# MICROLENSING EVENTS FROM MEASUREMENTS OF THE DEFLECTION

Jordi Miralda-Escudé[1]
E-mail: jordi@ias.edu

Submitted to The Astrophysical Journal (Letters)
May 17, 1996

## ABSTRACT

Microlensing events are now regularly being detected by monitoring the flux of a large number of potential sources and measuring the combined magnification of the images. This phenomenon could also be detected directly from the gravitational deflection, by means of high precision astrometry using interferometry. Relative astrometry at the level of $10\,\mu$as may become possible in the near future. The gravitational deflection can be measured by astrometric monitoring of a bright star having a background star within a small angular separation. This type of monitoring program will be carried out for the independent reasons of discovering planets from the angular motion they induce on the nearby star around which they are orbiting, and for measuring parallaxes, proper motions and orbits of binary stars. We discuss three applications of the measurement of gravitational deflections by astrometric monitoring: measuring the mass of the bright stars that are monitored, measuring the mass of brown dwarfs or giant planets around the bright stars, and detecting microlensing events by unrelated objects near the line of sight to the two stars. We discuss the number of stars whose mass could be measured by this procedure. We also give expressions for the number of expected microlensing events by unrelated objects, which could be stars, brown dwarfs, or other compact objects accounting for dark matter in the halo or in the disk.

*Subject headings*: gravitational lensing - astrometry - techniques: interferometric - stars: masses - stars: brown dwarfs - planetary systems

[1] Institute for Advanced Study, Princeton, NJ 08540

## 1. INTRODUCTION

The search for gravitational microlensing in stars of the Large Magellanic Cloud was suggested by Paczyński (1986) as a technique to discover compact objects that might account for part of the dark matter. Microlensing in the galactic bulge can be similarly used to study the distribution and mass function of stars (or dark matter) in our Galaxy (Paczyński 1991; Griest 1991; Kiraga & Paczyński 1994; Zhao, Spergel, & Rich 1995; Han & Gould 1996. See also Paczyński 1996 for a review). About 100 microlensing events have been detected so far over three years, mostly towards the bulge (Udalski et al. 1994a,b,c; Alcock et al. 1995, 1996a,b; Alard 1996). In principle, all the stars in our Galaxy can be microlensed by other stars in the foreground, although the optical depth is generally much lower than towards the bulge.

An alternative technique to monitoring the flux of a large number of potential sources to detect microlensing events is to search for candidate lenses, and then check if there are any sources along the path of the lens once the proper motion is known. This only works for lenses with high proper motion, in which case the positions of candidate sources can be measured and used to predict the event a reasonable time before it takes place. Paczyński (1995) has proposed to search for such high proper motion stars in the bulge fields, or elsewhere in the galactic plane; in many cases, these stars will be faint M dwarfs which, even if they are nearby, will still not be much brighter than many field stars at the distance of the bulge, and microlensing could be observed from the usual magnification lightcurve.

Microlensing can also be detected directly from the gravitational deflection, if the positions of the images can be monitored with very high accuracy using interferometry (Hog, Novikov, & Polnarev 1995; Miyamoto & Yoshii 1995; Gould 1996). Whereas the maximum magnification in a microlensing event goes as $\theta^{-4}$ when the impact parameter $\theta$ is high, the deflection decreases only as $\theta^{-1}$. Thus, with good astrometric accuracy the effect can be observed for very large impact parameters, increasing enormously the probability of detecting an event.

The very high astrometric accuracy required for microlensing can be achieved using interferometry in the near infrared to measure the relative position of a bright guide star (used to correct for the phase shift caused by seeing), and a reference star located within the isoplanatic angle of the guide star, which has a radius of $\sim 30''$, using the technique of closure phase (??? Shao & Colavita 1992a). The astrometric accuracy of ground-based interferometers is at present $\sim 50\,\mu$as in the Palomar Testbed Interferometer, with guide stars of magnitude $K \lesssim 6$ and reference stars with $K \lesssim 14$. However, using the two Keck telescopes or the VLT, this may be improved to 10 $\mu$as and down to guide and reference stars 2 to 4 magnitudes fainter (Shao & Colavita 1992a,b; Shao 1996, priv. communication). In this paper, the terms guide and reference star shall refer to any pair of stars that can be used for relative astrometry with this technique. It turns out that the observations required to find background stars and search for gravitational deflection are exactly the same as what is needed to discover planets or brown dwarfs around nearby stars. Any program to discover planets by direct imaging near stars bright enough to be used as guide stars will also identify any background stars adequate as references. Monitoring the relative position of the guide and reference stars may be done for the main purpose of discovering planets from the angular motion they induce on the star around which they orbit (or for measuring parallax, proper motion, or the orbit of a binary system in either the guide or the reference star), but will also reveal the presence of gravitational deflection.

This paper will describe three possible applications of an astrometric monitoring program related to microlensing: measuring the mass of the guide star, measuring the mass of a planet or brown dwarf near the guide star, and detecting microlensing events by other objects along the line of sight.

## 2. MICROLENSING WITH ASTROMETRY

We first consider the probability to observe a microlensing event when monitoring the position of a background star near another bright star. Astrometric monitoring of such a pair of stars would usually be done for the primary purpose of searching for planets, so the guide star will be chosen to be nearby to maximize the angular motion caused by a planet, and the fainter reference star will typically be more distant.

The microlensing optical depth towards the reference star, assumed to be at a distance $D_s$, is the fraction of the sky filled by the Einstein radii of all the lenses along the line of sight, which we assume to have a constant density $n = \int n(M)\, dM$, where $M$ is the mass of the lens. The Einstein radius is

$$\theta_E = \left( \frac{4GM}{c^2\, D} \frac{D_s - D}{D_s} \right)^{1/2} \ , \tag{1}$$

where $D$ is the distance to the lens, and the optical depth due to lenses of mass M is

$$\tau(M)\, dM = \int_0^{D_s} dD\, D^2\, n(M)\, \pi\theta_E^2\, dM = \frac{2\pi G D_s^2 \rho(M)}{3c^2}\, dM \ , \tag{2}$$

where $\rho(M) = Mn(M)$ is the density of lenses of mass $M$. This optical depth is the probability that the reference star is within an Einstein ring at any given time, where the total magnification is larger than $(9/5)^{1/2}$ (Paczyński 1986).

When we are searching for microlensing with astrometric measurements, where an event is detected if the maximum deflection is larger than $\theta_{min}$, the maximum impact parameter that allows a detection is $\theta = \theta_E^2/\theta_{min}$. The relevant quantity is then the fraction of the sky where the deflection is larger than $\theta_{min}$, which we call the "deflection optical depth", $\tau_d$:

$$\tau_d(M) \equiv \int_0^{D_s} dD\, D^2\, n(M)\, \pi\theta^2 = 5(\theta_{Es}/\theta_{min})^2\, \tau(M) \ , \tag{3}$$

where $\theta_{Es}^2 \equiv 4GM/(c^2 D_s)$.

As an example, we consider compact objects accounting for the dark matter in the halo, with a local density $\rho_{h0} = 0.01\, M_\odot\, \mathrm{pc}^{-3}$. The usual optical depth is $\tau = 2.5 \times 10^{-8} (D_s/5\,\mathrm{kpc})^2$, and from deflection the optical depth is $\tau_d = 2.3 \times 10^{-4} (M/\, M_\odot)\, (D_s/5\,\mathrm{kpc})\, (30\,\mu\mathrm{as}/\theta_{min})^2$. For known stars and dark matter in the disk, with local density $\rho_{d0} = 0.05\, M_\odot\, \mathrm{pc}^{-3}$ (see Gould, Flynn, & Bahcall 1996), an optical depth 5 times larger is inferred as long as the source star is in the plane, so that the assumption of a constant density of lenses is approximately valid. Because these observations would be made in the infrared, adequate sources could probably be found to considerably larger distances in the galactic plane than in the visual bands. Towards the bulge, the optical depth is known to be $\tau \sim 3 \times 10^{-6}$, but the Einstein radii are smaller ($\sim 0.3\,$mas, consistent with the observed durations and the velocity dispersion that the lenses and sources should have), so the deflection optical depth for the same events is $\tau_d \sim 3 \times 10^{-4} (30\,\mu\mathrm{as}/\theta_{min}^2)$. The durations of these events would be $\theta_E/\theta_{min}$ times longer than the magnification events, or 1 to 10 years.

The deflection angle observed during a microlensing event when the impact parameter is much larger than the Einstein radius is

$$\boldsymbol{\alpha} = \frac{\theta_E^2}{\theta_0^2 + (\mu t)^2}\, (\boldsymbol{\theta}_0 + \boldsymbol{\mu} t) \ , \tag{4}$$

where $\boldsymbol{\theta}_0$ is the impact parameter and $\boldsymbol{\mu}$ is the proper motion. If only this deflection is measured, only the parameters $\theta_E^2/\theta_0$ and $\mu/\theta_0$ are obtained. To measure the Einstein radius, an independent source of information is needed to obtain the impact parameter. When the impact parameter is small, the magnification is also measured and this gives the impact parameter (and the deflection angle in (4) is then also not exact, breaking the degeneracy), but in most cases the impact parameter will be too large. The maximum impact parameter where the magnification is measurable should be $\theta_0 \simeq 3\theta_E$, corresponding to $A_{max} = 1.008$. Thus, when the lens is unknown the only quantity independent of the impact parameter that is measured is $\theta_E^2/\mu$, a similar situation to the microlensing events where only the magnification is observed, and the event duration $t_E = \theta_E/\mu$ is the measured quantity. However, the gravitational deflection also allows us to determine the direction of the relative proper motion between the lens and the source. The deflection trajectory predicted by equation (4) must be observed with some minimum degree of sampling and accuracy to be confident that a microlensing event has been detected, since an apparent relative angular acceleration of the two stars could be due to several other causes. For example, the reference star might be a binary system (notice that this would be more difficult to distinguish from a planet orbiting the guide star).

In order to detect several microlensing events from the gravitational deflection, many thousands of stars would have to be monitored astrometrically over several years, with a frequency of a few observations per year. The total number of stars brighter than $K = 5$, to be used as guide stars, is $\sim 40000$ (similar to stars with $V < 8$), and the probability to find a reference star brighter than $K = 14$ in a field $30''$ in radius is $\sim 15\%$ (see Fig. 5 of Shao & Colavita 1992b). Therefore, only a few thousand pairs would be available for these magnitude limits (although the fact that *both* the potential guide stars and reference stars are concentrated to the galactic plane would increase the number of pairs available), and probably only a fraction of these can be observed given realistic observing times. Thus, it seems that detecting many events can only be done with more powerful interferometers than the present ones, and a large technological breakthrough would be required. Probably, the positions of many background stars would have to be measured simultaneously in crowded fields. Nevertheless, given that these observations will be done in any case in order to discover planets, one should keep in mind that the possibility to detect microlensing events is not negligible.

The optical depth is of course increased with higher astrometric accuracy. For a fixed mass of the lens, events of smaller deflections would also imply longer event timescales. However, if brown dwarfs with $M \sim 10^{-2}\,M_\odot$ account for dark matter in the disk with $\rho_{d0} = 0.05\,M_\odot\,\mathrm{pc}^{-3}$, then events with maximum deflection of $1\,\mu$as would still have timescales of $\sim 3$ years, and optical depth $\tau_d \sim 10^{-2}$.

Finally, we point out that the search for microlensing events using the deflection can detect extended objects of lower surface density than using the magnification, down to $\Sigma_{crit}\,(\theta_{min}/\theta_E)^2$. For objects of $M = 1\,M_\odot$ at distances of a few kpc, and $\theta_{min}/\theta_E \sim 10^{-2}$, this corresponds to densities of $\sim 10^{10}\,\mathrm{GeV\,cm^{-3}}$.

## 3. MEASUREMENTS OF GUIDE STAR MASSES

The position of the reference star (assumed to be much more distant than the guide star) will also be deflected by the guide star by an angle $\alpha = \theta_E^2/\theta$. Measurement of this deflection angle yields the mass of the guide star, since the parallax difference of the two stars (equal to $(D_s - D)/(D\,D_s)$) is the only other quantity that $\theta_E$ depends on, and is accurately measured by the astrometric monitoring (and in this case, the impact parameter is obviously known).

In order to measure the mass of the star, observations have to be done over a period $t \simeq \theta_0/\mu$, where $\mu$ is the proper motion 1. In addition, the impact parameter must be smaller than $\theta_E^2/\theta_{min}$, where $\theta_{min}$ is the minimum deflection angle that is measurable. The ratio

$$\frac{\theta_E^2}{\theta_{min}\,\mu t} = 2.6\,\frac{M}{M_\odot}\,\frac{50\,\mathrm{km\,s^{-1}}}{v}\,\frac{10\,\mathrm{yr}}{t}\,\frac{30\,\mu\mathrm{as}}{\theta_{min}}\,\frac{D_s - D}{D_s}\ , \qquad (5)$$

where $v$ is the transverse velocity of the guide star, will most often be greater than unity (except for high velocity, low-mass stars, which will rarely be bright enough for being used as guide stars). This implies that *if a microlensing event with a timescale* $\lesssim 10\,\mathrm{yr}$ *is predicted from the known positions and proper motions of two stars adequate for relative interferometry, then the deflection will in most cases be measurable with present interferometry techniques.*

The difficult challenge is to find the potential guide-reference star pairs that are sufficiently close to produce an event on a timescale less than some specified value $t$. To estimate the number of events that can be predicted by searching near all possible guide stars, we define $N(\mu, F)\,d\mu$ as the number of stars in the sky having proper motion $\mu$, with flux brighter than $F$. If an area $(\mu t)^2$ around each star is searched for potential reference stars, with average number density $n_{ref}$,

1 For a smaller observing time, only a small fraction of the deflection trajectory is observed, and the difference from a linear trajectory (which is the only information on the deflection) is of order $(\theta_E \mu t)^2/\theta_0^3$; moreover, accelerations due to orbiting companions cannot easily be distinguished

then the expected number of pairs (each of which will be a predicted event that can yield a mass measurement) is $N_{pair} = \int d\mu N(\mu, F)(\mu t)^2\, n_{ref}$.

We have used the Hipparcos Input Catalogue (Turon et al. 1992) to estimate this number of events. We use V magnitudes, since K magnitudes are unfortunately not available for all bright stars. This catalogue is complete down to $V = 7.3$. Figure 1 shows the sum of $\mu^2$ over all stars brighter than the indicated magnitudes. When multiplied by $t^2 n_{ref}$, this yields the number of expected pairs. We see that most of the area available for predicting events is in stars with $\mu \gtrsim 0.5''/\,\mathrm{yr}$, and it increases only slowly with magnitude. This can be simply understood as follows: Stars with $\mu \gtrsim 0.1\mathrm{arcsec}/\,\mathrm{yr}$ (corresponding to $50\,\mathrm{km\,s^{-1}}$ at $D = 100\,\mathrm{pc}$) should be nearby disk stars, or spheroid and thick disk stars at distances larger by no more than a factor $\sim 5$. Thus, their density and velocity distribution can be approximated as constant, which implies that $N(\mu, F)d\mu = G(F/\mu^2)\,(d\mu/\mu^4)$, where $G$ is a convolution of the cumulative luminosity function with the distribution of transverse velocities. The expected number of events goes as $\int (d\mu/\mu^2) G(F/\mu^2)$, which will converge at low $\mu$ when $G$ decreases faster than $\mu^{-1}$ with decreasing $\mu$, or equivalently when the cumulative luminosity function is steeper than $\phi(L) \sim L^{-1/2}$. This slope is achieved for luminosities $L \sim 0.1 - 1\, L_\odot$ in the V band (and lower luminosities in the K band), which for a limiting magnitude $V = 8$ would be at $D = 20\,\mathrm{pc}$, with typical proper motion $0.5''/\,\mathrm{yr}$.

For an observing time $t = 10\,\mathrm{yr}$, the number of events that can be predicted is $\sim N_{ref}/10^7$, where $N_{ref}$ is the total number of stars above the magnitude limit for reference stars (notice that high proper motion stars are isotropically distributed in the sky, so only the total number of reference stars available is relevant here). The total number of stars is $1.8 \times 10^8$ down to $V = 17$, and $6.5 \times 10^8$ down to $V = 20$ (see Table 4.2 in Mihalas & Binney 1981); the numbers are probably similar above $K = 14$ and $K = 17$, respectively (see also Figure 5 in Shao & Colavita 1992b). Thus, even with the limit $K = 14$, we would expect to predict $\sim 20$ events leading to mass measurements. Most of the events will be caused by nearby stars, which would be likely candidates for astrometric monitoring in a planetary search program in any case.

## 4. MEASUREMENT OF PLANET AND BROWN DWARF MASSES

The major reason to conduct astrometric monitoring programs will be to discover planets from the angular motion induced on the guide star. This angular motion is proportional to the mass of the planet, so the most massive planets are likely to be discovered. Giant planets and brown dwarfs may also be discovered by direct imaging. Mass measurements of these objects are important for testing theories of their structure and evolution (e.g., Marley et al. 1996). If the angular motion of the star is observed for a sufficiently long time to determine the orbit the mass of the planet can be derived, but the orbital period may be very long. So far, several giant planets have been discovered from radial velocity measurements (Mayor & Queloz 1995; Marcy & Butler 1996; Butler & Marcy 1996) and a brown dwarf by imaging (Nakajima et al. 1995, Oppenheimer et al. 1995). The brown dwarf (Gl 229B) should have an orbital period of several hundred years.

The mass of a companion of a guide star could also be measured from its gravitational deflection of the light of the reference star. The best case is for brown dwarfs. Assuming a mass $M_{bd} = 0.03\,M_\odot$ at a distance of 10 pc, the brown dwarf Einstein radius is 5 mas, and at a separation of 1" the deflection angle is $25\,\mu$as. The luminosity of this brown dwarf would be as low as $5 \times 10^{-7}\,L_\odot$ (or $K = 17$ at 10 pc) unless it is younger than a few billion years (e.g., Marley et al. 1996), so it should generally be detectable. Assuming the brown dwarf is detected prior to the microlensing event, the time and the maximum deflection of the event could be predicted.

For the low masses of brown dwarfs and giant planets, the maximum impact parameter of an event is probably limited by the astrometric accuracy, rather than the proper motion and the observing time. Therefore, the area where a background star would produce an observable deflection is $2\mu t\,\theta_{\rm max}$, or 20 arcsec$^2$ for typical brown dwarf parameters, giving an average probability of 1% for an event for a magnitude limit $K = 18$. The brown dwarf GL229B has $\mu = 0.74''/\,{\rm yr}$, $D = 7\,{\rm pc}$, and for a mass $M = 0.03\,M_\odot$ and $\theta_{min} = 10\,\mu$as an area of 40 arcsec$^2$ is swept by every ten years. Since the galactic latitude is 15°, the chance to find an adequate reference star in this area is not very small.

Discovering a previously unknown planet from the gravitational deflection is much more difficult, because many stars would have to be frequently monitored to search for rare and short events if they are not predicted in advance. However, the discovery of planets with microlensing over a wide mass range (down to much lower masses than those under discussion here) in distant systems using the traditional technique of measuring magnification lightcurves is very promising, as has been thoroughly discussed (Mao & Paczyński 1991; Gould & Loeb 1992; Bennett & Rhie 1996).

## 5. CONCLUSIONS

Accurate astrometric monitoring of pairs of guide and reference stars with interferometry will determine proper motions and parallaxes of high precision, and reveal extrasolar planets. Exactly the same observations should reveal gravitational deflection of the reference star (which is normally much more distant than the guide star) by the guide star, by any orbiting companions of the guide star, and by other objects near the line of sight to the reference star.

Initially, most of the stars that will be monitored will be nearby, because the angular motion caused by planets is larger for nearby stars. These are also the more likely stars to have adequate reference stars allowing for a mass measurement, given their high proper motions. They are also the best candidates for measuring the mass of a companion brown dwarf, for the same reason (notice also that the deflection angle at a fixed angular impact parameter scales as the inverse of the distance). To detect microlensing by other objects, the most important consideration is to find a distant reference star, which increases the optical depth. Any guide star is equally good (in fact, distant ones are best because events involving the guide star could also be detected). Although distant guide stars would not be chosen for discovering planets, their astrometric monitoring is also interesting for high precision measurement of parallaxes and proper motions.

At present, the detection of gravitational deflection is still difficult, and probably only a handful of stellar masses may be determined in the next $\sim 10$ years. Of course, the number of events that can be detected increases enormously with the astrometric accuracy and the total length of time of observation. Many binary star orbits are only known to us today because of the observations done over periods of 100 years or longer. Over the long term, the use of gravitational deflection to measure masses is likely to become of fundamental importance in astronomy.

**Acknowledgements** I thank Andy Gould for many comments that resulted in a much improved paper, and John Bahcall, Shri Kulkarni, Bohdan Paczyński and Michael Shao for useful comments. This work has used the Hipparcos Input Catalog, which was extracted from the ESO/ST-ECF/CADC astronomical database. This research was supported by a W. M. Keck Foundation grant.

## REFERENCES


Alard, C. 1996, in *Proc. IAU Symp. 173*, p. 215, eds. C. S. Kochanek, J. N. Hewitt; Kluwer Academic Publishers, Dordrecht/Boston/London

Alcock, C., et al. 1995, Phys. Rev. Lett., 74, 2867

Alcock, C., et al. 1996a, ApJ, in press (astro-ph 9512146)

Alcock, C., et al. 1996b, submitted to ApJ

Bennett, D. P., & Rhie, S. H. 1996, submitted to ApJ (astro-ph 9603158)

Butler, R. P., & Marcy, G. W. 1996, ApJ. Let., in press

Gould, A., & Loeb, A. 1992, ApJ, 396, 104

Gould, A. 1996, PASP, 108, in press (astro-ph 9604014)

Gould, A., Flynn, C., & Bahcall, J. N. 1996, ApJ, 465, in press

Griest, K. 1991, ApJ, 366, 412

Han, & Gould, A. 1996, ApJ, 468, in press

Hog, E., Novikov, I. D., & Polnarev, A. G. 1995, A&A, 294, 287

Kiraga, M. & Paczyński, B. 1994, ApJ, 430, 101

Mao, S., & Paczyński, B. 1991, ApJ, 388, L45

Marcy, G. W., & Butler, R. P. 1996, ApJ. Let., in press

Marley, M. S., Saumon, D., Guillot, T., Freedman, R. S., Burrows, A., & Lunine, J. I. 1996, submitted to Nature

Mayor, M., & Queloz, D. 1995, Nature, 378, 355

Mihalas, D., & Binney, J. 1981, *Galactic Astronomy* (W. H. Freeman and Co.: San Francisco)

Miyamoto, M., & Yoshii, Y. 1995, AJ, 110, 1427

Nakajima, T., et al. 1995, Nature, 378, 463

Oppenheimer, B. R., Kulkarni, S. R., Matthews, K., & Nakajima, T. 1995, Science, 270, 1478

Paczyński, B. 1986, ApJ, 304, 1

——— 1991, ApJ, 371, L63

——— 1995, Acta Astr., 45, 345

——— 1996, ARA&A, in press (astro-ph 9604011)

Shao, M., & Colavita, M. M. 1992a, ARA&A, 30, 457

——— 1992b, A&A, 262, 353

Turon, C., Gómez, A., Crifo, F., Creze, F., Perryman, M. A. C., Morin, D., Arenou, F., Nicolet, B., Chareton, M., & Egret, D. 1992, A&A, 258, 74

Udalski, A., et al. 1994a, ApJ, 436, L103

Udalski, A., Szymański, M., Kaluzny, J., Kubiak, M., Mateo, M., Krzemiński, W. 1994b, ApJ, 426, L69

Udalski, A., et al. 1994c, Acta Astr., 44, 227

Zhao, H. S., Spergel, D. N., & Rich, R. M. 1995, ApJ, 440, L13

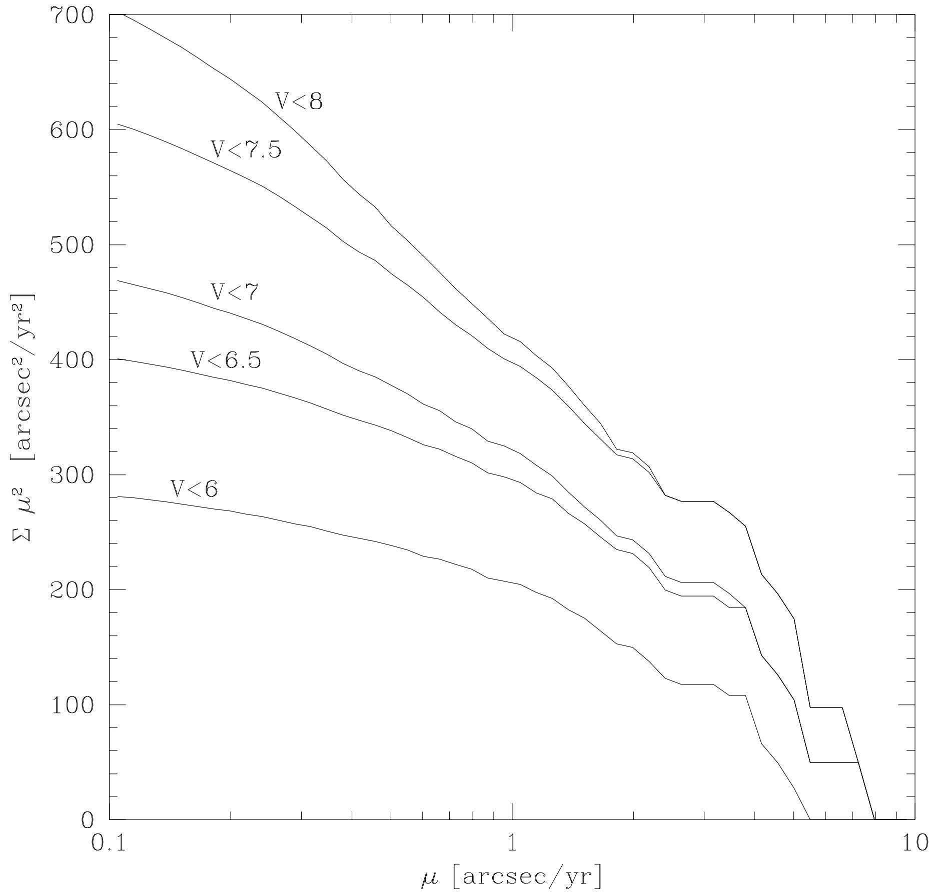


**Figure 1:** The vertical axis shows the sum of the square of the proper motions over all stars in the Hipparcos Input Catalog brighter than V=(8,7.5,7,6.5,6), and with proper motion higher than $\mu$. When multiplied by the square of the observing time in years, and by the average density of reference stars in arcsec$^{-2}$, this yields the expected number of stars whose mass can be measured from the gravitational deflection of the reference star light.