

Theories of the Cosmological Constant

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Abstract — This is a talk given at the conference *Critical Dialogues in Cosmology* at Princeton University, June 24–27, 1996. It gives a brief summary of our present theoretical understanding regarding the value of the cosmological constant, and describes how to calculate the probability distribution of the observed cosmological constant in cosmological theories with a large number of subuniverses (i. e., different expanding regions, or different terms in the wave function of the universe) in which this constant takes different values.

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1 Introduction

The problem of the cosmological constant looks different to astronomers and particle physicists. Astronomers may prefer the simplicity of a zero cosmological constant, but they are also prepared to admit the possibility of a cosmological constant in a range extending up to values that would make up most of the critical density required in a spatially flat Robertson–Walker universe. To a particle physicist, all the values in this observationally allowed range seem ridiculously implausible.

To see why, it is convenient to consider the effective quantum field theory that takes into account only degrees of freedom with energy below about 100 GeV, with all higher energy radiative corrections buried in corrections to the various parameters in the effective Lagrangian. In this effective field theory, the vacuum energy density that serves as a source of the long-range gravitational field may be written as

$$\rho_V = \frac{\Lambda}{8\pi G} + \frac{1}{2} \sum \hbar\omega, \quad (1)$$

where Λ is the cosmological constant appearing in the Einstein field equations, and the second term symbolizes the contribution of quantum fluctuations in the fields of the effective field theory, cut off at particle energies equal to 100 GeV. Now, we know almost everything about this effective field theory — it is what particle physicists call the standard model — and we know that the quantum fluctuations do not cancel, so that on dimensional grounds, in units with $\hbar = c = 1$, they yield

$$\frac{1}{2} \sum \hbar\omega \approx (100 \text{ GeV})^4 \quad (2)$$

On the other hand, observations do not allow ρ_V to be much greater than the critical density, which in these units is roughly 10^{-48} GeV^4 . Not to worry — just arrange that the Einstein term Λ has a value for which the two terms in Eq. (1) cancel to fifty-six decimal places. This is the cosmological constant problem: to understand this cancellation.

Here I will consider three main directions for solving this problem[1]:

- Deep Symmetries
- Cancellation Mechanisms
- Anthropic Constraints

By a ‘deep symmetry’ I mean some new symmetry of an underlying theory, which is *not* an unbroken symmetry of the effective field theory below 100 GeV (because we know all these symmetries), but which nevertheless requires ρ_V to vanish. In other contexts supersymmetry can sometimes play the role of a deep symmetry, in the sense that some dimensionless bare constants that are required

to vanish by supersymmetry can be shown to vanish to all orders in perturbation theory even though supersymmetry is spontaneously broken. Unfortunately the vacuum density is not a constant of this sort — it has dimensionality (mass)⁴ instead of being dimensionless, and it is a renormalized coupling rather than a bare coupling. Recently Witten has proposed a highly imaginative and speculative mechanism by which some form of supersymmetry makes ρ_V vanish[2]. I am grateful to the organizing committee of this conference for giving me only 15 minutes to talk, so that I don't have to try to explain Witten's idea. I turn instead to the other two approaches on my list.

2 Cancellation Mechanisms

The special thing about having $\rho_V = 0$ is that it makes it possible to find spacetime-independent solutions of the Einstein gravitational field equations. For such solutions, we have

$$\partial\mathcal{L}/\partial g_{\mu\nu} = 0, \quad (3)$$

where \mathcal{L} is the Lagrangian density for constant fields. The problem occurs only in the trace of this equation, which receives a contribution from ρ_V which for $\rho_V \neq 0$ prevents a solution. Many theorists have tried to get around this difficulty by introducing a scalar field ϕ in such a way that the trace of $\partial\mathcal{L}/\partial g_{\mu\nu}$ is proportional to $\delta\mathcal{L}/\delta\phi$:

$$g_{\mu\nu}\partial\mathcal{L}/\partial g_{\mu\nu} = f(\phi)\delta\mathcal{L}/\delta\phi, \quad (4)$$

with $f(\phi)$ arbitrary, except for being finite. Where this is done, the existence of a solution of the field equation $\delta\mathcal{L}/\delta\phi = 0$ for a spacetime-independent ϕ implies that the trace $g_{\mu\nu}\partial\mathcal{L}/\partial g_{\mu\nu} = 0$ of the Einstein field equation for a spacetime-independent metric is also satisfied. The trouble is that, with these assumptions, the Lagrangian has such a simple dependence on ϕ that it is not possible to find a solution of the field equation for ϕ . This is because Eq. (4), together with the general covariance of the action $\int d^4x \mathcal{L}$, tells us that, when the action is stationary with respect to variations of all other fields, it has a symmetry under the transformations

$$\delta g_{\lambda\nu} = 2\epsilon g_{\lambda\nu}, \quad \delta\phi = -\epsilon f(\phi), \quad (5)$$

which requires the Lagrangian density for spacetime-independent fields $g_{\mu\nu}$ and ϕ to have the form

$$\mathcal{L} = c \sqrt{\det g} \exp\left(4 \int^{\phi} \frac{d\phi'}{f(\phi')}\right), \quad (6)$$

where c is a constant whose value depends on the lower limit chosen for the integral. For $c \neq 0$, there is no solution at which this is stationary with respect

to ϕ . The literature is full of proposed solutions of the cosmological constant problem based on this sort of spontaneous adjustment of one or more scalar fields, but if you look at them closely, you will see that either they do not satisfy Eq. (4), in which case there may be a solution for ϕ but it does not imply the vanishing of ρ_V , or else they do satisfy Eq. (4), in which case a solution of the field equation for ϕ would imply a vanishing ρ_V , but there is no solution of the field equation for ϕ . To the best of my knowledge, no one has found a way out of this impasse.

3 Anthropic Considerations

Suppose that the observed subuniverse is only one of many subuniverses, in which ρ_V takes a variety of different values. This is the case for instance in theories of chaotic inflation[3], in which various scalar fields on which the vacuum energy depends take different values in different expanding regions of space. In a somewhat more subtle way, this can also be the case in some versions of quantum cosmology, where the wave function of the universe is a superposition of terms in which ρ_V takes different values, either because of the presence of some vacuum field (like the antisymmetric tensor gauge field $A_{\mu\nu\lambda}$ introduced for this purpose by Hawking[4]), or because of wormholes, as in the work of Coleman[5].

Some authors[4], [6], [7] have argued that in quantum cosmology the distribution of values of ρ_V is very sharply peaked at $\rho_V = 0$, which would immediately solve the cosmological constant problem. This conclusion has been challenged[8], and it will be assumed here that the probability distribution of ρ_V is smooth at $\rho_V = 0$, without any sharp peaks or dips.

In any theory of this general sort the measured effective cosmological constant would be much smaller than the value expected on dimensional grounds in elementary particle physics, not because there is any physical principle that makes it small in all subuniverses, but because it is only in the subuniverses where it is sufficiently small that there would be anyone to measure it. For negative values of ρ_V , this limitation comes from the requirement that the subuniverse must survive long enough to allow for the evolution of life[9]. For positive values of ρ_V (which are observationally more promising) the limitation comes from the requirement that large gravitational condensations like galaxies must be able to form before the subuniverse begins its final exponential expansion[10].

If you don't find this sort of anthropic explanation palatable, consider the following fable. You are an astronaut, sent out to explore a randomly chosen planet around some distant star, about which nothing is known. Shortly before you leave you learn that because of budget cuts, NASA has not been able to supply you with any life-support equipment to use on the planet's surface. You arrive on the planet, and find to your relief that conditions are quite tolerable —

the air is breathable, the temperature is about 300° K, and the surface gravity is not very different from what it is on earth. What would you conclude about the conditions on planets in general? It all depends on how many astronauts NASA has sent out. If you are the only one then it's reasonable to infer that tolerable conditions must be fairly common, contrary to what planetologists would have naturally expected. On the other hand, if NASA has sent out a million astronauts, then all you can conclude about the statistics of planetary conditions is that the number of planets with tolerable conditions is probably not much less than one in a million — for all you know, almost all of the astronauts have arrived on planets that cannot support human life. Naturally, the only astronauts in this program that are in a position to think about the statistics of planetary conditions are those like you who are lucky enough to have landed on a planet on which they can live; the others are no longer worrying about it.

In previous work[10] I calculated the anthropic *upper bound* on the cosmological constant, which arises from the condition that ρ_V should not be so large as to prevent the formation of gravitational condensations on which life could evolve. This bound is naturally larger than the *average* value of the cosmological constant that would be measured by typical observers, which obviously gives a better estimate of what we might find in our subuniverse. (Vilenkin[11] has advocated this point of view under the name of the ‘principle of mediocrity’, but did not attempt a detailed analysis of its consequences.) The difference is important, because the anthropic upper bound on ρ_V is considerably larger than the largest value of ρ_V allowed by observation.

I will leave the observational limits on the cosmological constant to Dr. Fukugita’s talk, but without going into details, it seems that for a spatially flat (i.e., $k = 0$) universe, ρ_V is likely to be positive and somewhat larger than the present mass density ρ_0 , but probably not larger than $3\rho_0$ [12]. On the other hand, we know that some galaxies were already formed at redshifts $z \approx 4$, at which time the density of matter was larger than the present density ρ_0 by a factor $(1 + z)^3 \approx 125$. It therefore seems unlikely that a vacuum energy density much smaller than $125\rho_0$ could have completely prevented the formation of galaxies, so the anthropic upper bound on ρ_V cannot be much less than about $125\rho_0$, which is much greater than the largest observationally allowed value of ρ_V .

In contrast, we would expect the anthropic *mean* value of ρ_V to be roughly comparable to the mass density of the universe at the time of the greatest rate for the accretion of matter by growing galaxies, because it is unlikely for ρ_V to be much greater than this and there is no reason why it should be much smaller. (I will make this more quantitative soon.) Although there is evidence that galaxy formation was well under way by a redshift $z \approx 3$, it is quite possible that most accretion of matter into galaxies continues to lower redshifts, as seems to be indicated by cold dark matter models. In this case the anthropic mean value $\langle \rho_V \rangle$ will be considerably less than the anthropic upper bound, and perhaps within the range allowed observationally.

I would like to present an illustrative example of a calculation of the whole probability distribution of the cosmological constant that would be measured by observers, weighted by the likelihood that there are observers to measure it. Instead of the very simple model[13] of galaxy formation from spherically symmetric pressureless fluctuations used previously[10], here I will rely on the well-known model of Gunn and Gott[14], which also assumes spherical symmetry and zero pressure, but takes into account the infall of matter from outside the initially overdense core. This is still far from realistic, but it will allow me to make four points about such calculations, which should be more generally applicable.

As shown in earlier work[10], the condition for a spherically symmetric fluctuation to recondense is that

$$\frac{500 (\Delta\rho)^3}{729 \rho^2} > \rho_V . \quad (7)$$

where ρ and $\Delta\rho$ are the average density and the overdensity in the fluctuation at some early initial time, say the time of recombination. Previously $\Delta\rho$ was assumed to be uniform within a spherical fluctuation, but Eq. (7) actually applies to any sphere, with $\Delta\rho$ understood to be the spatially averaged initial overdensity within the sphere.

Suppose that the fluctuation at recombination consists of a finite spherical core of volume V with positive average overdensity $\delta\rho$, outside of which the density takes its average value ρ . (This picture is appropriate for well separated fluctuations. The effects of crowding and underdense regions will be considered in a future paper.) Then the average overdensity within a larger volume V' centered on this core is $\Delta\rho = \delta\rho V/V'$. Assuming that Eq. (7) is satisfied by the average overdensity $\delta\rho$ within the core,

$$\left. \frac{500 (\delta\rho)^3}{729 \rho^2} \right|_{\text{recomb}} > \rho_V , \quad (8)$$

the average overdensity $\Delta\rho$ will satisfy the condition (7) out to a volume

$$V_{\text{max}} = \left(\frac{500}{729 \rho_V} \right)^{1/3} \rho^{-2/3} \delta\rho V$$

so the total mass that will eventually collapse is

$$M = \delta\rho V + \rho V_{\text{max}} = V \delta\rho \left[1 + \left(\frac{500 \rho}{729 \rho_V} \right)^{1/3} \right] . \quad (9)$$

Once a galaxy forms, the subsequent evolution of stars and planets and life is essentially independent of the cosmological constant (*this is point 1*), so the number of independent observers arising from a given fluctuation at the time of

recombination is proportional to the mass (9) for those fluctuations satisfying Eq. (8), and is otherwise zero. Of course, the value of the cosmological constant might be correlated with the values of other fundamental constants, on which the evolution of life does depend, but the range of anthropically allowed cosmological constants is so small compared with the natural scale (2) of densities in elementary particle physics that within this range it is reasonable to suppose that all other constants are fixed. (*This is point 2.*) The range of values of ρ_V for which gravitational condensations are possible is also so much less than the average density at the time of recombination, that the number of fluctuations $\mathcal{N}(\delta\rho, V) dV d\delta\rho$ with volume between V and $V + dV$ and average overdensity between $\delta\rho$ and $\delta\rho + d\delta\rho$ should be nearly independent of ρ_V . (*This is point 3.*) If $\mathcal{P}(\rho_V) d\rho_V$ is the *a priori* probability that a random subuniverse has vacuum energy density between ρ_V and $\rho_V + d\rho_V$, then according to the principles of Bayesian statistics, the probability distribution for *observed* values of ρ_V is

$$\begin{aligned} \mathcal{P}_{\text{obs}}(\rho_V) &\propto \mathcal{P}(\rho_V) \int_0^\infty dV \int_{(729\rho_V\rho^2/500)^{1/3}}^\infty d\delta\rho \mathcal{N}(\delta\rho, V) \\ &\quad \times V\delta\rho \left[1 + \left(\frac{500\rho}{729\rho_V} \right)^{1/3} \right] \\ &\propto \mathcal{P}(\rho_V) \left[1 + \left(\frac{500\rho}{729\rho_V} \right)^{1/3} \right] \int_{(729\rho_V\rho^2/500)^{1/3}}^\infty d\delta\rho \mathcal{N}(\delta\rho)\delta\rho \end{aligned} \quad (10)$$

where

$$\mathcal{N}(\delta\rho) \equiv \int_0^\infty dV V \mathcal{N}(\delta\rho, V). \quad (11)$$

Finally, the range of values of ρ_V for which gravitational condensations are possible is so small compared with the natural scale of densities in elementary particle physics that within this range the *a priori* probability $\mathcal{P}(\rho_V)$ may be taken as constant. (*This is point 4.*) The factor $\mathcal{P}(\rho_V)$ may therefore be omitted in the probability distribution (10). Also, all anthropically allowed values of ρ_V are much smaller than the mass density ρ at recombination, so we may neglect the 1 in the square brackets in Eq. (10), which now becomes

$$\mathcal{P}_{\text{obs}}(\rho_V) \propto \rho_V^{-1/3} \int_{(729\rho_V\rho^2/500)^{1/3}}^\infty d\delta\rho \mathcal{N}(\delta\rho)\delta\rho. \quad (12)$$

Strictly speaking, this gives the probability distribution only for $\rho_V > 0$. For $\rho_V < 0$ and $k = 0$, all mass concentrations that are large enough to allow pressure to be neglected will undergo gravitational collapse. The number of astronomers is instead limited[9] for $\rho_V < 0$ by the fact that the subuniverse itself also collapses, in a time

$$T(|\rho_V|) = \frac{2\pi}{3} \sqrt{\frac{3}{8\pi G|\rho_V|}}. \quad (13)$$

In contrast, the probability distribution for $\rho_V > 0$ is weighted by an ρ_V -independent factor, the average time \mathcal{T} in which stars provide conditions favorable for intelligent life. The probability distribution for negative values of ρ_V is small except for values of $|\rho_V|$ that are small enough so that $T(|\rho_V|)$ is less than or of order \mathcal{T} . It will be assumed here that \mathcal{T} is very large, so that $\mathcal{P}_{\text{obs}}(\rho_V)$ is negligible for $\rho_V < 0$ except in a small range near zero, and may therefore be neglected in calculating the mean value of ρ_V .

Using the probability distribution (12) and interchanging the order of the integrals over $\delta\rho$ and ρ_V , we easily see that the mean value of *observed* values of ρ_V is

$$\langle \rho_V \rangle = \frac{200 \langle \delta\rho^6 \rangle}{729 \langle \delta\rho^3 \rangle \rho^2}, \quad (14)$$

with all quantities on the right-hand side evaluated at the time of recombination, and the brackets on the right-hand side (unlike those in $\langle \rho_V \rangle$) indicating averages over fluctuations:

$$\langle f(\delta\rho) \rangle \equiv \int_0^\infty d\delta\rho \mathcal{N}(\delta\rho) f(\delta\rho). \quad (15)$$

It remains to use astronomical observations to calculate the fluctuation spectrum $\mathcal{N}(\delta\rho)$ for the density fluctuations at recombination, which can then be used in Eq. (12) to calculate the probability distribution for ρ_V . Here I will just give one example of how information about the time of formation of galaxies can put constraints on $\langle \rho_V \rangle$. With a positive ρ_V , the core of a fluctuation with average overdensity $\delta\rho$ at recombination will collapse at a time when the average cosmic density ρ_{coll} is less than it would be at the time of core collapse for $\rho_V = 0$: [10]

$$\rho_{\text{coll}} < \frac{500 \delta\rho^3}{243 \pi^2 \rho^2}, \quad (16)$$

with ρ and $\delta\rho$ on the right-hand side evaluated at recombination. Using this in Eq. (14) gives a mean vacuum density

$$\langle \rho_V \rangle > \frac{2\pi^2 \langle \rho_{\text{coll}} \rangle}{15}. \quad (17)$$

Even if we suppose for example that core collapse occurs for most galaxies at a redshift as low as $z \approx 1$, then $\rho_{\text{coll}} \approx 8\rho_0$, so Eq. (19) gives $\langle \rho_V \rangle > 10\rho_0$, which exceeds current experimental bounds on ρ_V . On the other hand, the median value of ρ_V is less than the mean value, so the discrepancy is less than this. Even so, it seems that most galaxies must be formed quite late in order for the value of ρ_V in our universe to be close to the value that is anthropically expected.

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At the meeting in Princeton I learned of an interesting paper by Efstathiou [15], in which he calculated the effect of a cosmological constant on the present num-

ber density of L_* galaxies, which he took as a measure of the distribution function $\mathcal{P}_{\text{obs}}(\rho_V)$. In this calculation he adopted a standard cold dark matter model for matter density fluctuations, with amplitude at long wavelengths fixed by the measured anisotropy of the cosmic microwave background. Efstathiou found that for a spatially flat universe the galaxy density falls off rapidly (say, by a factor 10) for values of ρ_V around 7 to 9 times the present mass density ρ_0 , so that $\langle \rho_V \rangle / \rho_0$ should be less than of order 7 to 9, giving a contribution $\Omega_0 = \rho_0 / (\rho_0 + \rho_V)$ of matter to the total density somewhat greater than around 0.1, which is consistent with lower bounds on the present matter density.

At first sight this seems encouraging, but there are a few problems with Efstathiou's calculation. For one thing, as pointed out by Vilenkin[11], the probability distribution of observed values of ρ_V is related to the number of galaxies (or, more accurately, the amount of matter in galaxies) that *ever* form, rather than the number that have formed when the age of the universe is at any fixed value, as assumed by Efstathiou. However, this will not make much difference if most galaxy formation is complete in typical subuniverses when they are as old as our own subuniverse. Efstathiou also encountered another problem that is endemic to this sort of calculation. The cosmological parameters that can reasonably be assumed to be uncorrelated with the cosmological constant are the baryon-to-entropy ratio and the spectrum of density fluctuations at recombination, because these are presumably fixed by events that happened before recombination, when any anthropically allowed cosmological constant would have been negligible. But the only way we know about the spectrum of density fluctuations at recombination is to use observations of the present microwave background (or possibly the numbers of galaxies at various redshifts), and unfortunately the results we obtain from this for $\mathcal{N}(\delta\rho)$ depend on the value of the cosmological constant in *our* subuniverse. In calculating $\mathcal{P}_{\text{obs}}(\rho_V)$ one should ideally make some assumption about the value of ρ_V in our subuniverse, then use this value to infer a spectrum of density fluctuations at recombination from the observed microwave anisotropies, and then calculate the number of galaxies that ever form as a function of ρ_V , with the spectrum of density fluctuations at recombination held fixed. Instead, Efstathiou calculated the number of L_* galaxies as a function of ρ_V , with the microwave anisotropies held fixed, which gave $\mathcal{P}_{\text{obs}}(\rho_V)$ an additional spurious dependence on ρ_V . This problem was known to Efstathiou, and apparently did not produce large errors.

There is one other problem, that did have a significant effect in Efstathiou's calculation. He relied on the standard method[16] of calculating the evolution of density fluctuations using linear perturbation theory, and declaring a galaxy to have formed when the fractional overdensity $\Delta\rho/\rho$ reaches a value δ_c , which is taken as the fractional overdensity of the linear perturbation at a time when a nonlinear pressureless spherically symmetric fluctuation would recollapse to infinite density. He took the effective critical overdensity for spatially flat cosmologies as $\delta_c = 1.68/\Omega_0^{0.28}$, with $\Omega_0 \equiv 1 - \rho_V/\rho_{\text{crit}}$, so that $\delta_c = 3.2$ for $\Omega_0 = 0.1$. But numerical calculations of Martel and Shapiro[17] show that for all fluctua-

tions that result in gravitational recollapse, δ_c is in a range from 1.63 to 1.69. The upper bound 1.69 is the well-known result $\delta_c = (3/5)(3\pi/2)^{3/2} = 1.6865$ for $\rho_V = 0$. The lower bound 1.63 can also be understood analytically[10]: it is the critical overdensity for the case where ρ_V has a value that just barely allows gravitational recollapse

$$(\delta_c)_{\min} = \frac{2}{\sqrt{\pi}} \left(\frac{729}{500} \right)^{1/3} \Gamma \left(\frac{11}{6} \right) \Gamma \left(\frac{2}{3} \right) = 1.629 . \quad (18)$$

With δ_c always between these bounds, it is impossible that the effective value of δ_c for any ensemble of fluctuations could be greater than 1.69. Overestimating δ_c biases the calculation toward late galaxy formation, with a corresponding increased sensitivity to relatively small values of ρ_V . Efstathiou has now redone his calculations with δ_c given the constant value 1.68, which should be a good approximation, and, as I interpret his results, he finds that this change in δ_c roughly doubles the value of ρ_V at which the present density of L_* galaxies drops by a factor 10, with a corresponding reduction in the expected value of Ω_0 . It remains to be seen whether this change in his results will lead to a conflict with observational bounds on Ω_0 and ρ_V .

At present Martel and Shapiro are carrying out a numerical calculation of \mathcal{P}_{obs} using Eq. (12).

I am grateful for helpful discussions with George Efstathiou and Paul Shapiro.

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