

Ferromagnetism of dense matter and magnetic properties of neutron stars

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received 5 March, 1996; accepted 22 April, 1996

Abstract. Possible consequences of ferromagnetic transition in dense matter suggested recently by Kutschera and Wójcik, for the magnetic properties of neutron stars, are studied. Specific model of dense matter, in which a small admixture of protons is completely polarized due to their interaction with neutrons, is considered. Magnetic field of neutron stars with a ferromagnetic core are calculated within the framework of general relativity. Two types of boundary conditions at the ferromagnetic core edge are considered, corresponding to normal and superconducting liquid envelope, respectively. Numerical results for the neutron star magnetic dipole moment are confronted with pulsar timing. To be consistent with observations, ferromagnetic cores surrounded by a non-superconducting envelope, should consist of weakly ordered ferromagnetic domains. If domains are highly ordered, ferromagnetic core should be screened by a superconducting envelope.

Key words: dense matter – stars: neutron – stars: pulsars

1. Introduction

Observations of radio and X – ray pulsars indicate, that the surface magnetic field of neutron stars can be as high as $10^{12} - 10^{13}$ G. The strength of the internal magnetic field of neutron stars is unknown; it could be much higher than that of the surface one.

Standard present day models of magnetic field identify its source with long-lived electric currents flowing in highly conductive neutron star matter. However, a complete scheme, based on such a model, which would explain both the origin and the evolution of neutron star magnetic field, is still lacking (recent review on the present status of this topic can be found in Bhattacharya & Srinivasan 1995).

A very different type of model of neutron star magnetic field is based on the hypothesis of existence of a ferromagnetic core in the liquid interior of neutron star. First models of this type were proposed just after the discovery of pulsars (Brownell & Callaway 1969, Rice 1969, Silverstein 1969). Neutron star matter, approximated by a pure neutron matter, was assumed to undergo a transition to a ferromagnetic state above some critical density. Ferromagnetic neutron matter, carrying high magnetization density, was there a permanent source of a superstrong magnetic field. This field, which would permeate stellar interior, was expected to extend outside the stellar surface as an observable (mostly dipole) magnetic field of a pulsar. The qualitative argument in favor of a ferromagnetic transition in sufficiently dense neutron matter was based on Pauli exclusion principle, combined with the repulsive character of the short range $n - n$ interaction. At high density, the spin singlet ($S = 0$) $n - n$ interaction becomes strongly repulsive, while the $S = 1$ one, due to Pauli principle, avoids the most repulsive, shortest range contribution from the $l = 0$ state (only odd l states are allowed in the $S = 1$ channel). So, although in the ferromagnetic transition the kinetic energy of the system increases, at sufficiently high density this was believed to be more than balanced by the removal of the repulsive interactions in the $S = 0$ states (because of complete spin polarization, all neutron pairs in a ferromagnetic phase are in the spin triplet state). The argument seemed to be particularly convincing for the schematic $n - n$ interaction of the infinite hard-core type (hard spheres gas). Some early calculations, based on the schematic models of the $n - n$ interaction, seemed to show the existence of spontaneous spin polarization in dense neutron matter above some critical density (Brownell & Callaway 1969, Rice 1969, Silverstein 1969, Østgaard 1970; see, however, Clark & Chao 1969, Clark 1969). However, further calculations, based on more realistic $n - n$ interactions and/or more precise methods of solution of the many body problem, ruled out possibility of

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ferromagnetism of dense neutron matter (Pandharipande et al. 1972, Haensel 1975, Modarres & Irvine 1979). Such a possibility was ruled out even in the most favorable case of the hard-sphere model of neutron matter, where the contribution from the $l = 1$ states was shown to be very important at superhigh densities (Dąbrowski et al. 1978, 1979).

It should be mentioned, that in contrast to the realistic calculations described above, the calculations done within the Hartree-Fock approximation using schematic *effective* nucleon Hamiltonian (or Lagrangian) often showed a ferromagnetic transition in neutron matter at a few times nuclear matter density. Vidaurre et al. (1984) used non-relativistic effective $n - n$ interaction of the Skyrme type, and found a ferromagnetic transition at about 1.5 times nuclear density. However, their result might be just a consequence of the specific density dependence of the spin dependent terms in the Skyrme interaction, which in most cases leads to unphysical collapse (no lower bound for the energy density) of dense polarized neutron matter (Kutschera & Wójcik 1994). The spin stability of dense neutron matter was also studied within the relativistic Dirac-Hartree-Fock approximation, with an effective nucleon-meson Lagrangian, by Marcos et al (1991). These authors find a ferromagnetic transition at several times nuclear density, and point out that the presence of isovector mesons in the mean-field Lagrangian is crucial for the possibility of ferromagnetic transition. Let us notice, that the importance of vector meson-coupling for the occurrence of ferromagnetism was pointed out some 20 years earlier (Kalman & Lai 1971). In general, it seems that the ferromagnetic transition in the mean-field theory models of dense neutron matter is a consequence of a very specific density dependence of the effective interactions characteristic of these models.

In all studies reviewed above, neutron star matter was approximated by pure neutron matter. The importance of the presence of a small admixture of protons in neutron star matter for the spin stability was pointed out by Kutschera and Wójcik (1989). They assumed that neutrons and protons in neutron star matter form normal Fermi liquids. The ferromagnetic state, corresponding to completely polarized protons and weakly polarized neutrons, was shown to be energetically preferred over the non-polarized one in two cases: for sufficiently strong spin-spin component of the neutron-proton quasiparticle interaction and/or for localized protons. Such conditions were shown to be satisfied for numerous models of the neutron star matter with low proton fraction. With their models of ferromagnetic cores in neutron stars, Kutschera and Wójcik were able to make specific predictions, concerning dependence of the strength of the surface magnetic field on the neutron star mass (Kutschera & Wójcik 1992). The calculations performed for strongly asymmetric nuclear matter within the relativistic Dirac-Hartree-Fock approximation, using the mean field nucleon-meson

Lagrangian, confirmed that the presence of an admixture of protons favors the ferromagnetic instability of dense matter (Bernardos et al. 1995).

At the densities exceeding the critical density for the onset of ferromagnetism, the ground state of neutron star matter corresponds to a non-zero spin polarization. However, the direction of the *local* spin polarization could vary within the ferromagnetic core, which might have a domain structure. The macroscopic magnetization, relevant for the stellar magnetic field, may be expected to result from some volume averaging, and should be consistent with hydrostatic equilibrium of the star. We present in this paper a self-consistent calculation of the magnetic field of neutron star, produced by a liquid, ferromagnetic core. Our calculations, which are done within the framework of general relativity, take into account constraints resulting from the requirement of the hydrostatic equilibrium, as well as from realistic boundary conditions.

The paper is organized as follows. In Section 2 we present the microscopic model of nucleon ferromagnetism. A model of ferromagnetic neutron star core is described in Section 3. Physical conditions within the neutron star interior, relevant for the properties of ferromagnetic neutron stars, are discussed in Section 4. The calculation of magnetic field of neutron star with a ferromagnetic core, performed in general relativity, is described in Section 5. Numerical results, obtained using specific assumptions concerning physical conditions in neutron star interiors, are presented in Section 6. In Section 7 we confront our results with observations of radio pulsars. Our conclusions are presented in Section 8.

2. Ferromagnetism of dense matter

The possibility of ferromagnetism of dense nucleon matter results from the existence of the spin-spin component of the quasiparticle nucleon-nucleon interaction. We consider the simplest model of neutron star matter, consisting of neutrons, protons and electrons. At given baryon density, n_b , the composition of matter can be characterized by the proton fraction x , such that proton density $n_p = xn_b = n_e$, while neutron density $n_n = (1 - x)n_b$. The degree of spin polarization of nucleons can be measured by the parameters

$$\begin{aligned} s_n &= \frac{n_{n\uparrow} - n_{n\downarrow}}{n_n}, \\ s_p &= \frac{n_{p\uparrow} - n_{p\downarrow}}{n_p}, \end{aligned} \quad (1)$$

where $n_{n\uparrow}$ is the number density of neutron quasiparticles with $+1/2$ spin projection of the spin on the spin quantization axis, etc. In the normal (i.e., non-ferromagnetic) state, the energy density reaches its minimum at $s_n = s_p = 0$.

Our discussion of the properties of the spin-polarized neutron star matter will follow the paper of Kutschera

and Wójcik (1989). We assume that nucleons are normal (possible effects of nucleon superfluidity will be discussed in Sections 4, 6). We neglect the thermal effects, and calculate all thermodynamic quantities at the $T = 0$ approximation. The change in the nucleon contribution to the energy density, implied by a small spin polarization of nucleons, δE_{pol}^N , can be calculated using the methods of the theory of Fermi liquids,

$$\delta E_{\text{pol}}^N = \frac{n_n^2}{2N_n}(1 + G_0^{\text{nn}})s_n^2 + \frac{n_p^2}{2N_p}(1 + G_0^{\text{pp}})s_p^2 + n_n n_p g_0^{\text{np}} s_n s_p, \quad (2)$$

where G_0^{nn} , G_0^{pp} and g_0^{np} are Fermi liquid parameters, describing the spin–spin terms in the quasiparticle nucleon–nucleon interaction in nucleon matter, and N_n , N_p are the densities of states at the corresponding Fermi surfaces for neutrons and protons, respectively. While first and second terms on the r.h.s. of Eq.(2) are positive (G_0^{nn} , $G_0^{\text{pp}} > -1$), the last term can be always made negative, irrespectively of the sign of the g_0^{np} parameter. Notice, that the third term, which results from the spin–spin interaction between neutron and proton quasiparticles, vanishes for a non–interacting system.

If $\delta E_{\text{pol}}^N < 0$, the system is unstable with respect to spin polarization. For small proton fraction ($x \ll 1$), the approximate expression for δE_{pol}^N reads (Kutschera & Wójcik 1989, hereafter referred to as KW89)

$$\delta E_{\text{pol}}^N \simeq \frac{n_p^2}{2} \left(\frac{1}{N_p} - \frac{N_n}{1 + G_0^{\text{nn}}} (g_0^{\text{np}})^2 \right) s_p^2. \quad (3)$$

Spontaneous spin polarization takes place for sufficiently strong g_0^{np} . The approximate condition for nucleon ferromagnetism reads thus

$$(g_0^{\text{np}})^2 > \frac{1 + G_0^{\text{nn}}}{N_n N_p}. \quad (4)$$

If condition (4) is satisfied, the ground state of matter contains then completely polarized protons ($s_p = 1$), and weakly polarized neutrons,

$$s_n = -x \frac{g_0^{\text{np}} N_n}{1 + G_0^{\text{nn}}}. \quad (5)$$

The values of the Fermi liquid parameters in strongly asymmetric nuclear matter, as well as their density dependence, are very uncertain. It seems to be reasonable to say, that their present knowledge does not exclude the situation, in which Equation (4) is satisfied above some critical density n_* .

For sufficiently low proton fraction (such that protons can be considered as impurities in neutron matter), the ground state of nucleon matter could correspond to a non-uniform distribution of neutrons, with proton localized (bound) in the neutron density minima (Kutschera &

Wójcik 1990). In the case of localized protons, the whole second term on the r.h.s. of Eq.(2) vanishes, and nucleon matter is always unstable with respect to full polarization of protons (KW89). Such a case corresponds most probably to the solid ferromagnetic core (Kutschera & Wójcik 1995).

It should be stressed, that the results described above can be directly used in the neutron star calculations only under a rather unrealistic assumption of perfect space homogeneity of a macroscopic element of matter. Only in such an idealized case (which was actually assumed in Kutschera & Wójcik 1992) the ‘microscopic’ and ‘macroscopic’ magnetizations will coincide. It seems to be quite realistic to assume, that the ferromagnetic phase of dense matter has some ‘domain structure’. In such a case, microscopic results would be valid only within a single domain. The macroscopic values, relevant for the stellar structure, would then correspond to an average over a sufficiently large number of domains. To be more precise, the macroscopic value of a quantity is then defined as an average over volume element, which contains a sufficient number of domains, so that the calculated value does not depend on the specific averaging procedure. Of course, the element should be still small on the stellar scale, so that the macroscopic value has a local character. The macroscopic quantities would have to be consistent with our assumption about the axial symmetry of the star.

It is well known, that ferromagnetism is an example of a second-order phase transition, and can be treated within the general scheme of the theory of second-order phase transitions, formulated by Landau (see, e.g., Section 39 of Landau et al. 1984). However, Landau theory of second-order phase transitions assumes expansion of E (or other suitable thermodynamic potential) up to fourth order in polarization parameter. Approximate expression for δE_{pol}^N , Eq. (2), is truncated at terms quadratic in polarization parameters. Such an approximation is characteristic of the Fermi liquid theory, in which only terms quadratic in deviations from the normal, reference state, are conserved. Therefore, the model proposed in KW89, if taken literally, cannot predict the threshold behavior s_p , $s_n \propto (n - n_*)^{1/2}$ for $n \rightarrow n_* + 0$, characteristic of a general case of spontaneous polarization in ferromagnetic phase.

The case of ferromagnetism with localized protons deserves a special comment. The threshold density for proton localization, n_{loc} , corresponds to a phase transition from a liquid mixture of protons, neutron and electrons to a proton crystal immersed in a neutron and electron liquid. Such a liquid–solid phase transitions is expected to be the first-order one (i.e., accompanied by the density and order parameter discontinuity). However, for normal neutrons, localization implies complete proton polarization, so that $n_* = n_{\text{loc}}$. In such a case, even for $n_b \rightarrow n_* + 0$ protons remain fully polarized.

3. Ferromagnetic neutron star cores

For simplicity, we assume that magnetization of the ferromagnetic core represents the only source of stellar magnetic field (i.e., we neglect contribution to magnetic field resulting from electric currents).

We consider a simple model for the magnetization density of polarized matter, \mathbf{m} , within the ferromagnetic core ($n_b > n_*$), as a function of baryon density, n_b . In general, the spin contribution to the z-component of \mathbf{m} in the ferromagnetic phase is given by

$$(m_z)_{\text{spin}} = n_p s_p \mu_p + n_n s_n \mu_n, \quad (6)$$

where μ_p , μ_n are intrinsic magnetic moments of protons and neutrons, respectively, and we assumed that z-axis is the spin quantization axis. We modeled $(m_z)_{\text{spin}}$ by a simple expression

$$(m_z)_{\text{spin}} = \alpha m_z^{\text{micr}}, \quad (7)$$

where m_z^{micr} corresponds to perfectly ordered ferromagnetic state, considered by Kutschera & Wójcik (1992, hereafter referred to as KW92), and dimensionless factor $\alpha \leq 1$ describes the degree of macroscopic polarization (degree of ordering of ferromagnetic domains). Our formula for \mathbf{m} , Eq.(7), takes thus into account a possibility of partial macroscopic ordering of nucleon spins (partial ordering of ferromagnetic domains), including the case of vanishing macroscopic magnetization within the ferromagnetic core ($\alpha = 0$).

Macroscopic magnetization density \mathbf{m} is the source of magnetic field within ferromagnetic neutron star cores. On the other hand, the presence of non-vanishing \mathbf{m} modifies the energy density, due to coupling to magnetic field. Consistent derivation of this coupling from microscopic theory shows, that the additional term to be included is $-\mathbf{m}_{\text{spin}} \cdot \mathbf{B}$ (de Groot & Suttrop 1972, Chapter X; Carter 1982). If we take the z-axis along \mathbf{B} , then minimization of energy requires $m_z B_z \geq 0$. The consequences of this condition will be discussed in Section 6.

4. Physical conditions in neutron star interior

In what follows, we consider the interior of neutron star, of density $n_b > n_h = 0.1 \text{ fm}^{-3}$. With our assumptions, this interior consists of a ferromagnetic core of the density $n_b > n_*$, and a non-ferromagnetic liquid envelope of the density $n_h < n_b < n_*$.

Neutron stars are believed to be born as very hot objects, with initial internal temperatures above 10^{11} K. Young neutron star cools via neutrino emission. Initial temperature of neutron star core can be expected to be larger than, or of the order of, the Curie temperature for the ferromagnetic transition, $T_{\text{ferro}} \sim$ few times 10^{10} K (Haensel 1995, unpublished). Another important critical temperature is that for nucleon superfluidity, $T_{\text{sup}} \sim$

10^9 K. It is thus reasonable to expect, that the nucleon ferromagnetic transition occurs before superfluidity sets in, i.e., that ferromagnetism takes place in normal Fermi liquids. Because $T_{\text{ferro}} > T_{\text{sup}}$, completely polarized proton component cannot undergo singlet pairing at T_{sup} . Also, a fraction of polarized neutrons remains in the normal state, due to their coupling to completely polarized protons. In such a way, protons, and a small fraction (a few percent) of neutrons may be expected to be locked in a polarized state, due to energy barrier against the depolarization of the ferromagnetic component.

It should be stressed, that in view of the uncertainties in the many body theory of dense matter at supranuclear density, even the very existence of proton superconductivity cannot be considered as absolutely certain. In view of this, we may contemplate several possibilities. In the first case, considered by KW92, ferromagnetic core is surrounded by a liquid envelope of normal matter with $n_h < n_b < n_*$, containing normal (i.e., non-superconducting) protons. In the second case, liquid envelope is assumed to contain superconducting protons. The superconductivity of the envelope could strongly influence magnetic properties of ferromagnetic neutron star (see Section 5, 6).

5. Calculation of magnetic field

The calculation of the stellar magnetic field has been done neglecting sources other than magnetization of matter within the ferromagnetic core. We took into account space curvature, implied by the neutron star gravity.

The first stage consisted in the calculation of the configuration of hydrostatic equilibrium for a non-rotating neutron star with a ferromagnetic core. In this calculation, we neglected the influence of the presence of magnetic field on the equation of state of the matter; this approximation will be justified quantitatively in Section 6. The magnetic field vectors H^j , B^j were calculated by solving the equations of magnetostatics in curved space. These equations are deduced from the Maxwell equations in the curved space in the static case (see, e.g., Eqs. (7.6) - (7.7) of Carter (1980), or Thorne & Macdonald (1982)),

$$\nabla_i B^i = 0, \quad (8)$$

$$\epsilon^{ijk} \nabla_j (N H^k) = 0, \quad (9)$$

where ∇_i denotes the covariant derivative on the $t = \text{const}$ hyperspaces, N is the lapse function, which in the static case is given by $N = \sqrt{-g_{00}}$, and ϵ^{ijk} is the antisymmetric Levi-Civita tensor associated with the 3-metric γ_{ij} on the $t = \text{const}$ hypersurfaces. Equation (9) will be satisfied, if and only if, $N H^i$ is the gradient of some scalar function Ψ , so that

$$H^i = \frac{1}{N} \nabla_i \Psi. \quad (10)$$

Inserting (10) into (8) we get, using $B^i = H^i + 4\pi m^i$,

$$\tilde{\Delta}\Psi = -4\pi\nabla_i m^i + \frac{1}{N}\nabla_i N\nabla^i\Psi, \quad (11)$$

where $\tilde{\Delta}$ is the Laplacian operator in the space curved by neutron star gravity, given by

$$\tilde{\Delta}\Psi = \frac{1}{\sqrt{\gamma}}\partial_i(\sqrt{\gamma}\gamma^{ij}\partial_j\Psi), \quad (12)$$

where $\gamma \equiv \det(\gamma_{ij})$. In view of the smallness of the deformation of the star, implied by the presence of magnetic field, we can consider the non-trivial elements of the metric γ_{ij} as depending on the radial coordinates only,

$$\begin{aligned} \gamma_{11} &= A^2(r), & \gamma_{22} &= r^2 A^2(r), \\ \gamma_{33} &= r^2 \sin^2\theta A^2(r). \end{aligned} \quad (13)$$

Within this approximation, equation (12) can be rewritten as

$$\begin{aligned} \tilde{\Delta}\Psi &= \frac{1}{r^2 A^3} \frac{\partial}{\partial r} \left(r^2 A \frac{\partial}{\partial r} \Psi \right) \\ &+ \frac{1}{A^2 r^2} \left(\frac{\partial^2}{\partial \theta^2} \Psi + \frac{\cos\theta}{\sin\theta} \frac{\partial}{\partial \theta} \Psi \right). \end{aligned} \quad (14)$$

In what follows, we will make a simplifying assumption that $\mathbf{m}(r)$ is parallel to the symmetry axis of the star. The non-vanishing normal components of magnetization in the local coordinate system are then: $m_r = m \cos\theta$, $m_\theta = -m \sin\theta$ (where $m = |\mathbf{m}|$ depends on the r coordinate only), so that $m_1 = Am_r$, $m^1 = m_r/A$, $m_2 = Arm_\theta$, $m^2 = m_\theta/(Ar)$. The divergence $\nabla_i m^i$ reads, in our approximation,

$$\nabla_i m^i = \frac{\cos\theta}{A^3} \frac{\partial}{\partial r} (A^2 m). \quad (15)$$

The Eq. (11) can thus be rewritten as

$$\begin{aligned} &\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 A \frac{\partial}{\partial r} \Psi \right) + \frac{A}{r^2} \left(\frac{\partial^2}{\partial \theta^2} \Psi + \frac{\cos\theta}{\sin\theta} \frac{\partial}{\partial \theta} \Psi \right) \\ &= -4\pi \cos\theta \frac{\partial}{\partial r} (A^2 m) + \frac{A}{N} \frac{\partial N}{\partial r} \frac{\partial \Psi}{\partial r}. \end{aligned} \quad (16)$$

Let us remind, that within our approximations A , N , and m are functions of r only. Putting $\Psi = \Phi(r) \cos\theta$ we rewrite Eq. (16) in a more suitable form:

$$\begin{aligned} \frac{1}{r^2} \left[\frac{d}{dr} \left(r^2 A \frac{d}{dr} \Phi \right) - 2A\Phi \right] &= -4\pi \frac{d}{dr} (A^2 m) \\ &+ \frac{A}{N} \frac{dN}{dr} \frac{d\Phi}{dr}. \end{aligned} \quad (17)$$

As a consequence of general properties of the second-order phase transitions, the derivative of m with respect to r is singular at the edge of ferromagnetic core, $r = r_*$. This is implied by the behavior $m \rightarrow \text{const.}(n_b - n_*)^{1/2}$

for $n_b \rightarrow n_* + 0$. To avoid numerical problems stemming from this singularity, we rewrite Eq. (17) as

$$\begin{aligned} \frac{d}{dr} \left[Ar^2 \frac{d}{dr} (r^2 \Phi) \right] &= -4\pi \frac{d}{dr} (A^2 m) \\ &+ \frac{2}{r} \Phi \frac{d}{dr} A + \frac{A}{N} \frac{dN}{dr} \frac{d\Phi}{dr}, \end{aligned} \quad (18)$$

which implies

$$\begin{aligned} \frac{A}{r^2} \frac{d}{dr} (r^2 A \Phi) &= -4\pi A^2 m \\ + \int_0^r \left[2 \frac{\Phi(r')}{r'} \frac{d}{dr'} A(r') + A(r') \frac{dN}{dr'} \frac{d\Phi}{dr'} \right] dr' &, \end{aligned} \quad (19)$$

so that

$$\begin{aligned} \Phi(r) &= -\frac{4\pi}{r^2} \int_0^r r'^2 A^2(r') m(r') dr' \\ &+ \frac{1}{r^2} \int_0^r \frac{r'^2}{A(r')} dr' \int_0^{r'} \times \\ &\times \left[\frac{2}{r''} \Phi(r'') \frac{d}{dr''} A(r'') + A(r'') \frac{dN}{dr''} \frac{d\Phi}{dr''} \right] dr'' . \end{aligned} \quad (20)$$

Let us consider first the case of normal envelope of the ferromagnetic core. Then, Equation (20) can be solved by iterations in all the space, $0 \leq r < \infty$, with regularity condition $\Phi(0) = 0$, and with boundary condition $\Phi(\infty) = 0$. Solutions were obtained using pseudospectral method, with three spatial grids, in each of the three regions $0 < r < r_*$, $r_* < r < R$ and $R < r < \infty$, respectively. The method used for compactification of the space is the same as that used for the calculation of the metric g_{ij} (see Bonazzola et al. 1993). Once Φ is known, the vectors H^i and B^i are calculated from

$$\begin{aligned} H_r(r, \theta) &= \frac{1}{AN} \Phi'(r) \cos\theta, \\ H_\theta(r, \theta) &= -\frac{1}{ANr} \Phi(r) \sin\theta, \\ B_r(r, \theta) &= \frac{1}{AN} \Phi'(r) \cos\theta + 4\pi m_r, \\ B_\theta(r, \theta) &= -\frac{1}{ANr} \Phi(r) \sin\theta + 4\pi m_\theta. \end{aligned} \quad (21)$$

Magnetic permeability of normal neutron star matter is close to unity; for simplicity, we replaced it by one.

In the second case, we assume that the ferromagnetic core is surrounded by liquid superconducting envelope (with superconducting protons). Here, we can deal with two distinct situations. If protons form a type-I superconductor, then $\mathbf{B} = 0$ for $n_b < n_b < n_*$ (Meissner effect). This would correspond to a perfect screening of the ferromagnetic core. It seems more likely, however, that the magnetic field nucleated within the superconducting envelope as an array of fluxoids, a situation characteristic for a type-II superconductor (Baym et al. 1969, Sauls 1989). Such a superconducting shell would behave as a diamagnetic envelope of ferromagnetic core.

6. Magnetic field under various physical conditions

Our models of neutron star were calculated assuming the BJI model of the equation of state (EOS) of neutron star matter (Malone et al. 1975). With this EOS, we obtain neutron star models which are quite similar (for $M \sim 1.4 - 1.8 M_\odot$) to those calculated using more recent EOS, such as the UV14+TNI or FPR models considered by KW92. For neutron star mass $M = 1.41 M_\odot$, we get the radius $R = 12.11$ km, and central density $n_{\text{centr}} = 0.58 \text{ fm}^{-3}$.

In our calculations of magnetic field, we assumed constant value of the degree of macroscopic polarization, α , and a constant, small value of the proton fraction, $x = 0.05$. In view of linearity of Eq. (17), the values of \mathbf{B} , \mathbf{H} for a specific value of α can be obtained from the $\alpha = 1$ ones using $\mathbf{B} = \alpha \mathbf{B}(\alpha = 1)$, $\mathbf{H} = \alpha \mathbf{H}(\alpha = 1)$. For the model of KW92, the microscopic magnetization m_z^{micr} has a specific dependence on the nucleon density, $n_b = n_n + n_p$. We fitted the density dependence of m_z^{micr} by an analytic formula, which exhibited correct threshold behavior for $n_b \simeq n_*$. We assumed $n_* = 0.3 \text{ fm}^{-3}$. The specific density dependence of m_z^{micr} at intermediate densities results from the density dependence of Fermi liquid parameters, combined with opposite signs of μ_n and μ_p , as well as from the assumed dependence of the proton fraction on the nucleon density. The quantity m_z^{micr} , calculated in KW92 using Eq. (6), with $s_p = 1$ and s_n given by Eq. (5), changes sign at some n_r . However, the condition $\mathbf{mB} \geq 0$ puts a definite condition on the sign of m_z on the stellar symmetry axis, which reads $m_z B_z \geq 0$. In view of this, vanishing of m_z at some point (with $B_z \neq 0$) should not be accompanied by the change of sign of m_z in the vicinity of this point. In view of this, passing through n_r does not change the sign of m_z , but requires the reversal the signs of s_n and s_p . Let us notice, that the nuclear contribution to the polarization energy, δE_{pol}^N , is invariant with respect to the change of the orientation of spins, $s_n \rightarrow -s_n$, $s_p \rightarrow -s_p$, while such an inversion of spins changes the sign of m_z^{micr} . Therefore, imposing the constraint $m_z^{\text{micr}} B_z \geq 0$ in the case of $\alpha = 1$, when $m_z = m_z^{\text{micr}}$, has no effect on the nuclear part of the energy density. However, it will have important consequences for the magnetic properties of the ferromagnetic core.

Our basic calculations have been performed, assuming $\alpha = 1$ and $M = 1.41 M_\odot$. We considered the case of a liquid envelope with normal protons. The radial distribution of m_z is displayed in Fig. 1. In Fig. 2 we show the the values of H_z and B_z , respectively, on the symmetry axis, as functions of z .

Let us notice, that relativistic effects in the ferromagnetic core are quite large. From equations (19) and (20), we obtain relation between the values of B_z, H_z and the lapse function, N , at the center of the star,

$$\frac{B_z(0)}{H_z(0)} = 1 - 3N(0). \quad (22)$$

For our neutron star model we get $B_z(0)/H_z(0) = -1.17$, to be compared with flat space-time value $(B_z(0)/H_z(0))_{\text{flat}} = -2$ (M. Kutschera, private communication). The importance of relativistic effects results from the fact, that the lapse function at the star center is significantly lower than unity, $N(0) = 0.618$.

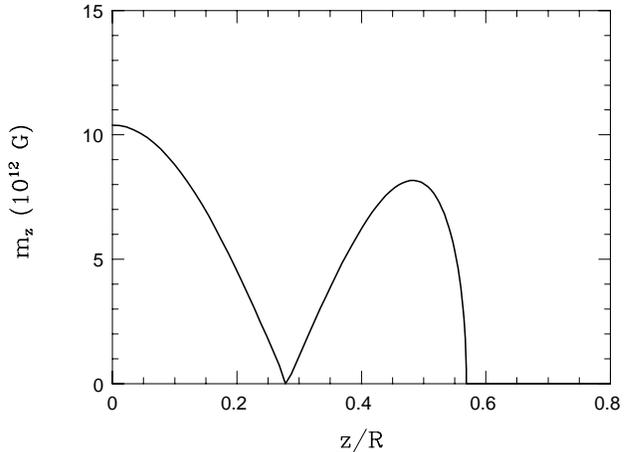


Fig. 1. Magnetization of matter within the core versus distance from the star center, for the $1.41 M_\odot$ model. Perfect ordering of ferromagnetic domains ($\alpha = 1$) has been assumed.

Let us notice, that the value of B_z at magnetic poles ($z = \pm R$) is significantly smaller (\sim four times) than its value at the ferromagnetic core edge. The dipole magnetic moment of the star is $\mathcal{M}_{30}^{\text{ferro}} = 15.1$ (here \mathcal{M}_{30} is the magnetic dipole moment in the units of 10^{30} G cm^3). At fixed M , the values of $B_z(\text{pole})$ and $\mathcal{M}^{\text{ferro}}$ for different values of the spin ordering parameter α can be calculated by multiplying by α the values obtained for $\alpha = 1$.

Both $B_z(\text{pole})$ and $\mathcal{M}^{\text{ferro}}$ depend on the mass (central density) of the neutron star model. Obviously, we have $\mathcal{M}^{\text{ferro}} = 0$ for $n_{\text{centr}} < n_*$. However, with our choice of $n_* = 0.3 \text{ fm}^{-3}$ and with our EOS, this implies that ferromagnetic core is present in neutron stars with $M > M_* = 0.7 M_\odot$ - a condition likely to be satisfied by radio pulsars. Both $\mathcal{M}^{\text{ferro}}$ and $B_z(\text{pole})$ increase with increasing stellar mass. We have $\mathcal{M}_{30} = 11.8$, $B_{z,12}(\text{pole}) = 11.7$ for $M = 1.2 M_\odot$, and both quantities increase with increasing mass up to $\mathcal{M}_{30} = 108$ and $B_{z,12} = 174$ for the maximum allowable mass, $M_{\text{max}} = 1.85 M_\odot$. Here, B_{12} is the magnetic field induction in the units of 10^{12} G .

It should be stressed, that in all cases the deformation of neutron star due to the presence of magnetic field is indeed negligibly small. For our model, the maximum energy density of magnetic field created via the ferromagnetic magnetization is comparable to the gain in the internal nuclear energy density resulting from the ferromagnetic transition. A simple estimate, based on the formulae of Sect.

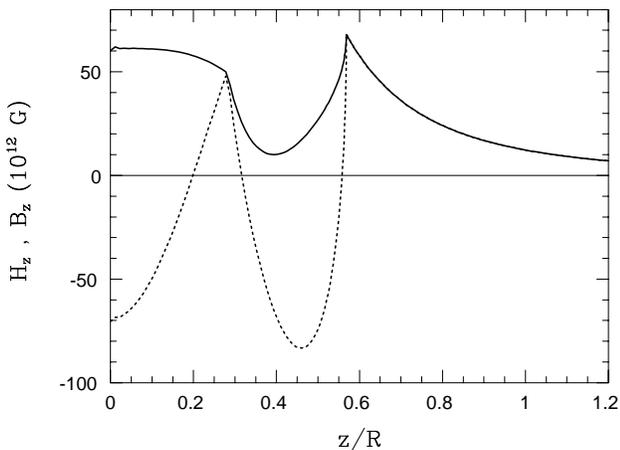


Fig. 2. The values of H_z (dotted line) and B_z (solid line) (in 10^{12} G) on the z -axis, as a function of the distance from the star center, for the $1.41 M_\odot$ star. Perfect ordering of ferromagnetic domains ($\alpha = 1$) and normal liquid envelope have been assumed.

2, gives $\delta E_{\text{pol}}^N \approx -3 \cdot 10^{31} (n_b/n_0)^{2/3} (x/0.05)^{5/3}$ erg/cm³, which is very small compared to the values of pressure near the neutron star core, $P \sim 10^{35}$ erg/cm³.

7. Confronting models with observations

Results described in the preceding section should be confronted with existing information on the magnetic field of pulsars. When confronting ferromagnetic neutron star models with observations, one should be aware of the fact, that even if some pulsars do contain ferromagnetic cores, such a core would be most probably only one of the possible sources of pulsar magnetic field, the other one being the long-living electric currents in the neutron star interior.

We will use a standard assumption, that the slowing down of pulsar rotation is due to the loss of its rotational kinetic energy, implied by the emission of the low frequency dipole radiation. We can then express the dipole magnetic moment a pulsar of period P and the time derivative of the period, \dot{P} , as

$$\mathcal{M}_{\text{PSR}} = \left(\frac{3c^3 I P \dot{P}}{8\pi^2 \sin^2 \beta} \right)^{\frac{1}{2}}, \quad (23)$$

where β is the angle between the magnetic and rotation axis, and I is the moment of inertia of neutron star. Assuming canonical mass of radio pulsars, $1.4 M_\odot$, which for our EOS corresponds to $I = 1.45 \times 10^{45}$ g cm², we see that pulsar timing puts limits on the magnetic mo-

ment of ferromagnetic core (expressed here in the units of 10^{30} G cm³),

$$\mathcal{M}_{30}^{\text{ferro}} |\sin \beta| < 14 \left(\frac{P}{10 \text{ ms}} \right)^{\frac{1}{2}} \left(\frac{\dot{P}}{10^{-13}} \right)^{\frac{1}{2}}. \quad (24)$$

Available timing data from Taylor et al. (1993) yield a very large range of $\mathcal{M}_{30} |\sin \beta|$: from 2.3×10^{-3} for second most rapid millisecond pulsar PSR B1957+20, to 0.83 for PSR B0154+61. Excluding the case of nearly perfect alignment of magnetic and rotation axes, we find two situations consistent with observations. In the first case, ferromagnetic core is characterized by a low degree of macroscopic ordering, which corresponds to $\alpha = 10^{-4} - 10^{-1}$. In the second case, the ferromagnetic core is screened by the superconducting (type-I) envelope, and therefore constraint (24) is irrelevant there. The case, when protons in the liquid envelope form a type-II superconductor would be an intermediate one; observational constraint on $\mathcal{M}^{\text{ferro}}$, Eq. (22), would then be weakened due to partial screening of ferromagnetic core.

8. Discussion and conclusions

In the ferromagnetic model of Kutschera and Wójcik (1992), consistency of the obtained values of surface magnetic field with pulsar timing was obtained via cancellations of the contributions to the stellar dipole magnetic moment coming from the inner and outer regions of the ferromagnetic cores. In particular, the low values of B for millisecond pulsars were explained by assuming that the magnetic moment of their ferromagnetic cores was some three orders of magnitude smaller than that of ordinary pulsars. According to Kutschera and Wójcik (1992), such a situation was made possible due to the cancellations, resulting from the change of sign within the ferromagnetic core. This was possible only within a very narrow interval of neutron star masses ($\Delta M \sim 0.1 M_\odot$).

Consistent calculations of the magnetic field produced by the ferromagnetic core of neutron star, performed in the present paper, show that the projection of the macroscopic magnetization onto the symmetry axis cannot change sign within the core. In view of this, consistency with pulsar timing can be achieved only under some specific conditions. If the liquid envelope surrounding ferromagnetic core is non-superconducting, than the ferromagnetic phase has to be highly disordered, with the degree of macroscopic polarization varying from $\sim 10^{-2}$ for young pulsars, to $\sim 10^{-4}$ for the millisecond pulsars. In the case, when the liquid envelope is a type-I superconductor, the ferromagnetic core produces no external magnetic field, which could be then produced exclusively by the currents within the neutron star crust.

Recently, Kotlorz & Kutschera (1994) pointed out an interesting possibility of existence of ferromagnetic pion-condensed quark cores, within sufficiently massive neutron stars. Their results imply $\mathcal{M}_{30}^{\text{ferro}}(\text{quark}) \sim 10^3$. Such huge values of $\mathcal{M}^{\text{ferro}}$ can be reconciled with pulsar timing only if either pion-condensed quark core has very low degree of macroscopic spin ordering, $\alpha < 10^{-4}$, or if this core is screened by the superconducting nucleon envelope.

Acknowledgements. We express our deep gratitude to Ericourgoulhon for his helpful remarks concerning Maxwell equations in the curved space, and for the careful reading of the manuscript. We are also grateful to Marek Kutschera for pointing out the importance of relativistic effects. This research has been supported in part by the Polish State Committee for Scientific Research (KBN) grant and by PICS/CNRS 198 ‘Astronomie Pologne’.

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