Ω_0 and λ_0 from galaxy and quasar clustering: cosmic virial theorem and cosmological redshift-space distortion

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I discuss two cosmological tests to determine the cosmological density parameter Ω_0 the cosmological constant λ_0 , which make use of the anisotropy of the two-point correlation functions due to the peculiar velocity field and the cosmological redshift-space distortion.

I. INTRODUCTION

The three-dimensional distribution of galaxies in the redshift surveys differ from the true one since the distance to each galaxy cannot be determined by its redshift z only; for $z \ll 1$ the peculiar velocity of galaxies, typically $\sim (100 - 1000)$ km/sec, contaminates the true recession velocity of the Hubble flow, while the true distance for objects at $z \gtrsim 1$ sensitively depends on the (unknown and thus assumed) cosmological parameters. This hampers the effort to understand the true distribution of large-scale structure of the universe. Nevertheless such redshift-space distortion effects are quite useful since through the detailed theoretical modelling, one can derive the peculiar velocity dispersions of galaxies as a function of separation, and also can infer the cosmological density parameter Ω_0 and the dimensionless cosmological constant λ_0 , for instance. In what follows, I will present two specific topics concerning the redshift distortion; the small-scale pair-wise peculiar velocity dispersions of galaxies [\[1](#page-9-0)], and anisotropies in the two-point correlation functions [\[2](#page-9-0)] at high redshifts.

II. Ω_0 from nonlinear galaxy clustering

Assuming that particle pairs in expanding universes are in "statistical equilibrium" on small scales, their relative peculiar velocity dispersion can be computed as a function of their

separation. The result is called the cosmic virial theorem[[3](#page-9-0),[4\]](#page-9-0) (CVT, hereafter) which predicts the one-dimensional pair-wise relative peculiar velocity dispersion $\sigma_{1D,CVT}$ as a function of Ω_0 .

The observed two- and three-point correlation functions of *galaxies*, $\xi_{\rm g}$ and $\zeta_{\rm g}$, are well approximatedby the simple form $|5,6|$:

$$
\xi_{g}(r) = \left(\frac{r_0}{r}\right)^{\gamma}, \qquad \zeta_{g}(r_1, r_2, r_3) = Q_g\left[\xi_{g}(r_1)\xi_{g}(r_2) + \xi_{g}(r_2)\xi_{g}(r_3) + \xi_{g}(r_3)\xi_{g}(r_1)\right] \tag{1}
$$

where $r_0 = (5.4 \pm 0.3)h^{-1}\text{Mpc}$, $\gamma = 1.77 \pm 0.04$, $Q_g = 1.29 \pm 0.21$. Thus it is reasonable to assume that the two- and three-point correlation functions of mass, ξ_{ρ} and ζ_{ρ} , also obey the same scaling except for the overall amplitude:

$$
\xi_{\rho}(r) = \frac{1}{b_{g}^{2}} \xi_{g}(r), \qquad \zeta_{\rho}(r_{1}, r_{2}, r_{3}) = \frac{Q_{\rho}}{Q_{g}b_{g}^{4}} \zeta_{g}(r_{1}, r_{2}, r_{3}). \tag{2}
$$

Then the CVT prediction of the small-scale peculiar velocity dispersion is given by [\[1](#page-9-0),[3](#page-9-0)]

$$
\sigma_{1D,CVT}(r) = 1460 \sqrt{\frac{\Omega_0 Q_\rho}{1.3b_g^2}} \sqrt{\frac{I(\gamma)}{33.2}} 5.4^{\frac{\gamma-1.8}{2}} \left(\frac{r_0}{5.4h^{-1} \text{Mpc}}\right)^{\frac{\gamma}{2}} \left(\frac{r}{1h^{-1} \text{Mpc}}\right)^{\frac{2-\gamma}{2}} \text{km/sec},\tag{3}
$$
\n
$$
I(\gamma) \equiv \frac{\pi}{(\gamma-1)(2-\gamma)(4-\gamma)}
$$
\n
$$
\times \int_0^\infty dx \frac{1+x^{-\gamma}}{x^2} \left\{ (1+x)^{4-\gamma} - |1+x|^{4-\gamma} - (4-\gamma)x \left[(1+x)^{2-\gamma} + |1+x|^{2-\gamma} \right] \right\},\tag{4}
$$

and numerically $I(1.65) \sim 25.4$, $I(1.8) \sim 33.2$, and $I(1.95) \sim 55.6$.

To what extent is the CVT prediction reliable ? In order to examine this, I compared $\sigma_{1D,CVT}(r)$ with the one-dimensional peculiar velocity dispersions of particle pairs with separation $r = |\mathbf{r}_1 - \mathbf{r}_2|$ directly computed from N-body simulations for cold dark matter models: $v_{12\parallel}(r) \equiv \langle [(v_1 - v_2) \cdot (r_1 - r_2)/|r_1 - r_2|]^2 \rangle^{1/2}$. Figure [1](#page-2-0) summarizes the comparison which implies that $\sigma_{1D,CVT}(r)$ reproduces the simulation result excellently for $0.1h^{-1}\text{Mpc} \lesssim r \lesssim 1h^{-1}\text{Mpc}$; CVT is quite reliable in predicting v_{12} (*r*) for $r < 1h^{-1}\text{Mpc}$. It should be stressed that the crucial assumption in deriving the prediction (3) is equation (2), and the result is independent of the theoretical model for dark matter. In this sense the prediction (3) is general, and the good agreement in CDM models should be ascribed to the fact that the CDM models actually satisfy the relation (3).

FIG. 1. Pairwise relative peculiar velocity dispersions: comparison between predictions of the CVT and simulations (Suto 1993). (a) Standard CDM ($\Omega_0 = 1$, $\lambda_0 = 0$, $h = 0.5$): Model EA employs $N = 128^3$ particles in comoving $(130h^{-1}\text{Mpc})^3$ box, while model EE employs $N = 128^3$ particles in comoving $(200h^{-1}\text{Mpc})^3$ box. The different symbols denote the results at different epochs for the models; EE at $b_{\rm g}=1$ (filled squares), EA at $b_{\rm g}=1$ (open triangles), EE at $b_{\rm g}=1.7$ (filled triangles), EA at $b_g = 1.7$ (open circles), and EA at $b_g = 1.7$ but for 2.5σ peak particles (crosses). The solid and dashed lines show the CVT prediction $\sigma_{1D,CVT}(r)$ for $b_g = 1.0$ and 1.7, respectively $(Q_\rho = 1.3)$. (b) Low-density CDM $(\Omega_0 = 0.2, b = 1.0)$: $(\lambda_0, h) = (0, 0.75)$ model with $N = 128^3$ particles in comoving $(300h^{-1}\text{Mpc})^3$ box (*open squares*), $(0.8, 0.75)$ model with $N = 128³$ particles in comoving $(300h^{-1}\text{Mpc})^3$ box (filled triangle), $(0.0, 1.0)$ model with 64³ particles in comoving $(100h^{-1}\text{Mpc})^3$ box (*crosses*), and $(0.8, 1.0)$ model with $N = 64^3$ particles in comoving $(100h^{-1}\text{Mpc})^3$ box (*open circles*).

Using the anisotropy of the two-point correlation function of the CfA1 galaxy redshift survey, Davis and Peebles [\[6](#page-9-0)] estimated the line of sight peculiar velocity dispersions $\sigma_{\text{g,obs}}^2(r_p)$ of galaxy pairs seen projected at separation r_p . This is related to the dispersions $\langle v_{3D,21}^2(r) \rangle$ for galaxy pairs with separation r in three dimension as

$$
\sigma_{\text{g,obs}}^2(r_p) = \frac{\int_0^\infty dy \,\xi_{\text{g}} \left(\sqrt{r_p^2 + y^2}\right) \left\langle v_{3D,21}^2 \left(\sqrt{r_p^2 + y^2}\right) \right\rangle}{3 \int_0^\infty dy \,\xi_{\text{g}} \left(\sqrt{r_p^2 + y^2}\right)}.
$$
\n(5)

Note that Davis and Peebles [\[6](#page-9-0)] found that $\sigma_{g,obs}^2(r_p) \propto r_p^{0.13\pm0.04}$ which justifies our assumption of the mass correlation function $\xi(r) \propto r^{-1.8}$ as $\xi_{\rm g}(r)$ from a dynamical point of view. If $\xi(r) \propto r^{-\gamma}$ and $\langle v_{3D,21}^2(r) \rangle \propto r^{2-\gamma}$, the corresponding CVT estimator corrected for the projection becomes

$$
\sigma_{1D,CVT,proj}(r_p) = C(\gamma) \sigma_{1D,CVT}(r_p) \qquad C(\gamma) \equiv \sqrt{\frac{\Gamma(\gamma/2) \Gamma(\gamma - 3/2)}{\Gamma(\gamma/2 - 1/2) \Gamma(\gamma - 1)}},\tag{6}
$$

where $\Gamma(x)$ is the Gamma function. Thus $C(\gamma)$ is the correction factor for the projection effect [\[1,6](#page-9-0)]; $C(1.65) \sim 1.36$, $C(1.8) \sim 1.11$, $C(1.95) \sim 1.02$.

Independent estimates of $\sigma_{\text{g,obs}}(r_p)$ are available from several galaxy redshift catalogues $[6,10-15]$ $[6,10-15]$ $[6,10-15]$. As pointed out earlier by Mo, Jing, and Börner $[12]$, the currently available redshift catalogues are far from the fair sample, and the observational estimate of $\sigma_{\text{g,obs}}^2(r_p)$ may differ from its true cosmic average due to the limited survey volume and the selection effect. Therefore it is meaningful to re-examine the problem in more details.

If the hierarchical relation([2\)](#page-1-0) holds as is indicated from the galaxy distribution, the amplitudes of the two- and three-point correlation functions, or equivalently $b_{\rm g}$ and Q_{ρ} , are the two uncertain parameters in the CVT prediction([3](#page-1-0)). Recent numerical and analytical studies in nonlinear gravitational clustering seem to indicate that Q_{ρ} can be approximated as a constant in the range of 0.5 and 2 which depends very weakly on the underlying cosmological model. Thus if the value of b_g is fixed from the COBE data assuming CDM models, for example, the CVT prediction [\(3](#page-1-0)) is completely specified.

Figure [2](#page-4-0) plots the empirical fit for the COBE normalized $\sigma(8h^{-1}\text{Mpc})$ and $b(8h^{-1}\text{Mpc}) \equiv$ $1/\sigma(8h^{-1}\text{Mpc})$ by Nakamura [[16](#page-9-0)] to the numerical computation by Sugiyama [[17](#page-9-0)]. There I adopt the baryon density parameter $\Omega_b = 0.015h^{-2}$. Incidentally it is amusing to note that $\Omega_0 = 0.2$, $\lambda_0 = 0.8$, and $h = 0.7$ CDM model just corresponds to $b_g = 1$, i.e., galaxies faithfully trace mass in this model according to the COBE normalization.

shows the dependence on Q_{ρ} and the right panel shows how the observational estimates which are shifted along the x-axis just for illustration. to their shows the dependence on observational estimates which are shifted along the x-axis just for illustration. The left panel $\sigma_{1D,CVI}$ Once the $\hat{r}_{\textit{prod}}(r)$ local $p_{\scriptscriptstyle \rm m}^0$ value, or sample-to-sample variation of the data. $= 1h^{-1}$ is specified, it is easy to compute the isspecified, it is easy to compute the CVT prediction (3) (3) . Figure 3 plots Mpc) (eq.[[6\]](#page-3-0)) as a function of Ω and the right panel shows how the CVT prediction is sensitive \circ . The symbols denote several recent CVT prediction (3). CVT prediction is sensitive ${\rm Figure}$ The left panel 3 plots

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bols) compared with the predictions of the cosmic virial theorem in FIG. FIG. 3. Pairwise relative peculiar velocity dispersions at $\ddot{\circ}$ Pairwise relative e peculiar $1h^{-1}$ Mpc from the observations (sym- \geq $\,$ $\,$ $\,$ $\,$ − Ω_{0} (thick curves) and

 \geq

= 0 (thin curves) CDM models.

Figure [3](#page-4-0) exhibits that both observational estimates and CVT predictions of $\sigma_{1D}(r =$ $1h^{-1}\text{Mpc}$) vary by a factor of 2 ~ 3 depending on the local clustering degree of each sample. Even allowing for these, however, it looks unlikely that high-density CDM models ($\Omega_0 \gtrsim 0.5$) are compatible with the currently observed values of $\sigma_{1D}(r = 1h^{-1}\text{Mpc})$. In a similar argument, one may obtain strong upper limits on Ω_0 in general cosmological scenarios as long as $b_{\rm g}$ is not much greater than unity.

III. Ω_0 AND λ_0 FROM QUASAR CLUSTERING AT HIGH REDSHIFTS

As shown in the previous section, small-scale velocity dispersions of galaxies place strong upper limits on Ω_0 , fairly independently of λ_0 . In turn, detailed analysis of clustering of objects at high redshifts will place a potentially important constraint on λ_0 via an effect which we call the *cosmological redshift distortion* [[2](#page-9-0)]. A very similar idea was put forward independently by Ballinger, Peacock and Heavens[[19](#page-9-0)] although they work entirely in k-space. The result that I describe below considers the analysis of two-point correlation functions, which would be more straightforward to derived from the quasar catalogues.

Let us consider a pair of objects located at redshifts z_1 and z_2 whose redshift difference $\delta z \equiv z_1 - z_2$ is much less than the mean redshift $z \equiv (z_1 + z_2)/2$. Then the observable separations of the pair parallel and perpendicular to the line-of-sight direction, $s_{\scriptscriptstyle\parallel}$ and $s_{\perp},$ are given as $\delta z/H_0$ and $z\delta\theta/H_0$, respectively, where H_0 is the Hubble constant and $\delta\theta$ denotes the angular separation of the pair on the sky. The cosmological redshift-space distortion originates from the anisotropic mapping between the redshift-space coordinates, $(s_{\parallel}, s_{\perp})$, and the real comoving ones [\[2](#page-9-0)], $(x_{\parallel}, x_{\perp}) \equiv (c_{\parallel}s_{\parallel}, c_{\perp}s_{\perp})$; c_{\perp} is written in terms of the angular diameter distance D_A as $c_{\perp} = H_0(1 + z)D_A/z$, and

$$
c_{\parallel}(z) = \frac{H_0}{H(z)} = \frac{1}{\sqrt{\Omega_0(1+z)^3 + (1-\Omega_0-\lambda_0)(1+z)^2 + \lambda_0}}.\tag{7}
$$

The relation between the two-point correlation functions of quasars in redshift space, $\xi^{(s)}(s_{\perp}, s_{\parallel})$ $\xi^{(s)}(s_{\perp}, s_{\parallel})$ $\xi^{(s)}(s_{\perp}, s_{\parallel})$, and that of *mass* in real space $\xi^{(r)}(x)$ can be derived in linear theory [[26,2\]](#page-9-0):

$$
\xi^{(s)}(s_{\perp}, s_{\parallel}) = \left(1 + \frac{2}{3}\beta(z) + \frac{1}{5}[\beta(z)]^2\right)\xi_0(x)P_0(\mu) - \left(\frac{4}{3}\beta(z) + \frac{4}{7}[\beta(z)]^2\right)\xi_2(x)P_2(\mu) + \frac{8}{35}[\beta(z)]^2\xi_4(x)P_4(\mu),
$$
\n(8)

where $x \equiv \sqrt{c_1^2 s_1^2 + c_1^2 s_1^2}$, $\mu \equiv c_1 s_1/x$, P_n 's are the Legendre polynomials,

$$
\beta(z) \equiv \frac{1}{b(z)} \frac{d \ln D(z)}{d \ln a}, \qquad \xi_{2l}(x) = \frac{(-1)^l}{x^{2l+1}} \left(\int_0^x x dx \right)^l x^{2l} \left(\frac{d}{dx} \frac{1}{x} \right)^l x \xi^{(r)}(x), \tag{9}
$$

and $D(z)$ is the linear growth rate.

For specific examples, we compute $\xi^{(s)}(s_\perp, s_\parallel)$ in linear theory applying equations (8) and (9) in CDM models with $H_0 = 70 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ [[18\]](#page-9-0). The resulting contours are plotted in Figure 4. The four sets of values of Ω_0 and λ_0 are indicated at the top of each panel. We adopt the COBE normalization [\[16,17,27](#page-9-0)].

FIG. 4. The contours of $\xi^{(s)}(s_\perp, s_\parallel)$ in CDM models at $z = 0$ (upper panels) and $z = 3$ (lower panels). Solid and dashed lines indicate the positive and negative $\xi^{(s)}$, respectively. Contour spacings are Δ log₁₀| ξ | = 0.25.

In order to quantify the cosmological redshift distortion in Figure 4, let us introduce the anisotropy parameter $\xi_{\parallel}^{(s)}(s)/\xi_{\perp}^{(s)}(s)$, where $\xi_{\perp}^{(s)}(s) \equiv \xi^{(s)}(s,0)$ and $\xi_{\parallel}^{(s)}(s) \equiv \xi^{(s)}(0,s)$. The left four panels in Figure [5](#page-7-0) show the anisotropy parameter against z in CDM models. This clearly exhibits the extent to which one can discriminate the different λ_0 models on the basis of the anisotropies in $\xi^{(s)}$ at high redshifts.

FIG. 5. The anisotropy parameter $\xi_{\parallel}^{(s)}(s)/\xi_{\perp}^{(s)}(s)$ as a function of z. Upper and lower panels assume that $b = 1$ and 2, respectively. From left to right, the panel corresponds to CDM at $s = 10h^{-1}$ Mpc, CDM at $s = 20h^{-1}$ Mpc, a power-law model with $\gamma = 1.8$ and a power-law model with $\gamma = 1.0$.

IV. CONCLUSIONS

One can roughly divide the source of cosmological information into four regimes; (i) linear regime ($r \gtrsim 10h^{-1}\text{Mpc}$) at $z \sim 0$, (ii) nonlinear regime ($r \sim 1h^{-1}\text{Mpc}$) at $z \sim 0$, (iii) linear regime at $z \gg 1$, and (iv) nonlinear regime at $z \gg 1$.

The clustering feature at $z \sim 0$ is best probed by galaxy redshift surveys, and the current samples, albeit statistically limited by the number of galaxies, are heavily used for a variety of cosmological tests. The conventional redshift-space distortion analysis [\[25,26](#page-9-0)] in the regime (i) yields generally $\Omega_0 \ll 1$; $\Omega_0^{0.6}/b_g(z=0) = 0.55 \pm 0.12$ for instance from the Durham/UKST galaxy redshift survey of ∼ 2500 galaxies. I have shown in the first part of the talk that the small-scale peculiar velocity dispersions of galaxies in comparison withe

the CVT prediction constrains the value of $\sqrt{\Omega_0 Q_\rho}/b_g(z=0)$. Allowing for the sample-tosample variation of the available redshift surveys and the uncertainties for Q_{ρ} and $b_{\rm g}(z=0)$, I still conclude that the observation in the regime (ii) favors low-density universes $\Omega_0 \lesssim 0.5$ at most.

In contrast to the regimes (i) and (ii), the proper analysis of the regimes (iii) and (iv) requires a large number of quasars homogeneously samples which are not currently available. Therefore the cosmological tests in theses regimes have not been fully explored even theoretically. In the second part of the talk, I presented an example in this line of investigations, cosmological redshift-space distortion [\[2](#page-9-0)]. I derived the formula which describes the degree of the anisotropies of two-point correlation functions of quasars in linear regime (iii), and argued that it is a potentially powerful discriminator of λ_0 once Ω_0 is determined from the nearby observation. From the observational point of view, however, it would be much easier to detect the the anisotropies of two-point correlation functions in the regime (iv). We are currently approaching this important area of research using the nonlinear theory and simulations[[29,30\]](#page-9-0).

I do look forward to the next-generation redshift surveys which will definitely provide data catalogues and thus make feasible these precise cosmological tests.

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