) as the “inverse-Virgo-distance” or $1/r_V$ correction.

There are two stages of the procedure in which these corrections may be applied. First, there is the distance used in calculating an absolute magnitude, used to create a semi-volume limited catalog by eliminatating all galaxies with absolute magnitude $M_B > -18.5 + 5 \log h$.

Second, there is the velocity used in calculating the correlation function $\xi(r_p, \pi)$. The choice of which corrections are to be used in each of these quantities is important, as we shall see.

It could be argued that infall corrections should only be applied to the velocities used as distances for the volume limiting. The method we use here to extract σ_{12} relies on measuring the distortion of $\xi(r_p, \pi)$ due to peculiar velocities, so in general one would not want to use corrected velocities in the pair counting to compute $\xi(r_p, \pi)$. However, in the CfA1 sample the shear due to the presence of the Virgo cluster in the foreground is so large that it could be argued that some correction is needed even in the velocities used in pair-counting.

Because the inverse-Virgo-distance model is singular near Virgo, DP83 applied the infall corrections differently depending on which region of the survey a galaxy was in. Define the “inner Virgo core” as all galaxies with $\theta_V < 6^\circ$ and $v < 2500$ km/s, where θ_V is the angle with respect to the center of the Virgo cluster ($\alpha_V = 12^h.48$, $\delta_V = 12.67^\circ$) and v is the redshift, the “outer core” as galaxies with $6^\circ < \theta_V < 14^\circ$ and $v < 2500$, and the “field” as all the remaining galaxies. Note that the determination of whether a galaxy is assigned to the Virgo core depends on which of the velocities (uncorrected, dipole corrected or inverse-Virgo-distance corrected) is used for the velocity condition. DP83 used the inverse-Virgo-corrected velocity.

Table 2 defines various permutations of “cuts”, or ways of applying the different corrections in the different regions of the survey. The entry Cut 1 shows the choices necessary to reproduce the original results of DP83. This reflects our discovery, in the course of this reanalysis, that the inner core of Virgo was inadvertently deleted in the original analysis due to a typographical error in the computer program used to obtain the results presented in DP83. The entry Cut 2 is what DP83 intended to do: the mean velocity of the Virgo cluster ($\langle v_{virgo} \rangle = 1460$ km/s) is assigned to all galaxies in the inner core for the volume limiting, the dipole corrected velocities are used in pair counting in the inner core, and in both the volume limiting and pair counting in the outer core, and the inverse-Virgo-distance corrected velocities are used in the field. Note that this value of $\langle v_{virgo} \rangle$ is the measured mean redshift of Virgo, 1020 km/s, plus the dipole correction for the infall of the Local Group, 440 km/s. Cut 3 is the same as Cut 2 except that the inverse-Virgo-distance correction is not used in the pair counting. Cut 4 uses the mean Virgo redshift for the volume limiting in the inner core, but uses only the dipole correction in all the other regions. This is to enable us to disentangle the effect of using the mean Virgo velocity in the inner core from the effect of the inverse-Virgo-distance correction.

We now know that a simple spherical infall model is not a very good model for the flow field around the Virgo cluster. Even in the relatively low mass Virgo cluster favored

by the Faber-Burstein Great Attractor flow model, the redshift-distance relation is triple valued within 20° of Virgo (Nolthenius 1993). In order to study the effects of using of the spherical infall model, Cut 5 and Cut 6 include only the dipole correction in all the regions of the survey. Cut 5 includes the correction in the pair counting whereas Cut 6 does not. Finally, we show the result obtained if no corrections at all are applied to the redshifts at any stage in the procedure. We also show the result where no corrections are applied but all clusters with internal velocity dispersion greater than 500 km/s have been removed using an automated cluster-removing program (see Somerville, Primack, & Nolthenius 1996).

Figure 1 shows the results for $\sigma_{12}(r)$ for each of the cuts. As can be seen the results are very sensitive to the way in which the cuts are applied. Note that the scale dependence of σ_{12} with r also changes significantly depending on the way the corrections are applied. In all cases the slope is steeper than the original result of DP83. One can understand these results qualitatively as follows. In Cut 6, the dipole correction assigns larger distances to all the galaxies, especially those in the Virgo core. This makes the galaxies seem more luminous and more of them make it into the volume-limited catalog. In Cut 6, 42 galaxies from the Virgo core are included in the volume-limited catalog, as compared to 29 when no corrections are used. The galaxies in the Virgo core have large pairwise velocities, and σ_{12} is pair-weighted, so this increases σ_{12} considerably. Cut 5 consists of the same galaxies as Cut 6, but using the dipole corrected velocities to calculate $\xi(r_p, \pi)$ reduces the velocity dispersion of the Virgo cluster and hence the measured σ_{12} for the sample. The differences in σ_{12} for Cut 3, Cut 4, and Cut 5 are not significant - the correlation functions appear very similar, and it is only because the correlation functions are very noisy at large π that different values of σ_{12} are obtained. The values are within the (large) formal errors on the fit. This implies that using the mean Virgo redshift for the galaxies in the inner core of Virgo did not have a large effect on the results. In Cut 2, galaxies with $v > 2500$ km/s, the cutoff for the Virgo core, are assigned larger velocities due to the $1/r_V$ correction. This whole band of galaxies is therefore moved farther away from the Virgo core, reducing the number of pairs containing Virgo galaxies which fall in the $1 h^{-1}$ bin, and reducing $\sigma_{12}(1)$. Finally, not surprisingly, removing the Virgo core altogether reduces σ_{12} significantly, as can be seen in Cut 1.

Our analysis of the SSRS1 survey yields $\sigma_{12}(1) \sim 323 \pm 91$ km/s (SSRS1), which is in agreement with previous results (Davis 1988, Mo et al. 1993). The SSRS1 survey does not contain any rich foreground clusters, and therefore the original analysis was not complicated by any infall corrections. Unlike CfA1, when we remove the clusters from SSRS1 using the cluster-removal routine, the value of $\sigma_{12}(1)$ does not change significantly (see Figure 2). This suggests that σ_{12} for CfA1 is dominated by the Virgo cluster and that the SSRS1 value is more typical of the field. However, the analysis of existing redshift surveys does not yet

allow a definite conclusion. Removing clusters appears to reduce sample-to-sample variation in σ_{12} (see Table 3) but work on simulations (Somerville et al. 1996) suggests that this will also reduce the ability of the statistic to discriminate between different cosmological models.

4. Conclusions

We hope that this paper has removed the confusion regarding the value of the velocity dispersion in the CfA1 survey. We have shown that the value of the velocity dispersion is extremely sensitive to the way in which corrections for infall into the Virgo cluster are applied. This is because the Virgo cluster is in the foreground of the CfA1 survey and contains many intrinsically faint galaxies in a thermally hot region. Increasing the distance to these galaxies by a small amount results in inclusion of fewer of these galaxies in the volume limited sample. Because σ_{12} is pair weighted, including or leaving out even ~ 10 galaxies from the Virgo cluster can change σ_{12} by $\sim 100 - 200$ km/s. In addition, including corrections for cluster infall in the calculation of the correlation function in redshift space, $\xi(r_p, \pi)$, effectively removes part of the “finger of god” and reduces the velocity dispersion of the cluster. Once again the pair-weighted nature of the statistic means that this will result in a significant reduction in the overall value of σ_{12} for the sample.

However, no infall corrections were used in recent calculations of σ_{12} for other redshift surveys and yet a wide range of values for σ_{12} is obtained for different surveys. We have argued that this is because σ_{12} is extremely sensitive to clusters, and existing redshift surveys do not sample a large enough volume of space to represent a fair sample of these relatively rare objects. One approach to solving this problem is to remove the clusters from the sample before calculating σ_{12} ; however, this reduces the ability of the statistic to discriminate between cosmological models. In addition, our work suggests that the results are likely to be sensitive to the details of how the clusters are identified and removed. Analysis of larger volume redshift surveys will be necessary in order to obtain a robust value of σ_{12} which is useful in discriminating between cosmological models or for estimating Ω_0 . In the meantime, modified velocity statistics such as the galaxy-weighted velocity dispersion (Miller et al. 1996), a density dependent version of the pairwise velocity dispersion (Strauss 1996), or the median velocity of groups (Nolthenius, Klypin & Primack 1996) have been designed to be less sensitive to clusters and may be promising alternatives.

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Survey	$\sigma_{12}(1h^{-1}Mpc)$	Reference	Comments
CfA1	340 ± 40 km/s	Davis & Peebles 1983	
CfA1	276 ± 17 km/s	Mo et al. 1993	with infall correction
CfA1	433 ± 24 km/s	Mo et al. 1993	no infall correction
CfA1	~ 540 km/s	Zurek et al. 1994	with infall correction
CfA1	~ 580 km/s	Zurek et al. 1994	no infall correction
CfA1	$\sim 450 - 870$ km/s	Somerville et al. 1996	see text
SSRS1	~ 300 km/s	Davis 1988	
SSRS1	242 ± 28 km/s	Mo et al. 1993	
SSRS1	$323 \pm 91 \pm 65$ km/s	Somerville et al. 1996	
IRAS	317^{+40}_{-49} km/s	Fisher et al. 1994b	
CfA2 North	647 ± 52 km/s	Marzke et al. 1995	
CfA2 South	367 ± 38 km/s	Marzke et al. 1995	
SSRS2	272 ± 42 km/s	Marzke et al. 1995	
Perseus-Pisces	769^{+171}_{-342} km/s	Guzzo et al. 1995	

Table 1: Velocity dispersions at $1 h^{-1}$ Mpc for various redshift surveys.

	volume limit	pair count	$\sigma_{12}(1)$ (km/s)	N_{gal}	N_{virgo}
cut 1 inner core outer core field	deleted dipole $\frac{1}{r_V}$	deleted dipole $\frac{1}{r_V}$	346 ± 98	1235	0
cut 2 inner core outer core field	$\langle v_{virgo} \rangle$ dipole $\frac{1}{r_V}$	dipole dipole $\frac{1}{r_V}$	453 ± 118	1268	33
cut 3 inner core outer core field	$\langle v_{virgo} \rangle$ dipole $\frac{1}{r_V}$	dipole dipole dipole	666 ± 189	1268	33
cut 4 inner core outer core field	$\langle v_{virgo} \rangle$ dipole dipole	dipole dipole dipole	737 ± 229	1195	46
cut 5	dipole	dipole	646 ± 184	1191	42
cut 6	dipole	none	867 ± 233	1191	42
no corrections	none	none	618 ± 113	1021	29
clusters removed	none	none	406 ± 85	972	0

Table 2: Different ways of applying Virgo infall corrections. N_{gal} is the number of galaxies included in the volume-limited catalog. N_{virgo} is the number of galaxies in the Virgo core included in the volume-limited catalog. See the text for definition of the labels.

Survey	$\sigma_{12}(1h^{-1}Mpc)$	Reference
CfA1	406 ± 85 km/s	Somerville et al. 1996
SSRS1	321 ± 90 km/s	Somerville et al. 1996
IRAS	317^{+40}_{-49} km/s	Fisher et al. 1994b
CfA2 North	388 ± 56 km/s	Marzke et al. 1995
CfA2 South	253 ± 54 km/s	Marzke et al. 1995
SSRS2	275 ± 64 km/s	Marzke et al. 1995
Perseus-Pisces	613^{+73}_{-57} km/s	Guzzo et al. 1995

Table 3: Velocity dispersions at $1 h^{-1}$ Mpc for various redshift surveys, with clusters removed. Clusters with internal velocity dispersion greater than 500 km/s were removed from CfA1 and SSRS1 using an automated computer routine. IRAS galaxies avoid rich clusters so no cluster removal is necessary. Clusters with Abell richness $R \geq 1$ were removed from the CfA2/SSRS2 data by hand. In Perseus-Pisces, the sample was restricted to $RA \leq 3^h 10^m$, thus excluding the Perseus cluster, which is the richest cluster in the survey.

REFERENCES

- da Costa, L. N., Pellegrini, P. S., Davis, M., Meiksin, A., Sargent, W. L., & Tonry, J. L. 1991, *ApJS*, 75, 935
- Davis, M., Efstathiou, G., Frenk, C. S., White, S. D. M. 1985, *ApJ*, 292, 371
- Davis, M. & Huchra, J. 1982, *ApJ*, 254, 437
- Davis, M. & Peebles, P. J. E. 1977, *ApJS*, 34, 425
- Davis, M. & Peebles, P. J. E. 1983, *ApJ*, 267, 465 (DP83)
- Davis, M. 1988, in *Cosmology and Particle Physics*, ed. L.Z. Fang & A. Zee (New York: Gordon and Breach), 65.
- Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A. & Huchra, J. P. 1994a, *MNRAS*, 266, 50
- Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A. & Huchra, J. P. 1994b, *MNRAS*, 267, 927
- Gelb, & Bertschinger, 1994, *ApJ*, 436, 491
- Guzzo, L., Fisher, K. B., Strauss, M. S., Giovanelli, R. & Haynes, M. P. 1995, preprint
- Huchra, J. P., Davis, M., Latham, D. W. & Tonry, J. 1983, *ApJS*, 52, 89
- Marzke, R. O., Geller, M. J., daCosta, L. N. & Huchra, J. P. 1995, *ApJ*, 110, 477
- Miller, A., Davis, M., & White, S.D.M. 1996, preprint
- Mo, H. J., Jing, Y. P. & Borner, G. 1993, *MNRAS*, 264, 825
- Nolthenius, R. 1993, *ApJS*, 74, 1
- Nolthenius, R., Klypin, A. A., & Primack, J. R. 1996, *ApJ*, in press
- Peebles, P. J. E. 1976, *Ap&SS*, 45, 3
- Peebles, P.J.E. 1980, *The Large Scale Structure of the Universe* (Princeton: Princeton University Press)
- Strauss, M., 1996, personal communication
- Somerville, R. S., Primack, J. R., & Nolthenius, R., 1996, in preparation
- Zurek, W., Quinn, P. J., Salmon, T. K., & Warren, M. S. 1994, *ApJ*, 431, 559

Fig. 1.— The velocity dispersion σ_{12} for different ways of applying the corrections for Virgo infall. Cut 1 reproduces the results of DP83. See Table 2 and the text for a summary of the “cuts”.

Fig. 2.— The velocity dispersion σ_{12} for CfA1 and SSRS1, without any corrections for infall towards Virgo. Filled squares are results for CfA1, open squares are CfA1 with clusters removed using an automated cluster removing procedure. Filled triangles are results for SSRS1, open triangles are SSRS1 with clusters removed.



