

**THE X-RAY BACKGROUND AS A PROBE  
OF DENSITY FLUCTUATIONS AT HIGH REDSHIFT**

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**Abstract:** The X-Ray Background (XRB) probes structure on scales intermediate between those explored by local galaxy redshift surveys and by the COBE Microwave Background measurements. We predict the large scale angular fluctuations in the XRB, expressed in terms of spherical harmonics for a range of assumed power-spectra and evolution scenarios. The dipole is due to large scale structure as well as to the observer's motion (the Compton-Getting effect). For a typical observer the two effects turn out to be comparable in amplitude. The coupling of the two effects makes it difficult to use the XRB for independent confirmation of the CMB dipole being due to the observer's motion. The large scale structure dipole (rms per component) relative to the monopole is in the range  $a_{1m}/a_{00} \sim (0.5 - 9.0) \times 10^{-3}$ . The spread is mainly due to the assumed redshift evolution scenarios of the X-ray volume emissivity  $\rho_x(z)$ . The dipole's prediction is consistent with a measured dipole in the HEAO1 XRB map. Typically, the harmonic spectrum drops with  $l$  like  $a_{lm} \sim l^{-0.4}$ . This behaviour allows us to discriminate a true clustering signal against the flux shot noise, which is constant with  $l$ , and may dominate the signal unless bright resolved sources are removed from the XRB map. We also show that Sachs-Wolfe and Doppler (due to the motion of the sources) effects in the XRB are negligible. Although our analysis focuses on the XRB, the formalism is general and can be easily applied to other cosmological backgrounds.

## 1. Introduction

Although discovered before the Cosmic Microwave Background (CMB), the origin of the X-ray Background (XRB) is still unknown. But it seems likely that the XRB is due to sources at high redshift (for reviews see Boldt 1987; Fabian & Barcons 1992). Here we shall not attempt to speculate on the nature of the XRB sources. Instead, we *utilise* the XRB as a probe of the density fluctuations at high redshift. The XRB sources are probably located at redshift  $z < 5$ , making them convenient tracers of the mass distribution on scales intermediate between those in the CMB as probed by COBE ( $\sim 1000$  Mpc), and those probed by optical and IRAS redshift surveys ( $\sim 100$  Mpc). In terms of the level of anisotropy, the XRB is also intermediate between the tiny CMB fluctuations ( $\sim 10^{-5}$  on angular scales of degrees) and galaxy density fluctuations (of the order of unity on scale of  $8 h^{-1}$  Mpc).

In recent years the XRB has been studied by means of analysing the total intensity, the spectrum and the spatial fluctuations. In particular, the spatial fluctuations were analysed by: (i) Source identifications of high-flux regions (e.g. Shanks et al. 1991); (ii) Auto-correlation functions for which upper limits and marginal detections were reported (e.g. de Zotti et al. 1990, Jahoda & Mushotsky 1991, Carrera et al. 1993, Chen et al. 1994, Sołtan & Hasinger 1994); (iii) Cross-correlation of the XRB with galaxies and clusters for which the detections and interpretation are reasonably established (e.g. Lahav et al. 1993, Miyaji et al. 1994, Carrera et al. 1995, Barcons et al. 1995, Roche et al. 1995, Sołtan et al. 1996, Treyer & Lahav 1996).

The preliminary measurements of the dipole anisotropy in the XRB (Shafer 1983; Shafer & Fabian 1983, Boldt 1987) were discussed qualitatively by associating it with local clusters such as Virgo and the Great Attractor and by other cosmographical arguments (e.g. Rees 1979; Fabian & Warwick 1979; Warwick, Pye & Fabian 1980; Jahoda & Mushotzky 1989; Goicoechea & Martin-Mirones 1990). In this paper we treat the problem in a statistical rather than cosmographical way. We generalize the analysis for any spherical harmonic of order  $l$ , corresponding to angular resolution  $\theta \sim \pi/l$ . The predicted rms harmonics are derived in the framework of growth of structure by gravitational instability from density fluctuations drawn from a Gaussian random field. The harmonics are then expressed in terms of the power-spectrum of density fluctuations and for evolution scenarios which are consistent with recent measurements of galaxy clustering and the Cosmic Microwave Background. As there is quite a lot of freedom in the parameterization of the XRB sources we shall restrict ourselves in this paper to an Einstein-de Sitter universe ( $\Omega = 1$ ,  $\lambda = 0$ ), although some of the expressions evaluated below are also valid for other world models. The Hubble constant is given as  $H_0 = 100h$  km/sec/Mpc. In principle, the X-ray background(s) should be discussed in different frequency bands, e.g. in the hard band (2-10 keV, e.g. HEAO1) and in the soft band (0.5-2.0 keV, e.g. ROSAT), which exhibit different properties. However, the current uncertainty in measurements (e.g. Table 1 in Treyer & Lahav 1996) does not make it practical at present to distinguish between the different bands. The formalism is kept general and can be used for specific cases in the future. It can also be easily generalized to other cosmological backgrounds.

The outline of the paper is as follows. Section 2 presents the harmonic formalism and rms predictions. Our main result is given in equations 15 and 16 and the reader who is not interested in the mathematical details can skip directly to these equations. Numerical estimates based on these formulae are given in Section 3, and a comparison to the observed HEAO1 XRB dipole is discussed in Section 4. We discuss the results in Section 5. In Appendix A we show that the Sachs-Wolfe and Doppler effects for the XRB are negligible compared with the source density fluctuations.

## 2. Spherical Harmonic Expansion of Background Sources

We consider a cosmological population of XRB sources that trace the matter distribution and examine the angular fluctuations in the observed XRB surface brightness. For convenient comparison with the Cosmic Microwave Background (CMB) (e.g. Padmanabhan 1993) and with galaxy distributions at low redshift (e.g. Peebles 1973, Scharf et al.

1992, Fisher, Scharf & Lahav 1994) we expand the surface brightness of the XRB over the sky in spherical harmonics and we estimate the expected rms fluctuations of different multipoles. For large scales (low multipoles) these fluctuations might be larger than the Poisson noise, provided bright resolved sources are removed from the XRB map.

### 2.1 The Clustering Term

The XRB surface brightness  $I_{\nu_0}(\hat{\mathbf{r}})d\nu_0$  is observed in a *narrow* frequency band ( $\nu_0, \nu_0 + d\nu_0$ ). Hereafter we omit the frequency label (whenever it is not essential) to make the notation easier. We expand  $I(\hat{\mathbf{r}})$  in spherical harmonics <sup>\*</sup>:

$$I(\hat{\mathbf{r}}) = \sum_{lm} a_{lm} Y_{lm}(\hat{\mathbf{r}}). \quad (1)$$

The XRB most likely results from numerous discrete sources. In this case the harmonic coefficients,  $a_{lm}$ , can be derived by summing over the sources, each with observed flux  $f_i(\nu_0)$ .

$$a_{lm} = \sum_{sources} f_i(\nu_0) Y_{lm}^*(\hat{\mathbf{r}}_i). \quad (2)$$

The flux observed in the frequency band ( $\nu_0, \nu_0 + d\nu_0$ ) due to an individual source emitting at frequency  $\nu = \nu_0(1+z)$  at redshift  $z$  and luminosity distance  $r_L$  is :

$$f(\nu_0)d\nu_0 = \frac{L_\nu}{4\pi r_L^2} d\nu = \frac{L_{\nu_0(1+z)}}{4\pi r_L^2} (1+z)d\nu_0. \quad (3)$$

Note the extra  $(1+z)$  factor which is due to the observation being per unit frequency. The predicted harmonic coefficients are then:

$$a_{lm} = \int \int dV_c dL_\nu \frac{L_\nu(1+z)}{4\pi r_L^2} [1 + b_x \delta(r_c, \hat{\mathbf{r}})] \Phi(L_\nu, z) Y_{lm}^*(\hat{\mathbf{r}}), \quad (4)$$

where again  $\nu = \nu_0(1+z)$ . Here  $\delta$  is the mass density perturbation, and we have assumed that the *comoving* luminosity function  $\Phi(L_\nu, z)$  is independent of local overdensity. For an  $\Omega = 1$  universe the volume element is  $dV_c = r_c^2 dr_c^2 d\omega$ , where  $r_c = \frac{2c}{H_0} [1 - (1+z)^{-1/2}]$  and the luminosity distance is  $r_L = (1+z)r_c$ . We have also assumed that there is a linear biasing between the X-ray sources and the mass fluctuations:

$$\delta_x(r_c, \hat{\mathbf{r}}) = b_x \delta(r_c, \hat{\mathbf{r}}), \quad (5)$$

for all redshifts. This is of course a naive assumption, reflecting our poor knowledge of the way X-ray sources are formed with cosmic time relative to the mass fluctuations.

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<sup>\*</sup> We consider here an ideal detector with zero beam width. If the detector has a beam profile of size  $\theta_B$ , then harmonics of order  $l \sim \pi/\theta_B$  are washed out. Since we are mostly interested in lower harmonics this effect is negligible for our calculations.

The observed comoving luminosity density ('volume emissivity') due to sources at redshift  $z$  is:

$$\rho_x(z) = \int dL_\nu L_\nu \Phi(L_\nu, z)(1+z), \quad (6)$$

where again  $\nu = \nu_0(1+z)$ . Hereafter we assume a simple power-law evolution model for the X-ray light density:

$$\rho_x(z) = \rho_{x0}(1+z)^q. \quad (7)$$

For example if the spectral energy distribution of a source is  $L_\nu \propto \nu^{-\alpha}$  and the source density evolves like  $(1+z)^p$  then  $q = p - \alpha + 1$ .

For the monopole ( $l = 0$ ), with  $Y_{00} = (4\pi)^{-1/2}$ , we recover the 'Olbers integral' or Lookback factor (cf. e.g. Weinberg 1972, Boldt 1987). In the case  $\Omega = 1$ ,  $\Lambda = 0$ , the mean intensity out to redshift  $z_{max}$  is:

$$\bar{I} = \frac{a_{00}}{\sqrt{4\pi}} = \int dV_c \frac{\rho_x(z)}{4\pi r_L^2} = \frac{\rho_{x0}}{4\pi} \left(\frac{c}{H_0}\right) \int_0^{z_{max}} dz (1+z)^{q-7/2} \quad (8)$$

The *fluctuations* in the background are expressed by harmonics  $l > 0$  :

$$a_{lm} = \frac{1}{4\pi} \rho_{x0} \frac{c}{H_0} \int \int d\omega dz (1+z)^{q-7/2} b_x \delta(r_c, \hat{\mathbf{r}}) Y_{lm}^*(\hat{\mathbf{r}}). \quad (9)$$

It is convenient to expand the density contrast in Fourier modes (where  $k$  is in comoving coordinates) :

$$\delta(\mathbf{r}_c) \equiv \frac{\delta\rho}{\rho}(\mathbf{r}_c) = \frac{1}{(2\pi)^3} \int d^3k \delta_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}_c}, \quad (10)$$

and to use the Rayleigh expansion of a plane wave in spherical coordinates :

$$e^{i\mathbf{k}\cdot\mathbf{r}_c} = 4\pi \sum_{lm} i^l j_l(kr_c) Y_{lm}^*(\hat{\mathbf{r}}_c) Y_{lm}(\hat{\mathbf{k}}). \quad (11)$$

With the orthogonality condition  $\int d\omega Y_{lm}(\hat{\mathbf{r}}) Y_{l'm'}^*(\hat{\mathbf{r}}) = \delta_{ll'}^{mm'}$ , we get:

$$a_{lm} = (i^l)^* \frac{1}{2\pi^2} \frac{1}{4\pi} \rho_{x0} \frac{c}{H_0} \int d^3k b_x \delta_{\mathbf{k}}(z) Y_{lm}^*(\hat{\mathbf{k}}) \int dz (1+z)^{q-7/2} j_l(kr_c), \quad (12)$$

where  $j_l$  is the Bessel function of order  $l$ . It is convenient to parameterize the growth of density perturbations by:

$$\delta_{\mathbf{k}}(z) = \delta_{\mathbf{k}}(0) (1+z)^{-\mu}. \quad (13)$$

E.g. in linear theory in an Einstein-de Sitter universe  $\mu = 1$ , which is a reasonable parameterization for the low-order harmonics (i.e. the large scales). The power-spectrum is given in terms of the present day fluctuations  $\delta_{\mathbf{k}}$  as:

$$\langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'}^* \rangle = (2\pi)^3 P(k) \delta^{(3)}(\mathbf{k} - \mathbf{k}'). \quad (14)$$

Taking the mean-square values and using Parseval's theorem we obtain the prediction for the rms fluctuations per harmonic component (there are  $2l + 1$  components per  $l$  and as the model is isotropic they are all equal and independent of  $m$ ):

$$\langle |a_{lm}|^2 \rangle = \frac{1}{(2\pi)^3} \rho_{x0}^2 b_x^2 \left(\frac{c}{H_0}\right)^2 \int dk k^2 P(k) |\Psi_l(k)|^2. \quad (15)$$

Only fluctuations within the horizon grow. Hence unless the initial power spectrum is very peculiar most of the contribution to the integral in Eq. (15) is from short wavelengths. In this case assuming the specific evolution model of eqs. (7) and (13), we can write the window function as:

$$\Psi_l(k) \approx \int_{z_{min}}^{z_{max}} dz (1+z)^{q-\mu-7/2} j_l(kr_c), \quad (16)$$

In principle, there could be a Sachs-Wolfe (1967) contribution to the XRB harmonics due to the difference in potential between the sources and the observer (similar to the effect in the CMB fluctuations on scales  $> 10^\circ$ ), and a Doppler contribution due to the motions of the XRB sources and to our motion. However, as shown in Appendix A, the Sachs-Wolfe and Doppler (due to the XRB sources motion) effects are less than  $\sim 0.1\%$  of the clustering effect and can safely be ignored.

## 2.2 The Shot Noise Term

On the other hand, a significant signal may arise from shot noise, due to the discreteness of the objects:

$$\langle |a_{lm}|^2 \rangle_{sn} = \frac{1}{4\pi} \sum_{sources} f_i^2. \quad (17)$$

A flux cutoff  $f_m$  must be used to eliminate bright sources, to avoid divergence of eq. (17). In terms of the differential number-flux relation in Euclidean space,  $N(f) = N_0 f^{-2.5}$ , the shot noise is:

$$\langle |a_{lm}|^2 \rangle_{sn} = \int_0^{f_m} f^2 N(f) df = 2N_0 f_m^{0.5} \propto r_m^{-1}, \quad (18)$$

where  $r_m = \sqrt{L_*/4\pi f_m}$  is the effective cutoff distance for an  $L_*$  galaxy. We give estimates of the shot noise relative to the clustering signal at the end of the next section. The shot-noise term is constant with  $l$  unlike the clustering term, which allows us, at least in principle, to distinguish between the two.

## 3. Quantitative Predictions for the XRB

To visualize the scales probed by the XRB, we show in Figure 1 the product  $k^3 P(k) \sim \langle (\frac{\delta\rho}{\rho})^2 \rangle$  for the standard Cold Dark Matter (CDM) power-spectrum  $P(k)$  with  $\Gamma \equiv \Omega h = 0.5$  (Bardeen et al 1986) and for a low density CDM (LCDM) power-spectrum with

$\Gamma = 0.2$ . We regard the LCDM power-spectrum only as a phenomenological fit to the clustering of local galaxies (e.g. Fisher et al. 1993), and we retain  $\Omega = 1$  in the rest of the analysis. The window functions  $|\Psi_2(k)|^2$  are shown for the quadrupole ( $l = 2$ ) of: (i) the IRAS 1.2Jy redshift survey, (ii) the XRB (eq. 16 with  $q - \mu = 3$ ,  $z_{min} = 0$ ,  $z_{max} = 5$ ) and (iii) the CMB, where\*  $|\Psi_2(k)|^2 = k^{-4} j_2^2(2ck/H_0)$ . We see that the XRB indeed probes intermediate scales, filling in the ‘gap’ between local redshift surveys and COBE.

Figure 2 shows the prediction for the XRB rms harmonics (per  $\{l, m\}$  component) due to source clustering (eq. 15). We normalize the rms  $a_{lm}$  by the monopole  $a_{00} = \sqrt{4\pi\bar{I}}$  (eq. 8), so that the ratio is dimensionless, and we do not have to specify  $\rho_{x0}$ . We also divide by the unknown normalization  $b_x\sigma_8$ , the present-epoch rms fluctuation of X-ray sources in  $8h^{-1}$  Mpc sphere, which is probably of the order unity. The mass density fluctuation  $\sigma_8$  can be specified e.g. from the COBE CMB measurements (for standard CDM  $\sigma_8 = 1.35$ ; Sugiyama 1995). The normalization  $b_x\sigma_8$  can also be fixed by comparing the measurement of the XRB auto-correlation function (e.g. Sołtan & Hasinger) with the prediction in terms of the power-spectrum (e.g. Treyer & Lahav 1996), but this determination is beyond the scope of this paper.

The normalized harmonics are shown in Figure 2 for both standard and low density CDM models (both with density perturbation growth index  $\mu = 1$ ,  $z_{min} = 0$ , and  $z_{max} = 5$ ), for a rather extreme evolution parameter  $q = 4$ . The harmonics decline monotonically like  $a_l \propto l^{-0.4}$ . Values for the normalized dipole ( $a_{1m}/a_{00}$ ) and quadrupole ( $a_{2m}/a_{00}$ ) are given in Table 1 for  $q = 4$  and  $q = 0$  (no evolution). We see that the predictions are very sensitive to  $q$ , i.e. to the redshift evolution of  $\rho_x(z)$ . They are relatively little dependent on the assumed power-spectrum and on the maximal redshift  $z_{max}$ .

The observational test for the detection of a clustering signal can be obtained by looking for a monotonically declining signal with  $l$ , compared with a constant shot-noise term. For the hard-band (2-10 keV) we can estimate the shot noise (eq. 18) by adopting  $N_0 \approx 2 \times 10^{-15}$  (erg/sec cm<sup>-2</sup>)<sup>3/2</sup> str<sup>-1</sup> and  $f_m = 3 \times 10^{-11}$  erg/sec cm<sup>-2</sup>, above which sources were identified (Piccinotti et al. 1982). With the observed mean intensity (Boldt 1987)  $\bar{I} = a_{00}/\sqrt{4\pi} = 5.2 \times 10^{-8}$  ergs/sec/cm<sup>2</sup>/str we find that the shot-noise normalized to the monopole is  $\langle |a_{lm}|^2 \rangle_{sn}^{1/2}/a_{00} \approx 8.0 \times 10^{-4}$ . This shot-noise level is comparable to the predicted dipole and quadrupole in  $q = 4$  models but well below the expected LSS signal for  $q = 0$  model (see Table 1). Therefore the signal-to-noise ratio strongly depends on the redshift evolution of  $\rho_x(z)$ . It is important therefore to explore a range of models and procedures for shot-noise suppression. In particular, there is some freedom to choose  $f_m$  such that the signal to noise is maximized.

We illustrate this point by applying a similar calculation to the soft-band using the observed ROSAT source counts (Georgantopoulos et al. 1996) extrapolated to unresolved fluxes. We find that for the rather extreme evolution model  $q = 4$  the first 10 multipoles

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\*The CMB harmonics due to the Sachs-Wolfe effect in  $\Omega = 1$  universe are (e.g. Padmanabhan 1993):

$$\langle |a_{lm}|^2 \rangle_{SW} = \left(\frac{H_0}{2c}\right)^4 \frac{2}{\pi} \int dk k^{-2} P(k) [j_l(2ck/H_0)]^2 .$$

outreach the shot noise if sources brighter than  $f_m \approx 10^{-14} \text{erg cm}^{-2} \text{s}^{-1}$  are removed. Note that for consistency they must also be removed from the  $a_{lm}$ 's, hence reducing the clustering signal. Calculating the multipoles as a function of flux limit requires knowledge of the X-ray luminosity as a function of redshift. For our purposes, we simply apply a redshift (radius) cutoff to Eq. (16). We find that the low order multipoles still outreach the shot noise (assuming the above flux cutoff) if sources nearer than few  $100 \text{ h}^{-1} \text{ Mpc}$  are removed from the X-ray map, although the harmonic spectrum flattens significantly. Again, we emphasize that for other evolution models (e.g. the extreme no-evolution case  $q = 0$ ) the expected clustering signal is well above the shot-noise.

#### 4. XRB Dipole

The dipole pattern in the XRB is due to two effects: (i) the flux emitted by the XRB sources tracing the large scale structure (LSS), as we predicted above (eq. 15 for  $l = 1$ ); and (ii) the motion of the observer relative to the XRB. The second effect, first discussed by Compton & Getting (1935) for the cosmic-ray background, gives a dipole pattern of the form

$$\frac{\Delta I}{\bar{I}} = (3 + \alpha) \frac{V_{obs}}{c} \cos(\theta), \quad (19)$$

for an observer moving at velocity  $V_{obs}$  relative to an isotropic sea of radiation with spectrum  $I_\nu \propto \nu^{-\alpha}$ . This relation is most easily derived from Liouville's theorem, which implies that  $I_\nu/\nu^3$  is an invariant. This was accurately measured in the CMB (where  $\alpha = -2$ ), most recently using the COBE 4-year data (Lineweaver et al 1996), giving a solar motion of  $368.9 \pm 2.5 \text{ km/sec}$  relative to the CMB in the direction ( $l = 264^\circ; b = 48^\circ$ ). However, based on this measurement alone, we cannot rule out an entropy gradient origin for the CMB dipole (Pacýnski and Piran 1990; Langlois and Piran 1996). If the CMB dipole does arise from our motion relative to the CMB background, then we expect to find a similar contribution to the XRB dipole. For the hard XRB, with  $\alpha = 0.4$ , the expected excess in the direction of motion is  $\frac{\Delta I}{\bar{I}} = 4.2 \times 10^{-3}$ . But as we show below, it is unfortunately difficult to separate the Compton-Getting (CG) effect from the dipole due to large scale structure (LSS) in the distribution of the XRB sources, as the two effects have similar amplitudes.

The measurements of the XRB dipole are not accurate, due to contamination by Galactic emission and low resolution, but several studies reported a detection. The HEAO1 2-10 keV whole-sky map shows a dipole (Shafer 1983, Shafer & Fabian 1983, Boldt 1987) in the direction ( $l = 282^\circ; b = 30^\circ$ ) (the 90% confidence region is rather large and covers about 1/8 of the sky). If the entire signal is due to motion, then the inferred velocity is  $475 \pm 165 \text{ km/sec}$ . At higher energies (80-165 keV) the dipole's direction is ( $l = 304^\circ; b = 26^\circ$ ) and the derived velocity (again assuming the dipole is purely due to motion) is  $1450 \pm 440 \text{ km/sec}$  (Gruber 1991). It is perhaps not too surprising that (within the large error bars) the derived velocity is larger than that deduced from the CMB, as the XRB dipole may be 'contaminated' by LSS anisotropies. The importance of the LSS effect due to nearby unresolved sources is supported by several studies. Jahoda & Mushotzky (1989) found an enhancement in the direction of the Great Attractor (see also Goicoechea & Martin-Mirones

1990), Miyaji & Boldt (1990) derived from a sample of AGNs an acceleration dipole which is consistent with the CMB dipole's direction, and a cross-correlation signal was detected between the unresolved XRB and nearby galaxy catalogues (e.g. Lahav et al 1993, Miyaji et al. 1994, Carrera et al. 1995).

Our formalism allows us to estimate the strength of the two effects for a hypothetical random observer. As shown in Table 1 the expected LSS dipole is in the range  $a_{1m,LSS}/a_{00} \sim (0.5 - 9.0) \times 10^{-3}$ . To estimate the Compton-Getting (CG) effect we first calculate the mean square velocity for a random observer (e.g. Kaiser & Lahav 1989, Padmanabhan 1993):

$$\langle V_{rms}^2 \rangle = \frac{H_0^2 \Omega_0^{1.2}}{2\pi^2} \int dk P(K) e^{-k^2 R_*^2}, \quad (20)$$

assuming linear theory and that the density fluctuations causing the motion are at distances much smaller than the horizon. The region sharing the motion is modeled here as a Gaussian sphere with radius  $R_*$ . For a point ( $R_* = 0$ ) typically  $V_{rms} \approx 1000\sigma_8\Omega^{0.6}$  for the CDM models concerned. \* The expected CG dipole in the rms sense is then  $a_{1m,CG}/a_{00} = \frac{1}{3}(3 + \alpha)\frac{V_{rms}}{c}$ . As shown in Table 1 it is interesting that the CG and LSS are of comparable amplitude for a hypothetical observer, including the case of our Sun's motion.

It is important to verify that the XRB CG dipole agrees with the CMB dipole as a proof that the CMB dipole is due to motion. However, as the 'contamination' by the LSS effect is unknown, it is better to subtract the CG dipole (based on the CMB dipole) from the XRB map, and to look at the residual LSS effect. Jahoda (1993) removed the CG dipole from the HEAO1 (2-10 keV) map, and after correcting for Galactic emission (according to Iwan et al. 1982), found  $|D_{LSS}| \equiv |\sum f_i \hat{\mathbf{r}}_i| \approx 3 \times 10^{-9}$  erg/sec/cm<sup>2</sup> towards ( $l = 309^\circ; b = +45^\circ$ ). In our notation  $a_{1m,LSS}/a_{00} = D/(4\pi\bar{I})$ . We see from Table 1 that the observed residual LSS dipole is within the range of our model predictions for the LSS dipole. A more detailed estimation of the HEAO1 dipole is underway (Scharf et al., in preparation).

## 5. Discussion

This paper gives quantitative predictions for the fluctuations on large angular scales in the X-ray Background. The rms predictions are based on assumed power-spectrum and evolution scenarios, and are expressed in spherical harmonics. We stress that any application to whole-sky XRB maps (such as HEAO1 and ROSAT) requires careful treatment of the shot-noise (by removing bright sources) and the smearing by the beam size. Another major observational obstacle (in particular for estimating the quadrupole in the soft band) is Galactic emission, but it can be corrected for by inversion and filtering techniques.

Our main conclusions are:

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\* When compared with e.g. the motion of the Local Group relative to the CMB ( $\approx 600$  km/sec), one should take a filtering scale  $R_*$  of a few Mpc, leading to lower predicted rms velocities.

(i) The XRB is an important probe of density fluctuations on scales intermediate between scales explored by galaxy redshift surveys and by COBE.

(ii) For a range of cosmological and evolution models the shape of the harmonic spectrum drops with  $l$  like  $a_{lm} \sim l^{-0.4}$ . The amplitude of the harmonics is mainly sensitive to the redshift evolution the X-ray volume emissivity  $\rho_x(z) = \rho_{x0} (1+z)^q$ . We show that for some models (e.g.  $q = 4$ ) the signal is comparable to the shot-noise, while for others (e.g.  $q = 0$ ), the signal-to-noise ratio is  $\sim 10$ .

The assumed power-spectra and maximal redshift little affect the predictions.

(iii) For realistic models, the harmonic amplitudes for  $l < 10$  are above the shot-noise level (which is constant with  $l$ ), provided that bright resolved sources are removed from the XRB map.

(iv) Sachs-Wolfe and Doppler effects in the XRB are negligible compared to the clustering signal.

(v) The expected dipole amplitude due to large scale structure is comparable to the Compton-Getting dipole amplitude due to the observer's motion, and is consistent with a recently measured dipole in the HEAO1 XRB map. Unfortunately, the coupling of the two effects makes it difficult to detect the CG dipole in the XRB independently (unless the LSS dipole amplitude is predicted from a model or by extrapolation from higher harmonics).

As illustrated in this paper, important cosmological information can therefore be obtained by analysing whole-sky XRB maps, in particular the HEAO1 (2-10 keV) and ROSAT (0.5-2.0) surveys.

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## Appendix A: The Sachs-Wolfe and Doppler effects for the X-ray Background

The fluctuations in the gravitational potential and velocity field at the time the XRB is produced yield additional variations. Ignoring non-linear effects, if the fluctuations are small they are additive and we can modify Eq. (9) to be (for  $l > 0$ ):

$$a_{lm} = \frac{1}{4\pi} \rho_{x0} \frac{c}{H_0} \int \int d\omega dz (1+z)^{q-7/2} \left[ b_x \delta + (3+\alpha) \left( \frac{\delta\phi}{3c^2} + \frac{(\mathbf{V} - \mathbf{V}_{obs}) \cdot \hat{\mathbf{r}}}{c} \right) \right] Y_{lm}^*(\hat{\mathbf{r}}), \quad (\text{A1})$$

where  $\delta\phi$  is the perturbation in the potential,  $\mathbf{V}$  is the peculiar velocity of the XRB sources,  $\mathbf{V}_{obs}$  is the velocity of the observer in the same reference frame as  $\mathbf{V}$  and  $\alpha$  is the spectral energy distribution index. The factor  $(3+\alpha)$  was inserted according to the Compton-Getting formula (eq. 19) for the observer's motion. The same factor applies to the motion of the XRB sources (Doppler) and to the gravitational redshift (Sachs-Wolfe) effect as they are all redshift effects. The division of the effect between Sachs-Wolfe and Doppler which we have written here is in the synchronous gauge. This division is gauge dependent but the total result is not (Padmanabhan, 1993).

We shall assume  $\Omega = 1$  and linear theory. We expand  $\delta$  and  $\delta\phi$  in Fourier series and relate them via Poisson equation:

$$\delta\phi_{\mathbf{k}} = \frac{3}{2} \frac{H^2 a^2}{k^2} \delta_{\mathbf{k}}(z) = \frac{3}{2} H_0^2 (1+z) \frac{\delta_{\mathbf{k}}(z)}{k^2}. \quad (\text{A2})$$

The last equality follows as  $(Ha)^2 = H_0^2(1+z)$  in an Einstein de-Sitter universe.

The line-of-sight velocity can be written in terms of spherical harmonics and  $\delta_{\mathbf{k}}$  by decomposing the potential into Fourier components and using Rayleigh's expansion and linear theory (e.g. Fisher, Scharf & Lahav 1994):

$$U(\mathbf{r}) = \frac{Ha}{2\pi^2} \sum_{lm} (i^l)^* \int d^3k \frac{\delta_{\mathbf{k}}(z)}{k} j_l'(kr) Y_{lm}^*(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{r}}) \quad (\text{A3})$$

where  $j_l'(kr) = dj_l(kr)/d(kr)$  is the first derivative of the Bessel function.

Following the analysis in Section 2, with  $\delta_{\mathbf{k}}(z) = \delta_{\mathbf{k}}(0) (1+z)^{-1}$  and  $\rho_x(z) = \rho_{x0}(1+z)^q$ , we can write Eq. (A1) as:

$$a_{lm} = (i^l)^* \frac{1}{2\pi^2} \frac{\rho_{x0}}{4\pi} \int d^3k Y_{lm}^*(\hat{\mathbf{k}}) \delta_{\mathbf{k}} \left[ b_x \frac{c}{H_0} \Psi_{l,\delta} + \frac{(3+\alpha) H_0}{2k^2} \frac{H_0}{c} \psi_{l,SW} + \frac{(3+\alpha)}{k} \psi_{l,D} \right] - \frac{4\pi}{3} (3+\alpha) \frac{|\mathbf{V}_{obs}|}{c} \bar{I} Y_{lm}^*(\hat{\mathbf{v}}_{obs}) \delta_{l1} \quad (\text{A4})$$

The last term is due to the motion of the observer (cf. Compton-Getting eq. 19), where  $\bar{I}$  is the mean intensity (eq. 8), and  $\hat{\mathbf{v}}_{obs}$  is the direction of motion of the observer.

The window functions (under the assumption that fluctuations only grow within the horizon) are:

$$\Psi_{l,\delta}(k) \equiv \int_{z_{min}}^{z_{max}} dz (1+z)^{q-9/2} j_l(kr_c), \quad (\text{A5})$$

$$\Psi_{l,SW}(k) \equiv \int_{z_{min}}^{z_{max}} dz (1+z)^{q-7/2} j_l(kr_c), \quad (A6)$$

and

$$\Psi_{l,D}(k) \equiv \int_{z_{min}}^{z_{max}} dz (1+z)^{q-4} j'_l(kr_c). \quad (A7)$$

Finally, taking the mean square value of (A4) we obtain, for the first three terms:

$$\langle |a_{lm}|^2 \rangle = \frac{\rho_{x0}^2}{(2\pi)^3} \int dk k^2 P(k) \left[ b_x \frac{c}{H_0} \Psi_{l,\delta} + \frac{(3+\alpha) H_0}{2k^2} \frac{H_0}{c} \psi_{l,SW} + \frac{(3+\alpha)}{k} \psi_{l,D} \right]^2 \quad (A8)$$

The interpretation of the 3 terms that multiply  $k^3 P(k) \sim \langle (\frac{\delta\rho}{\rho})^2 \rangle$  can be understood as follows. Apart from the  $k$ -dependence of the window functions, the first term squared is constant with  $k$ , the second term (Sachs-Wolfe) squared scales like  $k^{-4}$  and the third term (Doppler) squared scales like  $k^{-2}$ . Hence they represent contributions from small, large and intermediate scales, respectively.

In addition to those terms we obtain three mixed terms that arise from ‘interferences’ between the different modes. Those terms depend on  $k^{-1}$ ,  $k^{-2}$  and  $k^{-3}$ . Figure 3 compares the window functions with and without the Sachs-Wolfe terms. Although the Sachs-Wolfe and Doppler effects change the window functions on very large scales, their contribution to the derived rms  $a_{lm}$  integral for the power-spectra considered are tiny relative to the  $a_{lm}$  arising from density fluctuations. The difference in  $a_{lm}/a_{00}$  is no more than 0.1 % over the harmonics range  $1 \leq l \leq 10$ . Therefore, the Sachs-Wolfe and Doppler effects can safely be ignored in our analysis.

**Table 1.** Dipole and Quadrupole moments in the X-ray Background

	Dipole $a_{1m}/a_{00}$	Quadrupole $a_{2m}/a_{00}$
Expected velocity dipole from COBE	$1.4 \times 10^{-3}$	
rms velocity CDM	$3.7 \times 10^{-3}$	
rms velocity LDCDM	$4.3 \times 10^{-3}$	
Observed HEAO1 LSS dipole	$4.6 \times 10^{-3}$	
rms LSS CDM ( $q = 0, z_{max} = 5$ )	$8.2 \times 10^{-3}$	$6.5 \times 10^{-3}$
rms LSS LDCDM ( $q = 0, z_{max} = 5$ )	$9.3 \times 10^{-3}$	$7.2 \times 10^{-3}$
rms LSS CDM ( $q = 4, z_{max} = 5$ )	$4.8 \times 10^{-4}$	$3.8 \times 10^{-4}$
rms LSS LDCDM ( $q = 4, z_{max} = 5$ )	$5.7 \times 10^{-4}$	$4.6 \times 10^{-4}$
rms LSS CDM ( $q = 4, z_{max} = 3$ )	$7.0 \times 10^{-4}$	$5.6 \times 10^{-4}$
rms LSS LDCDM ( $q = 4, z_{max} = 3$ )	$8.3 \times 10^{-4}$	$6.6 \times 10^{-4}$
rms Shot Noise (hard band)	$8.0 \times 10^{-4}$	$8.0 \times 10^{-4}$

Comments:

(i) The predicted velocity-induced dipole is based on the interpretation of the COBE dipole being due to the motion of the Sun at 369 km/sec relative to the CMB.

(ii) The rms velocity is calculated in linear theory for a point, assuming either Cold Dark Matter (CDM) or Low Density CDM (LDCDM) power-spectra. The value scales like the product  $\sigma_8 \Omega^{0.6}$ , taken here to be unity.

(iii) The observed HEAO1 dipole is from Jahoda (1993), after correcting for Galactic emission and the velocity-induced dipole.

(iv) The predictions due to large scale structure (LSS) assume either CDM or LDCDM power-spectra (normalized with  $b_x \sigma_8 = 1$ ) in  $\Omega = 1$  universe. The perturbations are assumed to grow like  $\delta(z) \propto (1+z)^{-1}$  and the comoving emissivity to evolve like  $\rho_x(z) \propto (1+z)^q$ , given here for  $q = 0$  and  $q = 4$  out to redshift  $z_{max} = 5$  or 3 (with  $z_{min} = 0$ ).

(v) The shot noise was estimated for the hard band using:  $N_0 \approx 2 \times 10^{-15}$  (erg/sec  $\text{cm}^{-2}$ )<sup>3/2</sup> str<sup>-1</sup> and  $f_m = 3 \times 10^{-11}$  erg/sec  $\text{cm}^{-2}$ , above which sources were identified (Piccinotti et al. 1982) and an observed hard-band mean intensity (Boldt 1987)  $\bar{I} = a_{00}/\sqrt{4\pi} = 5.2 \times 10^{-8}$  ergs/sec/ $\text{cm}^2$ /str.

## FIGURE CAPTIONS

**Figure 1:** The quadrupole window functions  $|\Psi_{l=2}(k)|^2$ , where

$$\langle |a_{lm}|^2 \rangle \propto \int dk k^2 P(k) |\Psi_l(k)|^2.$$

For the CMB,  $|\Psi_l(k)|^2 = k^{-4} j_l^2(2ck/H_0)$  is due to the Sachs-Wolfe effect. The quadrupole window function of the IRAS 1.2 Jy redshift survey is based on Fisher, Scharf & Lahav (1994), with a Gaussian radial function centred at 6000 km/sec with  $\sigma = 2000$  km/sec. That of the XRB is given by Eq. 16 in the text, assuming  $\Omega = 1$ ,  $q - \mu = 3$ ,  $z_{min} = 0$  and  $z_{max} = 5$ . The solid and dashed lines represent  $k^3 P(k)$  for a standard CDM model and the observed galaxy power spectrum (fitted by a low density CDM model) respectively.

**Figure 2:** XRB rms normalized harmonics  $a_{lm}/a_{00} b_x \sigma_8$ . The solid and dashed lines correspond to CDM and LDCDM models both with  $\Omega = 1$ , perturbation growth  $\delta \propto (1+z)^{-1}$  and evolution law  $\rho_x(z) \propto (1+z)^q$  out to  $z_{max} = 5$  with  $q = 4$ . The shot noise level normalized is shown for two flux limits above which sources are removed from the ROSAT map.

**Figure 3:** The Sachs-Wolfe and velocity effects on the quadrupole window function  $|\Psi_2(k)|^2$  (Eq. A8) are shown by the dashed lines. The solid lines represent the same functions when these effects are neglected. We assumed  $q - \mu = 3$  as in Fig. 1. As they only affect the largest scales, the SW and velocity effects are significantly reduced when weighted by  $k^3 P(k)$  (here a low density CDM model fitting the observed galaxy power spectrum) and their resulting contribution to the  $\langle |a_{lm}|^2 \rangle$  is negligible.