## <span id="page-0-0"></span>Roles of a coherent scalar field on the evolution of cosmic structures

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A coherently oscillating scalar field, an axion as an example, is known to behave as a cold dark matter. The arguments were usually made in the Newtonian context. Ratra proved the case in relativistic context using the synchronous gauge. In this paper we present another proof based on a more suitable gauge choice, the uniform-curvature gauge, which fits the problem. By a proper time averaging the perturbed oscillating scalar field behaves as a cold dark matter on the relevant scales including the superhorizon scale.

*1. Introduction:* There exists a large amount of literature concerning the role of a coherently oscillating scalar particle as a cold dark matter[[1–4\]](#page-3-0). Most of the arguments were based on the Newtonian or qualitative analyses [\[1](#page-3-0)]. A relativistic treatment can be found in[[2\]](#page-3-0), however, there also exist the mixed statements concerning the role [\[3](#page-3-0)]. B. Ratra presented a fully relativistic analyses based on the synchronous gauge choice [\[4](#page-3-0)]. In the synchronous gauge the analysis becomes unnecessarily complicated and one has to be careful of tracing the remaining gauge mode as the author of[[4\]](#page-3-0) did. In this paper we would like to derive the same results, now, based on a different gauge choice which simplifies both the analyses and the results. Results can be easily translated into solutions in any other gauge.

We consider a homogeneous, isotropic and flat cosmological model with general scalar type metric perturbations (we set  $c \equiv 1$ )

$$
ds^{2} = -(1+2\alpha) dt^{2} - a\beta_{,\alpha} dt dx^{\alpha}
$$

$$
+a^{2} [\delta_{\alpha\beta} (1+2\varphi) + 2\gamma_{,\alpha\beta}] dx^{\alpha} dx^{\beta}, \qquad (1)
$$

where  $\alpha(\mathbf{x},t)$ ,  $\beta(\mathbf{x},t)$ ,  $\varphi(\mathbf{x},t)$ , and  $\gamma(\mathbf{x},t)$  are perturbed order metric variables. We consider the model is supported by a coherently oscillating minimally coupled scalar field with  $V = \frac{1}{2}m^2\phi^2$ . The scale we are interested in is

$$
m \gg k_p \gg H,\tag{2}
$$

where  $k_p \equiv k/a$  and  $H \equiv \dot{a}/a$ . We have:

$$
\frac{m}{H} = 4.689 \times 10^{27} h^{-1} \left(\frac{m}{10^{-5} \text{eV}}\right) \left(\frac{t}{t_0}\right),
$$
  
\n
$$
\Gamma \equiv \frac{k_p^2}{mH} = 1.917 \times 10^{-15}
$$
  
\n
$$
\times \left(\frac{m}{10^{-5} \text{eV}}\right)^{-1} \left(\frac{1 \text{Kpc}}{k_{p0}^{-1}}\right)^2 \left(\frac{t}{t_0}\right)^{-1/3},
$$
 (3)

where  $H_0 \equiv 100h \text{km/secMpc}$  with a subindex 0 indicating the present epoch. In the following analysis we will strictly *ignore* H/m order higher terms. However, we will consider the *expansion* in  $\Gamma \equiv k_p^2/(mH)$ .

*2. Background evolution:* Since the scalar field oscillates with the frequency  $m$  which is much larger than the characteristic expansion rate of the universe  $H$ , the temporal averages of the oscillating scalar field will contribute to the background fluid quantities, and thus to the metric. The set of equations describing the background evolution with a general minimally coupled scalar field is given in Eqs. (5-6) of[[5\]](#page-3-0). Considering the time averaging we have:

$$
\mu = \frac{1}{2} \langle \dot{\phi}^2 + m^2 \phi^2 \rangle, \quad p = \frac{1}{2} \langle \dot{\phi}^2 - m^2 \phi^2 \rangle,
$$
  
\n
$$
H^2 = \frac{4\pi G}{3} \langle \dot{\phi}^2 + m^2 \phi^2 \rangle, \quad \dot{H} = -4\pi G \langle \dot{\phi}^2 \rangle,
$$
  
\n
$$
\ddot{\phi} + 3H\dot{\phi} + m^2 \phi = 0.
$$
 (4)

An angular bracket indicates an averaging over time scale of order  $m^{-1}$ . Ignoring  $H/m$  order higher terms we have an approximate solution,[[4\]](#page-3-0)

$$
\phi(t) = a^{-3/2} \Big[ \phi_{+0} \sin (mt) + \phi_{-0} \cos (mt) \Big], \tag{5}
$$

where  $\phi_{+0}$  and  $\phi_{-0}$  are constant coefficients. Equation (4) becomes:

$$
\mu = \langle m^2 \phi^2 \rangle = \langle \dot{\phi}^2 \rangle = \frac{1}{2} m^2 a^{-3} \left( \phi_{+0}^2 + \phi_{-0}^2 \right), \np = 0, \quad \langle \phi \dot{\phi} \rangle = 0, \n\dot{H} = -\frac{3}{2} H^2, \quad H^2 = \frac{8\pi G}{3} \mu, \quad a \propto t^{2/3}, \quad H = \frac{2}{3t}. \quad (6)
$$

Thus, the background medium behaves exactly like a pressureless ideal fluid [\[1](#page-3-0)].

*3. Perturbed equations in a gauge ready form:* A complete set of equations describing the perturbed evolution of a minimally coupled scalar field with a general potential is presented in a gauge ready form in[[5\]](#page-3-0). In our case, we should take into account of the fact that due to the rapid oscillation of the background scalar field, the perturbed part of the scalar field also oscillates, thus the properly time averaged quantities contribute to the evolution of the perturbed fluid quantities and metric. Perturbed fluid quantities are presented in Eqs. (7) of  $\vert 5 \vert$ :

$$
\varepsilon = \langle \dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\alpha + m^2\phi\delta\phi \rangle, \n\pi = \langle \dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\alpha - m^2\phi\delta\phi \rangle, \n\Psi = -\langle \dot{\phi}\delta\phi \rangle, \quad \sigma = 0.
$$
\n(7)

<span id="page-1-0"></span>Perturbed set of equations in a gauge ready form is presented in Eqs. (8-13) of[[5\]](#page-3-0). Considering the time averaging we have:

$$
\kappa = -3\dot{\varphi} + 3H\alpha + \frac{k^2}{a^2}\chi,\tag{8}
$$

$$
-H\kappa + \frac{k^2}{a^2}\varphi = 4\pi G\langle \dot{\phi}\delta\dot{\phi} - \dot{\phi}^2\alpha + m^2\phi\delta\phi \rangle, \tag{9}
$$

$$
\kappa - \frac{k^2}{a^2} \chi = 12\pi G \langle \dot{\phi} \delta \phi \rangle,\tag{10}
$$

$$
\alpha + \varphi = \dot{\chi} + H\chi,\tag{11}
$$

$$
\dot{\kappa} + 2H\kappa = \left(\frac{k^2}{a^2} - 4\pi G \langle \dot{\phi}^2 \rangle\right) \alpha + 8\pi G \langle 2\dot{\phi}\delta\dot{\phi} - m^2 \phi \delta\phi \rangle, \tag{12}
$$

$$
\delta \ddot{\phi} + 3H \delta \dot{\phi} + \left(\frac{k^2}{a^2} + m^2\right) \delta \phi = \dot{\phi} \left(\kappa + \dot{\alpha}\right) - \left(3H \dot{\phi} + 2m^2 \phi\right) \alpha.
$$
 (13)

 $\chi \equiv a(\beta + a\dot{\gamma})$  is a spatially gauge invariant combination. Every perturbed order variable in Eqs. (7-13) is spatially gauge-invariant, but depends on the temporal gauge condition. As a gauge fixing condition we can impose one condition in any of these temporally gauge dependent variables. Imposing any one of the following conditions is fine as a gauge condition:  $\alpha \equiv 0$  (synchronous gauge),  $\Psi \equiv 0$  (comoving gauge),  $\delta \phi \equiv 0$ (uniform-field gauge),  $\chi \equiv 0$  (zero-shear gauge),  $\varphi \equiv 0$ (uniform-curvature gauge),  $\kappa \equiv 0$  (uniform-expansion gauge),  $\varepsilon \equiv 0$  (uniform-density gauge), etc. The decision is up to us. Often the choice is made based on the author's taste. However, rather naturally, depending on the problem, the analysis in a certain gauge condition is more convenient than in the other. Sometimes, the advantage we get from the suitable gauge condition is exclusive. By writing the equations without choosing the gauge as in Eqs. (8-13) we can try different gauge conditions as we wish. We call this approach, which allows us to use the gauge issue not as a problem but as an advantage, a gauge ready method. The solutions in other gauge conditions can be derived from the known solutions in a gauge. Except for the synchronous gauge, any of other gauge conditions fixes the gauge mode completely. Thus, every variable in such a gauge has the corresponding gauge invariant combination. In this sense, the variables in such gauge conditions can be regarded as the equivalently gauge invariant ones. Our experience tells that the uniform-curvature gauge is convenient for problems involving the scalar field. In the following we will start by adopting the uniform-curvature gauge, or equivalently gauge invariant combinations.

*4. Analyses in the uniform-curvature gauge:* We impose the uniform-curvature gauge condition which takes  $\varphi \equiv 0$ . A gauge invariant combination is

$$
\delta\phi_{\varphi} \equiv \delta\phi - \frac{\dot{\phi}}{H}\varphi, \qquad (14)
$$

which becomes (thus, equivalent to)  $\delta\phi$  in the uniformcurvaturegauge. For  $\delta \phi_{\varphi}$  we take an *ansatz*, [[4\]](#page-3-0)

$$
\delta\phi_{\varphi}(\mathbf{k},t) = \delta\phi_{+}(\mathbf{k},t)\sin\left(mt\right) + \delta\phi_{-}(\mathbf{k},t)\cos\left(mt\right). \tag{15}
$$

In the analyses of the perturbed order equations we consider only the leading order terms in  $H/m$ . However, since we are considering the small scale limit compared with the horizon, we include higher order terms in the expansion of Γ. From Eqs.  $(8,10)$  we have

$$
\alpha_{\varphi} = \frac{4\pi G}{H} \langle \dot{\phi} \delta \phi_{\varphi} \rangle.
$$
 (16)

Using Eqs.  $(9,16)$ , Eq.  $(13)$  leads to

$$
\delta \ddot{\phi}_{\varphi} + 3H \delta \dot{\phi}_{\varphi} + \left(\frac{k^2}{a^2} + m^2\right) \delta \phi_{\varphi}
$$
  
= 
$$
-\frac{8\pi G}{H} m^2 \left(\phi \langle \dot{\phi} \delta \phi_{\varphi} \rangle + \dot{\phi} \langle \phi \delta \phi_{\varphi} \rangle\right)
$$
  
= 
$$
-\frac{3H}{\phi_{+0}^2 + \phi_{-0}^2} a^{3/2} \left[m\phi \left(\phi_{+0} \delta \phi_{-} - \phi_{-0} \delta \phi_{+}\right) + \dot{\phi} \left(\phi_{+0} \delta \phi_{+} + \phi_{-0} \delta \phi_{-}\right)\right],
$$
 (17)

where in the second step we used Eqs.  $(5.15)$ . Using Eqs. ([5,](#page-0-0)15), Eq. (17) leads to:

$$
t\delta\dot{\phi}_{-} + \frac{2\phi_{-0}^{2}}{\phi_{+0}^{2} + \phi_{-0}^{2}}\delta\phi_{-} = \left(-\frac{2\phi_{+0}\phi_{-0}}{\phi_{+0}^{2} + \phi_{-0}^{2}} + \frac{1}{3}\Gamma\right)\delta\phi_{+},
$$
  

$$
t\delta\dot{\phi}_{+} + \frac{2\phi_{+0}^{2}}{\phi_{+0}^{2} + \phi_{-0}^{2}}\delta\phi_{+} = \left(-\frac{2\phi_{+0}\phi_{-0}}{\phi_{+0}^{2} + \phi_{-0}^{2}} - \frac{1}{3}\Gamma\right)\delta\phi_{-}.
$$
  
(18)

The solution valid to second order in the *expansion* of Γ is (notice that  $\Gamma \propto t^{-1/3}$ )

$$
\delta\phi_{+}(\mathbf{k},t) = c_{1}(\mathbf{k}) \left( 1 + \frac{1}{5} \frac{\phi_{+0}}{\phi_{-0}} \Gamma + \frac{1}{10} \Gamma^{2} \right) \n+ c_{2}(\mathbf{k}) t^{-2} \left( 1 + \frac{1}{7} \frac{\phi_{-0}}{\phi_{+0}} \Gamma - \frac{1}{14} \Gamma^{2} \right), \n\delta\phi_{-}(\mathbf{k},t) = c_{1}(\mathbf{k}) \left( -\frac{\phi_{+0}}{\phi_{-0}} + \frac{1}{5} \Gamma - \frac{1}{10} \frac{\phi_{+0}}{\phi_{-0}} \Gamma^{2} \right) \n+ c_{2}(\mathbf{k}) t^{-2} \left( \frac{\phi_{-0}}{\phi_{+0}} - \frac{1}{7} \Gamma - \frac{1}{14} \frac{\phi_{-0}}{\phi_{+0}} \Gamma^{2} \right). (19)
$$

By the following identification of the coefficients

$$
C(\mathbf{k}) \equiv \frac{2}{3} \left( \frac{a^{3/2} t^{-1}}{\phi_{-0}} \right) \frac{1}{m} c_1(\mathbf{k}),
$$
  

$$
d(\mathbf{k}) \equiv \frac{27}{4} \left( \frac{a^{3/2} t^{-1}}{\phi_{+0}} \right) \left( \frac{H}{k_p} t^{1/3} \right)^2 c_2(\mathbf{k}),
$$
 (20)

<span id="page-2-0"></span>Eqs. $(15,19)$  $(15,19)$  lead to

$$
\delta\phi_{\varphi}(\mathbf{k},t) = -C(\mathbf{k})\frac{1}{H}\left(\dot{\phi} - \frac{1}{5}m\phi\Gamma + \frac{1}{10}\dot{\phi}\Gamma^{2}\right) + \frac{4}{27}d(\mathbf{k})t^{-5/3}\left(\frac{k_{p}}{H}\right)^{2}\left(\phi - \frac{1}{7}\frac{\dot{\phi}}{m}\Gamma - \frac{1}{14}\phi\Gamma^{2}\right).
$$
 (21)

In order to derive the complete set of solutions in the other gauge conditions we need solutions valid to second order in the expansion of  $\Gamma$ . Using the solution in Eq. (21) the other perturbed quantities follow from Eqs.([8-](#page-1-0) [12](#page-1-0)) as:

$$
\alpha_{\varphi} = -\frac{3}{2} \frac{H}{\mu} \Psi_{\varphi} = -\frac{3}{2} C \left( 1 + \frac{1}{10} \Gamma^2 \right) \n- \frac{2}{63} dt^{-5/3} \Gamma^2 \left( 1 + 0 \cdot \Gamma \right), \nH \chi_{\varphi} = \left( \frac{H}{k_p} \right)^2 \frac{\kappa_{\varphi}}{H} = -\frac{3}{2} \left( \frac{H}{k_p} \right)^2 \delta_{\varphi} \n= -\frac{3}{5} C \left( 1 + 0 \cdot \Gamma \right) - \frac{4}{9} dt^{-5/3} \left( 1 - \frac{1}{14} \Gamma^2 \right). \tag{22}
$$

In the pressureless limit the relativistic cosmological perturbation of an ideal fluid reduces to the Newtonian one. Newtonian hydrodynamics is described by the following quantities: the density, pressure, velocity, and gravitational potential. In Sec. 4 of [\[6](#page-3-0)] we made some arguments that in the Einstein gravity filled with an ideal fluid, the following gauge invariant combinations play the roles of the Newtonian relative density fluctuation  $(\delta \varrho/\varrho)$ , potential fluctuation  $(-\delta \Phi)$ , and velocity fluctuation  $(\delta v)$ , respectively:

$$
\delta_{\Psi} \equiv \frac{\varepsilon - 3H\Psi}{\mu}, \quad \varphi_{\chi} \equiv \varphi - H\chi,
$$
  

$$
v_{\chi} \equiv -\frac{k}{a} \frac{\Psi + (\mu + p)\chi}{\mu + p}.
$$
 (23)

 $\delta_{\Psi}$ ,  $\varphi_{\chi}$ , and  $v_{\chi}$  are the relative density fluctuation in the comoving gauge, the potential fluctuation in the zeroshear gauge, and the velocity fluctuation in the zero-shear gauge, respectively [\[7](#page-3-0)]. In our case, from Eq. (22), the gauge invariant combinations become:

$$
\delta_{\Psi} = \left(\frac{k_p}{H}\right)^2 \left[\frac{2}{5}C\left(1 + 0\cdot\Gamma\right) + \frac{8}{27}dt^{-5/3}\left(1 - \frac{1}{14}\Gamma^2\right)\right],
$$
  
\n
$$
\varphi_{\chi} = \frac{3}{5}C\left(1 + 0\cdot\Gamma\right) + \frac{4}{9}dt^{-5/3}\left(1 - \frac{1}{14}\Gamma^2\right),
$$
  
\n
$$
v_{\chi} = \frac{k_p}{H}\left[-\frac{2}{5}C\left(1 + 0\cdot\Gamma\right) + \frac{4}{9}dt^{-5/3}\left(1 - \frac{5}{42}\Gamma^2\right)\right].
$$
  
\n(24)

On the other hand, a thorough study of the evolution of perturbations in a pressureless ideal fluid was made in [[6\]](#page-3-0). From Table 2 of [\[6](#page-3-0)] we have (these solutions are valid in general scales):

$$
\delta_{\Psi} = \left(\frac{k_p}{H}t^{-1/3}\right)^2 \left(\frac{2}{5}Ct^{2/3} + \frac{8}{27}dt^{-1}\right),
$$
  
\n
$$
\varphi_{\chi} = \frac{3}{5}C + \frac{4}{9}dt^{-5/3},
$$
  
\n
$$
v_{\chi} = \left(\frac{k_p}{H}t^{-1/3}\right) \left(-\frac{2}{5}Ct^{1/3} + \frac{4}{9}dt^{-4/3}\right).
$$
 (25)

Thus, to the linear order in  $\Gamma$  Eq. (24) coincides with Eq. (25). Therefore, the perturbed part of the coherently oscillating scalar field behaves as a perturbed pressureless fluid, thus as a cold dark matter.

*5. Comparison with the general solutions in the largescale limit:* It is interesting to compare the solutions in Eqs. (21,22) with the general integral form solutions valid in the large scale limit  $(k_p \ll H)$ . A general equationfor  $\delta\phi_{\varphi}$  is derived in [[8\]](#page-3-0) as

$$
\delta \ddot{\phi}_{\varphi} + 3H \delta \dot{\phi}_{\varphi} + \left[ \frac{k^2}{a^2} + V_{,\phi\phi} + 2\frac{\dot{H}}{H} \left( 3H - \frac{\dot{H}}{H} + 2\frac{\ddot{\phi}}{\dot{\phi}} \right) \right] \delta \phi_{\varphi} = 0. \quad (26)
$$

This equation can be derived from Eqs.([8-10,13,](#page-1-0)[4\)](#page-0-0) for a general minimally coupled scalar field without the coherent oscillation. If we consider the coherent oscillation and the corresponding contribution of the time averaged quantities to the metric, the same equations instead will lead to Eq. [\(17](#page-1-0)). For  $k_p \ll H$  we have

$$
\delta\phi_{\varphi}(\mathbf{x},t) = -\frac{\dot{\phi}}{H} \left[ C(\mathbf{x}) - D(\mathbf{x}) \int_0^t \frac{H^2}{a^3 \dot{\phi}^2} dt \right].
$$
 (27)

Equations (26,27) are valid for an arbitrary  $V(\phi)$ . In Eq.  $(109)$ of  $[9]$  we proved that  $[10]$  $[10]$ 

$$
D(\mathbf{k}) = \frac{4}{3}k^2 \left(\frac{a}{t^{2/3}}\right) d(\mathbf{k}).
$$
 (28)

Thus, in the large scale limit the growing mode of Eq. (21) coincides with the one in Eq. (27). A complete set of the large scale asymptotic solutions is presented in[[5\]](#page-3-0). From the Table 1[[5\]](#page-3-0) we have [\[11](#page-3-0)]:

$$
\frac{H}{\mu}\Psi_{\varphi} = -\frac{2}{3}\alpha_{\varphi} = -\frac{H}{\dot{\phi}}\delta\phi_{\varphi} = \frac{1}{3}\delta_{\varphi} = -\frac{2}{9}\frac{\kappa_{\varphi}}{H} = C,
$$
  

$$
H\chi_{\varphi} = -\frac{3}{5}C - \bar{d}\frac{H}{a}.
$$
 (29)

The second term in  $\chi_{\varphi}$  corresponds to a decaying mode. The growing modes of  $\Psi_{\varphi}, \alpha_{\varphi}, \delta\phi_{\varphi}$ , and  $\chi_{\varphi}$  in Eq. (29) coincide with the ones in Eq. (22). However, behaviors of  $\delta_{\varphi}$  and  $\kappa_{\varphi}$  are different. Since we have derived Eqs. (21,22) based on Eq. [\(2](#page-0-0)) they are not necessarily valid in the  $k_p \ll H$  limit. Never the less, from Eq. (29) we can show that the gauge invariant combinations in Eq. (24) *remain valid* in the large scale limit; a proof in a more general ground can be found by comparing the <span id="page-3-0"></span>large scale solutions presented in the Table 1 of [5] with the exact solutions valid in the general scale presented in the Table 1 of [6]. Therefore, Eqs. [\(24](#page-2-0),[25\)](#page-2-0) are also valid in the superhorizon scale, which implies that, in this sense, the coherently oscillating scalar field behaves as a cold dark matter on general scales as long as we have  $\Gamma \ll 1$  (and  $H/m \ll 1$ ).

*6. Solutions in the other gauges:* From the complete set of solutions derived in the uniform-curvature gauge [Eqs. [\(21](#page-2-0),[22\)](#page-2-0)] we can derive the rest of the solutions in the other gauge conditions. This translation into other gauges can be done systematically using either the gauge transformation or various gauge invariant combinations; the latter method is much easier as shown in deriving Eq.([24\)](#page-2-0). In practice, it is convenient to derive any one variable in the gauge we are interested using any of the above methods, and then, to derive the rest of the perturbed metric and fluid variables using the fundamental set of equations in Eqs.([8-13](#page-1-0)). In the *zero-shear gauge* we set  $\chi \equiv 0$ . From Eqs. (17,19) of [12] we have  $\delta \phi_\chi \equiv \delta \phi - \dot{\phi} \chi = \delta \phi_\varphi - \dot{\phi} \chi_\varphi$  $\delta \phi_\chi \equiv \delta \phi - \dot{\phi} \chi = \delta \phi_\varphi - \dot{\phi} \chi_\varphi$  $\delta \phi_\chi \equiv \delta \phi - \dot{\phi} \chi = \delta \phi_\varphi - \dot{\phi} \chi_\varphi$ . Thus, from Eqs. ([21,22](#page-2-0)) we have

$$
\delta\phi_{\chi} = -\frac{2}{5H}C\left(\dot{\phi} - \frac{1}{2}m\phi\Gamma\right) +\frac{2}{3}dt^{-2/3}\left(\dot{\phi} + \frac{1}{3}m\phi\Gamma - \frac{5}{42}\dot{\phi}\Gamma^2\right).
$$
 (30)

The rest of the perturbed variables follow from Eqs.([8-](#page-1-0) [13](#page-1-0)). The *uniform-expansion gauge* takes  $\kappa = 0$ . From Eqs. (17,19) of [12] we have  $\delta\phi_{\kappa} \equiv \delta\phi + \dot{\phi}(3\dot{H} - k_p^2)^{-1}\kappa$ . Since  $H^2/k_p^2 = (H/m)/\Gamma$ , we neglect  $H^2$  term compared with $k_p^2$  one. From Eqs. ([22\)](#page-2-0) we have  $\chi_{\varphi} = \kappa_{\varphi}/k_p^2$ , thus we have  $\delta \phi_{\kappa} = \delta \phi - \dot{\phi}(\kappa / k_p^2) = \delta \phi - \dot{\phi} \chi = \delta \phi_{\chi}$ . Thus, the solution in Eq.  $(30)$  is also valid in the uniformexpansion gauge. The *synchronous gauge* takes  $\alpha = 0$ . From Eqs. (17,19) of [12] we have  $\delta\phi_{\alpha} \equiv \delta\phi - \dot{\phi} \int^t \alpha dt$ . The combination with a subindex  $\alpha$  is not gauge invariant. The lower bound of the integration in the right hand side leads to the behavior of the gauge mode. From Eqs. ([21,22\)](#page-2-0) we can derive

$$
\delta\phi_{\alpha} = \left(\frac{k_p}{H}\right)^2 \left[\frac{1}{5}C\left(\phi + \frac{\dot{\phi}}{m}\Gamma\right) + \frac{4}{27}dt^{-5/3}\left(\phi - \frac{1}{4}\frac{\dot{\phi}}{m}\Gamma - \frac{1}{14}\phi\Gamma^2\right)\right] + \text{const.} \times \dot{\phi}, \quad (31)
$$

where the last term is the gauge mode.

*7. Discussions:* When we have  $H/m \ll 1$  the homogeneous part of a coherently oscillating scalar field behaves as a pressureless medium  $(\S2)$ . In such a background, we have shown that on scales satisfying  $\Gamma \ll 1$  the perturbed part of a coherently oscillating scalar field behaves as a cold dark matter  $(\S4)$ . We have shown that this conclusion is valid even for scales larger than the horizon (§5). We followed the method suggested in [4], but took the uniform-curvature gauge which allows simpler analysis.

Solutions in any other gauge can be derived easily (§6). We expect the equations and results presented above will be useful for handling the reheating process based on the coherently oscillating scalar field [13]. Applications to the reheating process will be presented elsewhere.

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- [10] We note that d used in [9], let's denote it as  $\bar{d}$ , differs from our d as  $\bar{d} = \frac{2}{3} (a/t^{2/3})d$ .
- [11] As shown in Eq.  $(28)$  the decaying mode in Eq.  $(27)$  is higher order in the large scale expansion compared with the solutions in the other gauges, thus disappearing in Eq.([29\)](#page-2-0) which is valid in the large scale limit.
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