Heating of a Star by Disk Accretion

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ABSTRACT

We examine various ways in which disk accretion can heat an accreting star. These include 1) radiation emitted from the disk surface which is intercepted by the stellar surface, 2) radiative flux directly across the disk-star interface, and 3) advection of thermal energy from the disk into the star. For each of these, the physics of the boundary layer between the disk and the star is crucial to determining the amount of stellar heating that occurs.

We assess the importance of the methods listed above for heating the star in accreting pre-main-sequence stars and cataclysmic variables, using recent models of boundary layers in these systems. We find that intercepted radiation tends to be the most important source of stellar heating in thin disk systems such as T Tauri stars and high- \dot{M} cataclysmic variables. We argue that direct radiation across the disk-star interface will be unimportant in steady-state systems. However, it may be important in outbursting systems, where the disk temperature rises and falls rapidly. Advection of thermal energy into the star becomes the dominant source of stellar heating in thick disk systems such as FU Orionis objects.

Subject headings: accretion, accretion disks—radiative transfer—stars: novae, cataclysmic variables—stars: pre-main-sequence

1. Introduction

Disk accretion onto a star is important in many contexts in astrophysics. Viscous processes remove angular momentum from the disk material, allowing it to spiral inward and eventually reach the star. The gravitational potential energy released by the accreting material is viscously dissipated, and most of it is radiated from the disk surface. Some of this radiation is intercepted by the accreting star. Also, some energy and angular momentum remains in the accreting material that reaches the surface of the accreting star. We have explored the problem of angular momentum accretion in disks elsewhere (Popham & Narayan 1991; Popham 1996; Popham *et al.* 1996). Here we will discuss the transfer of energy to the accreting star that results from disk accretion.

Heating of the accreting star by disk accretion can have important effects. In pre-main-sequence accretion disk systems such as FU Orionis objects, heating due to disk accretion appears to expand the stellar envelope. FU Orionis outbursts appear to arise in disks around T Tauri stars, but accretion disk models for FU Orionis objects find disk solutions where the stellar radii are 2–3 times larger than the radii of T Tauri stars (Kenyon, Hartmann, & Hewett 1988; Popham *et al.* 1996). The expanded stellar radii result in significantly lower disk luminosities and temperatures. The addition of accretion energy to these stars will also alter their evolutionary tracks in the HR diagram (Hartmann, Cassen, & Kenyon 1996).

In cataclysmic variables, ultraviolet observations of some dwarf novae have revealed that the white dwarfs are the dominant sources of ultraviolet flux from these systems during quiescent periods. Moreover, the white dwarfs in these systems have been observed to cool and fade with time after an outburst ends (Hassall, Pringle, & Verbunt 1985; Verbunt *et al.* 1987; Kiplinger, Sion, & Szkody 1991; Long *et al.* 1994; Gänsicke & Beuermann 1996). The white dwarfs in these systems are clearly heated during outbursts, and this implies that disk accretion is responsible for the heating, since the accretion rate and the disk temperature both increase dramatically during outbursts.

Various authors have envisioned several ways in which an accretion disk may heat the accreting star. The accretion disk has a luminosity which in many cases greatly exceeds the luminosity of the accreting star. Some fraction of the radiation from the disk surface will be intercepted by the star (Adams & Shu 1986). Also, some fraction of the energy dissipated in the innermost part of the disk may be radiated inward across the disk-star boundary into the star (Bertout & Regev 1992; Lioure & Le Contel 1994; Regev & Bertout 1995). Thermal energy may also be advected across this boundary by the accreting material (Popham & Narayan 1995; Popham *et al.* 1996).

These methods for heating the star depend strongly on the structure of the boundary layer between the disk and the star, where up to half of the accretion luminosity will be released. The proximity of the boundary layer to the star means that a large fraction of the boundary layer luminosity will be intercepted by the stellar surface. The temperature and density structure of the boundary layer region will also determine whether the luminosity released there goes inward into the star, outward into the disk, or simply upward to be radiated from the disk surface. Finally, the efficiency of the boundary layer in radiating away the energy dissipated there will determine the thermal energy content of the accreting material flowing into the star. Recently, we have constructed steady-state models of boundary layers in cataclysmic variables (CVs) and accreting pre-main sequence stars (Narayan & Popham 1993; Popham *et al.* 1993; Popham & Narayan 1995; Popham 1996; Popham *et al.* 1996). These models follow the flow of the accreting material through the disk and boundary layer and into the outer layers of the accreting star. They are based on the slim disk equations (Paczyński & Bisnovatyi-Kogan 1981; Muchotrzeb & Paczyński 1982; Abramowicz *et al.* 1988), which include a number of terms which are ignored in the standard thin disk treatment. These include radial pressure gradients and radial energy transport by radiation and by advection of thermal energy.

Our solutions directly calculate the radial distribution of flux from the surface of the boundary layer and disk. Most of the solutions include a small settling zone inside the boundary layer which we take to represent the outer layers of the star in an approximate way. The radiative energy flux between this zone and the boundary layer gives us some indication of the flux across the disk–star interface. Finally, our solutions calculate the amount of thermal energy advected into the star, based on the midplane properties of the disk at the stellar surface. These features of our boundary layer solutions permit us for the first time to calculate the importance of each of these types of stellar heating by the accretion disk. We can also compare stellar heating for different types of accreting stars.

Section 2 discusses heating of the star by intercepted disk radiation. We calculate the disk luminosity intercepted by the star for standard disk effective temperature profiles, and for our disk models which include the boundary layer region. In §3, we argue that direct radiative heating across the disk-star boundary will be unimportant in steadily accreting systems, but may be important in outbursting systems where the boundary layer temperature changes on a timescale comparable to the heating timescale. §4 discusses heating by advection of thermal energy into the star. In §5, we compare the relative importance of these types of stellar heating in various accreting systems, including T Tauri and FU Orionis stars and cataclysmic variables.

2. Stellar Heating by Intercepted Disk Radiation

2.1. Stellar Heating for Standard Disk Temperature Profiles

In most accretion disk systems, the accretion luminosity exceeds the intrinsic luminosity of the accreting star, often by a large factor. Therefore, if a substantial fraction of the radiation from the disk is intercepted by the stellar surface, it can heat the star to a significantly higher temperature than it would have in the absence of disk accretion. Adams & Shu (1986) derived expressions for the heating of the star by radiation from the disk. These are based on the assumptions that the star is spherical and the disk is flat and infinitely thin. The incident flux on the stellar surface is then

$$F_{inc}(\theta) = \frac{\cos\theta}{\pi} \int_{r_{min}}^{r_{out}} \sigma T_d^4(r) r dr \int_{-\phi_{max}}^{\phi_{max}} (r\sin\theta\cos\phi - 1)(r^2 - 2r\sin\theta\cos\phi + 1)^{-2} d\phi,$$

where θ is the polar angle of the point on the stellar surface, ϕ is the azimuthal angle between this point and the point on the disk, $T_d(r)$ is the effective temperature of the disk, r is the radius R of the disk point in units of the stellar radius R_* , r_{out} is the radius of the outer edge of the disk, $r_{min} = 1/\sin\theta$ is the innermost radius of the disk seen by the point on the star, and $\phi_{max} = \arccos[1/(r\sin\theta)]$ is the largest azimuthal angle seen by the point on the star. For a given variation of disk temperature with radius $T_d(r)$, we can integrate to find the θ -dependence of the incident flux on the stellar surface and then integrate over the stellar surface to find the total luminosity L_{inc} intercepted by the star.

Adams & Shu (1986) calculated that for a disk with a simple $T_d(r) \propto r^{-3/4}$ temperature profile, $L_{inc} = (1/2 - 4/3\pi)L_{disk} = 0.0756L_{disk}$. Note that the standard thin disk temperature distribution $T_d(r) \propto r^{-3/4}(1 - r^{-1/2})^{1/4}$ reaches a maximum at $r \simeq 1.36$ and then decreases in the innermost part of the disk (Shakura & Sunyaev 1973). Numerically integrating the equation above with this temperature distribution, we find that the luminosity intercepted by the star is much smaller, only amounting to $L_{inc} = 0.0216L_{disk}$. This difference clearly shows the importance of the inner part of the disk in heating the star. This suggests that when the boundary layer luminosity is included in the effective temperature distribution, a substantially larger fraction of the total accretion luminosity will be intercepted by the star.

2.2. Stellar Heating from Disk and Boundary Layer Radiation in CVs and Pre-Main-Sequence Stars

We can calculate the amount of stellar heating by disk radiation directly, using numerical solutions for $T_d(r)$ in the boundary layer and disk. Popham & Narayan (1995) calculated boundary layer and disk solutions for high- \dot{M} cataclysmic variables. We select a typical solution with $\dot{M} = 10^{-8} \,\mathrm{M_{\odot}} \,\mathrm{yr^{-1}}$, $M_* = 0.6 \,\mathrm{M_{\odot}}$, shown in Fig. 1 of Popham & Narayan (1995). By numerically integrating the equation given above, we find that the disk luminosity incident on the stellar surface is $L_{inc} \simeq 1.24 \times 10^{34} \mathrm{ergs s^{-1}}$, which is a fraction 0.217 of the total disk luminosity of $5.73 \times 10^{34} \mathrm{ergs s^{-1}}$. The reason for this very large incident luminosity is the presence of the boundary layer, which radiates approximately half of the total accretion luminosity from disk radii which are only a few percent larger than R_* . Nearly half of this boundary layer luminosity hits the stellar surface, so the overall fraction of the total luminosity which reaches the stellar surface is close to 1/4.

A similar situation occurs in T Tauri star accretion. Here we use the $\dot{M} = 10^{-7} \,\mathrm{M_{\odot} \ yr^{-1}}$ solution from Popham *et al.* (1993) to calculate the luminosity incident on the stellar surface, and find that $L_{inc} \simeq 1.39 \times 10^{33} \mathrm{ergs \ s^{-1}}$, which is 0.224 of the total disk luminosity of $6.20 \times 10^{33} \mathrm{ergs \ s^{-1}}$. Again, this is due to the boundary layer luminosity being radiated from a small region close to the stellar surface.

In FU Orionis objects, the situation is quite different. Here, we use a numerical solution from Popham *et al.* (1996) which fits the spectral energy distribution and line profiles observed from V1057 Cygni. This solution has a high accretion rate $\dot{M} = 10^{-4} \,\mathrm{M_{\odot} \ yr^{-1}}$, and as a result the disk is quite thick, with the disk height $H \simeq 0.4R$. The boundary layer luminosity is spread out over a large region of the inner disk, so that a much smaller fraction of it reaches the stellar surface. We find that $L_{inc} \simeq 2.96 \times 10^{34} \mathrm{ergs \ s^{-1}}$, which is only 0.0388 of the total disk luminosity of $7.62 \times 10^{35} \mathrm{ergs \ s^{-1}}$. In fact, this fraction of the disk luminosity is only about half that which would reach the star from a disk with $T_{eff} \propto r^{-3/4}$, as discussed above. It is still larger than the fraction from a standard thin disk, where no boundary layer is included.

The reason for the large variation in the fraction of the disk luminosity which reaches the stellar surface can be seen clearly in Fig. 1a, where we show $T_d(r)$ for the solutions described above. The CV and T Tauri solutions show an obvious peak in $T_d(r)$ near $R = R_*$, whereas the FU Orionis solutions shows no such peak, since the boundary layer luminosity is spread out over a large emitting area.

2.3. The Fraction of the Disk Flux Incident on the Star

For each disk radius, a certain fraction of the flux radiated from the disk surface reaches the stellar surface. We can define a spherical coordinate system centered at a point on the disk at radius r, where the polar axis points toward the center of the star. We can then integrate the flux from that point on the disk which is incident upon the star. The star subtends a portion of the sky which extends out to $\theta_0 = \arcsin(1/r)$, and the cosine of the angle to the normal to the disk surface is $\sin \theta \cos \phi$. Thus we find the fraction of the flux which hits the star to be

$$f_F(r) = \frac{\int_0^{\theta_0} \int_{-\pi/2}^{\pi/2} \sin \theta (\sin \theta \cos \phi) d\phi d\theta}{\int_0^{\pi} \int_{-\pi/2}^{\pi/2} \sin \theta (\sin \theta \cos \phi) d\phi d\theta} = \frac{1}{\pi} \left[\arcsin(1/r) - \frac{(r^2 - 1)^{1/2}}{r^2} \right].$$

This is plotted in Figure 1b.

We can integrate the flux which reaches the star $f_F F$ over the disk surface to find the total luminosity incident on the star, and divide this by the total disk luminosity to find the fraction of the disk luminosity f_L which reaches the star. This allows us to check the values of f_L which were calculated above by integrating the incident flux over the stellar surface. If we do this for a disk with $T_d(r) \propto r^{-3/4}$, so that $F \propto r^{-3}$, we find

$$f_L = \frac{\int_1^\infty \frac{1}{\pi} \left[\arcsin(1/r) - \frac{(r^2 - 1)^{1/2}}{r^2} \right] r^{-3} r dr}{\int_1^\infty r^{-3} r dr}$$
$$= -\frac{1}{\pi} \left[\frac{\arcsin(1/r)}{r} + \frac{(r^2 - 1)^{1/2}}{r} + \frac{(r^2 - 1)^{3/2}}{3r^3} \right]_1^\infty = \frac{1}{2} - \frac{4}{3\pi} = 0.0756.$$

This agrees with the value obtained above by integrating over the stellar surface. We can follow the same procedure for the standard thin disk flux distribution $F \propto r^{-3}(1 - r^{-1/2})$, and we find $f_L = 0.0216$ as before. Similarly, we can numerically integrate the fluxes radiated from our disk and boundary layer solutions for CVs and pre-main-sequence stars, weighting them by f_F as given above, and again we find values of f_L which agree with those derived above.

2.4. Distribution of Incident Flux on the Stellar Surface

The different disk temperature distributions also produce rather different distributions of the incident flux $F_{inc}(\theta)$ on the stellar surface. F_{inc} is always largest close to the equatorial plane, and decreases to zero at the pole; however, the degree to which F_{inc} is concentrated toward the equator depends strongly on the disk temperature distribution. In general, the incident flux distribution on the star mirrors the flux distribution radiated by the disk. A disk which radiates a large flux from the inner disk or boundary layer region will produce an incident flux distribution which is concentrated near the equatorial plane. The incident flux distributions produced by $T_d \propto r^{-3/4}$ and by standard thin disk temperature distributions are shown in Figure 2a. They are normalized by $F_* = \sigma T_*^4 = 3GM_*\dot{M}/8\pi R_*^3$. Note that F_{inc} drops to zero at both $\theta = 0$ and $\theta = \pi/2$. For the $r^{-3/4}$ distribution, F_{inc} peaks only about 0.6° above the equatorial plane. For the standard thin disk temperature distribution, which peaks at $r \simeq 1.36$, the incident flux distribution peaks farther above the disk, at about 7° above the equatorial plane.

The distribution of incident flux on the stellar surface for our disk and boundary layer solutions is shown in Figure 2b. The CV and T Tauri disks have a sharp peak in the radiated flux near the stellar surface due to the boundary layer, and these disks produce an incident flux distribution on the stellar surface which peaks sharply near the disk surface. The FU Orionis disk has a much smoother distribution of flux, and the incident flux on the stellar surface is similarly rather smooth. For comparison, we have plotted the incident flux on the stellar surface which would result from disks with a standard Shakura-Sunyaev temperature distribution, for the same values of \dot{M} , M_* , and R_* . Over most of the surface of the star, the incident flux distributions are nearly identical. However, near the equatorial plane, the boundary layer radiation increases the incident flux dramatically, particularly in the CV and T Tauri cases. The increase in the incident flux in the FU Orionis system is much smaller, and is spread over a larger region of the stellar surface, due to the larger radial extent of the boundary layer region.

The incident fluxes on the stellar surface shown in Fig. 2b are quite large in some cases; to make them easier to compare to the intrinsic fluxes from the accreting stars, we have included a vertical scale showing the effective temperature corresponding to F_{inc} , $T_{inc} = (F_{inc}/\sigma)^{1/4}$. For all three systems, T_{inc} falls somewhat below the peak boundary layer effective temperature; also T_{inc} exceeds the stellar temperature near the equatorial plane, but drops below the stellar temperature away from the equatorial plane. For the CV solution, the peak value of T_{inc} is 1.58×10^5 K. This is somewhat below the peak boundary layer effective temperature of 2.25×10^5 K, but much hotter than normal white dwarf temperatures, which are probably around 2×10^4 K. T_{inc} stays higher than 2×10^4 K over most of the stellar surface (Fig. 2b). For the T Tauri solution, the peak T_{inc} is about 6430 K, the peak boundary layer temperature is 8530 K and the stellar temperature is about 4000 K. Here T_{inc} only exceeds the stellar temperature in a small equatorial region, and drops off rapidly away from the equator. For the FU Orionis solution, the peak T_{inc} is about 5790 K, the peak boundary layer temperature is about 7110 K, and the stellar temperature can be estimated at 5000 K. Again T_{inc} only exceeds the stellar temperature in a small equatorial zone, but here T_{inc} falls off more slowly with distance above the equatorial plane.

Note that in calculating the incident fluxes and luminosities, we have continued to assume that the disk is flat and infinitesimally thin. This is a fairly good approximation for the CV and T Tauri disks. These are quite thin, and while they have a concave surface, this should only produce substantial changes in the flux coming from the outer parts of the disk, which make a very small contribution to the total flux reaching the star. The FU Orionis disk has a fractional thickness $H/R \sim 0.4$, where H is the vertical pressure scale height; the photosphere of the disk is probably a few scale heights above the midplane. Thus, in this case the disk photosphere may have a conical shape, and the disk will intercept radiation from itself. Since the disk will presumably cover much of the star, only a small portion of the stellar surface will intercept radiation from the disk.

3. Direct Radiative Heating Across the Disk–Star Boundary

3.1. The Inner Boundary Condition

Direct radiative heating of the star across the disk-star boundary is the most problematic of the heating terms we consider here. Current boundary layer models assume a steady state, making it difficult to calculate the radial energy transport between the boundary layer and the star. The steady-state assumption requires that the mass accretion rate \dot{M} be constant for all radii. This is a reasonable assumption to make for the disk and boundary layer, but it is probably not realistic for the accreting star, since the accreted matter may accumulate at the surface of the star or cause changes in the stellar radius. Since steady-state boundary layer models cannot treat the star realistically, in general the inner edge of the computational grid is placed at or near the boundary layer-star interface. The radiative energy transport across the inner boundary must then be specified by a boundary condition, but it is not obvious what condition to use.

In our models (Narayan & Popham 1993; Popham et al. 1993; Popham & Narayan 1995), we try to avoid this problem by setting the inner boundary of our calculation just below the stellar surface, at $R \simeq 0.95 - 0.98R_*$. The region between the inner boundary and the BL-star interface at $R = R_*$ is a pressure-supported settling flow rotating at the stellar rotation rate, with the same M as the disk and boundary layer; we take this settling region to represent the outer layers of the accreting star. We impose a boundary condition on the radially-directed radiative energy flux at the inner boundary, which matches the flux from the surface of the star in the absence of accretion. However, the presence of the settling region between the inner boundary and the boundary layer isolates the boundary layer from the inner boundary condition on the radial flux. Thus, the radiative energy transport across the BL-star interface is determined primarily by the conditions in the settling region, unless the radial luminosity entering or leaving at the inner boundary is comparable to the boundary layer luminosity. In general, our solutions have flux radiated from the settling region into the boundary layer, but the luminosity across the BL-star interface is generally much less than the accretion luminosity, so that the heating of the boundary layer by the star is insignificant. The BL luminosity in these solutions travels radially outward into the inner disk as it diffuses toward the disk surface, producing a "hot" boundary layer. Thus, in our steady-state models, there is no heating of the star by radiative flux across the boundary layer-star interface.

Other steady-state boundary layer models (Bertout & Regev 1992; Lioure & Le Contel 1994; Regev & Bertout 1995) have put a boundary condition on the temperature of the accreting material at the boundary layer-star interface at $R = R_*$, which they take to be

equal to the assumed photospheric temperature of the star in the absence of accretion. In general, this is substantially lower than the temperature of the boundary layer gas, so that there is a strong temperature gradient with the temperature decreasing inward. This produces a large inward flux of radiative energy, which can carry a large fraction of the energy dissipated in the boundary layer into the star. Solutions for boundary layers around T Tauri stars which use this boundary condition have found that the luminosity radiated into the star L_{in} is a substantial fraction of the total accretion luminosity L_{acc} ; Bertout & Regev (1992) found $L_{in} = 0.2L_{acc}$, Lioure & Le Contel (1994) $L_{in} = 0.286L_{acc}$, and Regev & Bertout (1995) $L_{in} = 0.3L_{acc}$. These luminosities represent an even larger fraction of the rate at which energy is dissipated in the boundary layer. For an accreting star rotating at a fraction f of the Keplerian velocity, the energy dissipation rate in the boundary layer is $\simeq 0.5(1-f)^2 L_{acc}$. All of the solutions discussed use f = 0.1, so that the dissipation rate is $\simeq 0.4L_{acc}$, and the fraction of the dissipated energy which is radiated into the star ranges from $\simeq 0.5 - 0.75$. As a result, these are "cool" boundary layer solutions, where the boundary layer effective temperature is substantially lower than it would be in solutions where little or no luminosity goes into the star.

Godon, Regev, & Shaviv (1995) suggested that the differences between the two types of solutions are due to a fundamental difference between placing boundary conditions on the flux and placing them on the temperature. They compared the effects of flux and temperature boundary conditions using four boundary layer solutions for accretion disks in CVs. Two of the solutions used an inner boundary condition on the midplane temperature at R_* , $T_* = 2 \times 10^5$ K, and the other two instead used a condition on the radial radiation flux at R_* , $F_* = \sigma T_{eff}^4$, where $T_{eff} = 2 \times 10^4$ K. The angular velocity profiles of the solutions show little dependence on the type of boundary condition used. The temperature profiles differ; in all of the solutions, the temperature of the accreting material rises as it moves inward, and exceeds 2×10^5 K. In the solutions with a temperature boundary condition, the temperature then drops back down to the assumed boundary value of 2×10^5 K at R_* , while in the solutions with a flux condition, the temperature continues to rise gradually inward to R_* . Godon, Regev, & Shaviv (1995) claim that this difference reflects the existence of two different families of solutions.

However, it seems clear that if appropriately higher values of T_* were chosen, the resulting solutions would be identical to the flux-condition solutions, and conversely, if an appropriate negative radial flux (carrying energy into the star) were chosen, the flux-condition solutions could be made identical to the temperature-condition solutions shown by Godon, Regev, & Shaviv (1995). We have verified this by calculating two solutions, one using a flux condition, and the other a temperature condition where the temperature has been chosen to match that produced by the flux solution. The resulting solutions are identical in all respects. Thus the nature of the solutions depends not on the type of boundary condition, but rather whether the flux or temperature they impose results in an inward or an outward radiative flux.

Solutions which have a large inward flux, whether it results from a boundary condition on the flux or the temperature, face a serious problem. The inward flux results from the temperature decreasing inward. This flux cools the boundary layer by transferring heat to the star, but in doing so, it rapidly heats the outer layers of the star. This reverses the temperature gradient that produced the inward flux in the first place. For instance, if the boundary layer–star interface is assigned the photospheric temperature in the absence of accretion, it is cooler than both the boundary layer and the stellar envelope. Thus, it represents a minimum in the temperature as a function of radial distance from the center of the star. Therefore, flux from both the star and the boundary layer will heat this region; radiative diffusion will fill in the minimum in the temperature. If a large fraction of the boundary layer luminosity goes into the star, the outer layers of the star will be heated until the temperature gradient is reversed. Here we show that in fact, this heating will occur quickly, so that it is unrealistic to use a boundary condition that results in a large inward flux in steady-state boundary layer models.

3.2. The Heating Timescale

One can estimate the time to heat the outer layers of the star as follows. The temperature gradient, and therefore the flux, will go to zero when the outer layers of the star are heated to the same temperature as the boundary layer gas, T_{BL} . Thus, if we take the mass m_{heat} of the portion of the star which is cooler than T_{BL} , we can derive the energy required to heat that mass of gas to T_{BL} . From this, we can estimate the heating timescale; in general, the boundary layer luminosity should be the dominant source of heating. Assuming that all of the boundary layer luminosity goes into heating the boundary layer–star interface region, we find a heating timescale

$$t_{heat} = c_P T_{BL} m_{heat} / L_{BL}$$

First, we must estimate m_{heat} . This can be done simply by taking the standard equation for diffusion of radiative energy, and transforming it so that the mass m is used as the independent variable. This gives

$$\frac{dT}{dm} = -\frac{3\kappa T_{eff}^4}{64\pi R^2 T^3},$$

where T_{eff} is the effective temperature of the star. This gives us an estimate for m_{heat} ,

$$m_{heat} \simeq \Delta m \simeq \frac{64\pi R^2 T^3}{3\kappa T_{eff}^4} \Delta T \simeq \frac{16\pi R^2}{3\kappa T_{eff}^4} \Delta(T^4) \simeq \frac{16\pi R_*^2 T_{BL}^4}{3\kappa T_{eff}^4}.$$

This estimate for the mass to be heated assumes that the entire surface of the star will be heated by the boundary layer. In fact only the equatorial region will be heated directly. The remainder of the stellar surface would have to be heated indirectly, by radiative diffusion or fluid motions which would transfer heat from the equatorial region toward the poles. Radiative diffusion is unlikely to be able to accomplish this, since the radiative energy will escape from the stellar surface as it moves toward the poles. Since the boundary layer heating of the equatorial region only extends to a modest depth in the stellar atmosphere, the timescale for diffusion to the surface is probably much shorter than that for diffusion to the poles. Diffusion of the radiative energy by fluid motions would suffer from the same problem; in order to spread the boundary layer heating evenly around the stellar surface, these motions would have to move the heated material to the poles before it could cool. Here we consider both alternatives: heating of the surface layers of the entire star, or of only the equatorial regions. The boundary layer extends to a height H_{in} above and below the equatorial plane, so it directly heats an area $4\pi R_*H_{in}$ of the stellar surface, which is a fraction H_{in}/R_* of the total stellar surface area. Therefore, for heating of the equatorial regions only, we reduce the estimate of the mass to be heated by this factor.

3.3. Heating Timescales for CVs and Pre-Main-Sequence Stars

We can use the boundary layer temperatures taken from our numerical solutions to estimate heating timescales for CVs and accreting pre-main-sequence stars. For a cataclysmic variable, we take as an example the solution with $\dot{M} = 10^{-8} \,\mathrm{M_{\odot}} \,\mathrm{yr^{-1}}$, $M_* = 0.6 \,\mathrm{M_{\odot}}, \,\Omega_* = 0, \,\alpha = 0.1$, presented in Popham & Narayan (1995). The white dwarf has a radius $R_* = 8.7 \times 10^8$ cm, and we adopt a white dwarf temperature $T_{eff} = 10,000$ K. The boundary layer has $T_{BL} \simeq 350,000$ K and $H_{in}/R_* \simeq 0.025$. If we again assume $\kappa = 1 \,\mathrm{cm^2 g^{-1}}$, we find $m_{heat} \simeq 2 \times 10^{25}$ g if the whole stellar surface is heated. To heat this amount of gas to 350,000 K will require $\sim 2.4 \times 10^{39}$ ergs. The accretion luminosity for these parameters is $\sim 6 \times 10^{35} \mathrm{ergs s^{-1}}$, so half of the accretion luminosity will heat the gas in $8 \times 10^4 \,\mathrm{s} \simeq 1$ day. If only the equatorial region is heated, we find $m_{heat} \simeq 5 \times 10^{23}$ g, and a heating timescale of 2×10^3 s.

For a T Tauri star, we adopt typical values of $R_* = 2 R_{\odot}$, $T_{eff} = 4000$ K. We take $T_{BL} \simeq 20,000$ K and $H_{in}/R_* \simeq 0.06$ from a T Tauri boundary layer solution with

 $\dot{M} = 10^{-7} \,\mathrm{M_{\odot} \ yr^{-1}}, M_* = 1 \,\mathrm{M_{\odot}}, \Omega_* = 0, \alpha = 0.1$ (Popham *et al.* 1993). The opacity κ varies strongly with temperature and density in this regime. The boundary layer solution gives densities $\rho \simeq 10^{-7} - 10^{-8} \,\mathrm{g \ cm^{-3}}$ in the inner boundary layer. At $T = 20,000 \,\mathrm{K}$, this would give $\kappa \sim 100 \,\mathrm{cm^2 \ g^{-1}}$ or more, but at lower temperatures, the opacity drops off rapidly, reaching $\kappa \sim 0.01 \,\mathrm{cm^2 \ g^{-1}}$ at $T = 4000 \,\mathrm{K}$ (Alexander & Ferguson 1994). If we assume for simplicity that $\kappa = 1 \,\mathrm{cm^2 \ g^{-1}}$, we find $m_{heat} \simeq 2 \times 10^{26} \,\mathrm{g}$ if the whole stellar surface is heated (if we instead approximate the opacity as a power law $\kappa = 10^{-38}T^{10}$, we find $m_{heat} \simeq 2 \times 10^{25} \,\mathrm{g}$). To heat this amount of gas to $T_{BL} = 20,000 \,\mathrm{K}$ requires energy $c_P T_{BL} m_{heat} \simeq 1.4 \times 10^{39} \,\mathrm{ergs}$. The accretion luminosity for these parameters is $L_{acc} = G M_* \dot{M} / R_* \simeq 5.6 \times 10^{33} \mathrm{ergs} \,\mathrm{s^{-1}}$. If we assume that half of this luminosity goes into heating the star, we find a heating timescale $t_{heat} \simeq 5 \times 10^5 \,\mathrm{s} \simeq 6$ days. If only the equatorial regions are heated, we find $m_{heat} \simeq 1.2 \times 10^{25} \,\mathrm{g}$, and a heating timescale of $3 \times 10^4 \,\mathrm{s} \simeq 0.35 \,\mathrm{days}$.

In FU Orionis objects, the heating timescale is much longer than in T Tauri stars, due primarily to the much higher boundary layer temperature. Our solution for V1057 Cygni has a midplane temperature of $T_{BL} \simeq 2 \times 10^5$ K. Therefore the mass of the portion of the stellar envelope which has a temperature below T_{BL} is much larger; taking $R_* = 3.5 \times 10^{11}$ cm and the star's effective temperature to be $T_{eff} = 5000$ K, we find $m_{heat} \simeq 5 \times 10^{30}$ g. Half of the accretion luminosity is $0.5L_{acc} \simeq 6.3 \times 10^{35}$ ergs s⁻¹, so the heating timescale is $t_{heat} \simeq 5.7 \times 10^8$ sec $\simeq 18$ years.

The estimates derived here are quite rough. They depend on the fourth power of the ratio T_{BL}/T_{eff} , so they are quite sensitive to variations in these temperatures. The boundary layer temperature can vary significantly for different values of M, M_*, Ω_* , and α (Popham & Narayan 1995). They also assume that the dominant means of energy transfer in the boundary layer-star interface region is radiative diffusion. In some cases (e.g. T Tauri stars), the accreting star may have a convective envelope. If convection is present in the heated zone, it would lead to a different expression for the heating timescale, and it could produce more efficient sideways transport of the boundary layer energy within the star. However, the assumed boundary layer heating will reverse the direction of energy transfer in the adjoining portion of the stellar envelope, which should suppress convection in this region. Prialnik & Livio (1985) found that for simple spherical accretion at moderate rates onto an initially fully convective 0.2 M_{\odot} star, the convective zone recedes from the stellar surface. For accretion rates of $10^{-7} \,\mathrm{M_{\odot}} \,\mathrm{yr^{-1}}$ or below, their results indicate that this occurs roughly when the inward energy flux due to accretion exceeds the outward flux from the star's inherent luminosity. A T Tauri star has an effective temperature around 4000 K, so the stellar flux is about 1.5×10^{10} ergs cm⁻² s⁻¹. If the boundary layer luminosity, $0.5L_{acc} = GM\dot{M}/2R_* \simeq 3 \times 10^{33} \text{ergs s}^{-1}$ is radiated into the equatorial region,

which has a surface area $4\pi R_* H_{in} \simeq 1.5 \times 10^{22} \text{ cm}^2$, the inward flux of accretion energy is $\simeq 2 \times 10^{11} \text{ ergs cm}^{-2} \text{ s}^{-1}$. This is much greater than the stellar flux, and should be sufficient to suppress convection in the equatorial surface layers of the star.

3.4. Direct Radiative Heating in Steadily Accreting Systems

The heating timescales derived above for CV and T Tauri accretion disks are very short. If the mass accretion rate remains steady over this short timescale, the stellar envelope will be heated to the boundary layer temperature, and the inward radiative flux will vanish as the temperature gradient flattens out. The system should then reach a steady state where there is no inward radiative flux across the disk–star boundary.

The use of a boundary condition which sets the temperature at the disk-star interface to the photospheric temperature of the unperturbed star inevitably results in a large inward flux, since the temperature is forced to drop to an unnaturally low value, and the resulting temperature gradient forces the radiative flux to be directed inward (Lioure & Le Contel 1994; Regev & Bertout 1995). The short heating timescales for the stellar envelope show that this temperature gradient would quickly be eliminated by the rapid heating. Thus, such boundary conditions (or, equivalently, boundary conditions which directly specify a large inward flux) should not be applied to steadily accreting systems.

The use of a flux boundary condition offers a significant advantage over a temperature condition. If the radial flux is chosen to have a small value and be directed outward, the resulting temperature gradient will be small, and there will be no minimum in the temperature at the disk-star boundary. If a temperature condition is used, one must guess the value of the temperature at the disk-star boundary, and even a relatively small error in the temperature can result in a large radial flux, which can heat or cool the boundary layer and change its structure substantially.

3.5. Reradiation of the Boundary Layer Luminosity by the Stellar Surface Layers

One way around the argument given above is to say that the surface layers can radiate the boundary layer luminosity away at the same rate as it enters, so that a steady state will be reached, as argued by Regev & Bertout (1995). However, for the boundary layer luminosity to be radiated inward toward the star and then reradiated from the stellar surface would require that the optical depth along this indirect path be lower than the vertical optical depth of the boundary layer itself. This is not the case, for the simple reason that below the boundary layer, the accreting material must be supported by the pressure gradient, so that the pressure and density of the accreting gas increase rapidly with decreasing radius. By contrast, the density is low immediately outside the boundary layer due to the large radial velocity of the accreting material. As a result, the vertical optical depth of the accreting material is smallest just outside the boundary layer, and increases rapidly as the material flows inward and settles onto the star. This leads to the natural conclusion that the boundary layer luminosity is radiated radially outward rather than inward, and emitted directly from the surface of the boundary layer and inner disk (the "thermal boundary layer"), rather than indirectly from the surface of the star. This is exactly what happens in our boundary layer solutions.

Regev & Bertout (1995) argued that most of the boundary layer luminosity in T Tauri stars would be radiated into the star and subsequently radiated vertically, away from the equatorial plane, finally emerging from the stellar surface above and below the disk. They first calculated the temperature distribution of the stellar atmosphere in the equatorial plane, using an expression for the variation of temperature in an atmosphere illuminated from above. However, this expression is inappropriate for calculating the temperature in the equatorial plane, since it treats the boundary layer only as a source of incident radiation, and assumes that the equatorial surface layers of the star can radiate freely into space. In fact, the equatorial surface layers of the star are buried beneath the boundary layer and disk, so the optical depth in the equatorial plane is very large. Regev & Bertout also claimed that the indirect emission from the stellar surface layers would be observationally indistinguishable from direct emission from the boundary layer. This was based on assuming that the vertical optical depth of the emitting region would be about one, but boundary layer solutions show that the vertical optical depth increases rapidly as the material moves inward through the boundary layer, and is much larger than one.

3.6. Direct Radiative Heating in Outbursting Systems

Although direct radiative heating of the star by the boundary layer is unlikely to be important in steady accretion, it could be significant in systems where the accretion rate varies on a short timescale. A number of accreting systems, such as dwarf novae and FU Orionis objects, experience outbursts where their brightness increases quite rapidly. The outbursts in these systems are generally interpreted as being due to an increase in the mass accretion rate, which would in turn increase the boundary layer temperature. If the outburst rise time is faster than the heating timescale of the stellar envelope derived above, then the hot boundary layer will radiatively heat the star.

For dwarf novae, the rise times of outbursts are generally around 1 day, comparable to the heating timescales derived above. This suggests that during the early phases of the outbursts, a large fraction of the boundary layer luminosity could go into heating the white dwarf surface layers. This could have two important effects. First, the increase in the boundary layer luminosity would be delayed until the white dwarf surface layers were heated, which might account for some of the observed delay between the optical and EUV fluxes during outbursts (e.g., Mauche, Raymond, & Mattei 1995). Second, the energy stored in the heated surface layers of the white dwarf would be radiated away during quiescence, as observed in a number of cases (Hassall, Pringle, & Verbunt 1985; Verbunt *et al.* 1987; Kiplinger, Sion, & Szkody 1991; Long *et al.* 1994; Gänsicke & Beuermann 1996). This is discussed further in §5.2.

For FU Orionis objects, we derived a heating timescale of ~ 18 years in §3.3. This is substantially longer than the rise times of FU Orionis and V1057 Cygni, which are ~ 1 year, and is probably a substantial fraction of the length of the outbursts, which are estimated to last ~ 100 years. Thus we expect that direct radiative heating across the star-disk boundary will heat the stellar envelope in the early part of the outburst, until the envelope temperature reaches T_{BL} . However, the fact that the stellar radii derived from disk models of FU Orionis objects are substantially larger than those of T Tauri stars suggests that the star expands rapidly in response to the high outburst accretion rate. Thus, the interaction between the disk and star is probably more complicated than our simple picture of radiative heating. In particular, the "stellar envelope" may in fact consist largely of accreted material which is already at a high temperature, so that little or no radiative heating would take place across the disk-star boundary.

4. Heating by Advection of Thermal Energy

The material accreted by the star carries thermal energy with it. In some cases this can be an important source of stellar heating. Advection of entropy is included in the slim disk energy equation (Paczyński & Bisnovatyi-Kogan 1981; Muchotrzeb & Paczyński 1982; Abramowicz *et al.* 1988). It may have an important effect in accretion disks around black holes, since thermal energy which is advected into the black hole will not be radiated. This changes the luminosity and spectral energy distribution of the black hole system (Narayan, McClintock & Yi 1996). In accretion onto a star, the thermal energy of the accreting material cannot be eliminated in this way. Instead, it is advected into the star.

A good indicator of the importance of energy advection in the disk is the vertical pressure scale height H of the disk. The standard expression for the disk height is $H \simeq c_s/\Omega_K$, which can be rewritten $H^2/R^2 \simeq c_s^2/v_K^2$. Thus the square of the disk height as a fraction of the disk radius R is proportional to the thermal energy content of the accreting material, as a fraction of the gravitational energy released $GM_*\dot{M}/R = \dot{M}v_K^2$. Therefore, we should expect that energy advection will be important in thick disks, like those found in systems with high accretion rates.

4.1. The Fraction of the Accretion Luminosity Advected Into the Star

A more accurate calculation of the energy advected into the star can be made by using the slim disk energy equation, one of the equations of our boundary layer model, which reads

$$\nu \Sigma \left(R \frac{d\Omega}{dR} \right)^2 - F_V - \Sigma v_R T_c \frac{dS}{dR} - \frac{1}{R} \frac{d}{dR} (RHF_R) = 0.$$

The first term in this equation represents the viscous dissipation within the disk; ν is the kinematic viscosity, Σ is the surface density of the disk, and $d\Omega/dR$ is the angular velocity gradient. The second term is the vertical flux F_V from the disk surface. The third term is the advected entropy; v_R , T_c , and S are the radial velocity, midplane temperature, and entropy of the accreting material. The final term is the divergence of the radial radiation flux F_R between adjacent disk annuli. If we integrate this equation over the surface of the disk, we have

$$L_{diss} - L + \dot{M} \int T_c dS - 4\pi [R_{out} H_{out} F_{R,out} - R_* H_* F_{R,*}] = 0.$$

Thus the overall energy balance is determined not only by the rate of energy dissipation within the disk L_{diss} and the rate at which energy is radiated away from the disk surface L, but also by the energy carried into and out of the disk by energy advection and radial radiation.

We can use the other slim disk equations, in particular the angular momentum equation and the radial momentum equation, to derive an expression for L_{diss} (see Popham & Narayan 1995 for details),

$$L_{diss} = \frac{GM_*\dot{M}}{R_*} \left(1 - j\frac{\Omega_*}{\Omega_K(R_*)} + \frac{1}{2}\frac{\Omega_*^2}{\Omega_K^2(R_*)}\right) + \dot{M}\int \frac{dP}{\rho}.$$

The first term is the accretion luminosity $L_{acc} = GM_*\dot{M}/R_*$, corrected for the rotational kinetic energy transferred between the star and the disk, where $j \equiv \dot{J}/\dot{M}\Omega_K(R_*)R_*^2$ is the

angular momentum accretion rate in standard units, and Ω_* and $\Omega_K(R_*)$ are the stellar rotation rate and the Keplerian rotation rate at R_* . The second term accounts for radial pressure support of the accreting material; P is the pressure.

The radial radiation fluxes at the inner and outer edges of the disk are small. At the inner edge, as discussed above, we assume that the flux entering the disk is σT_*^4 , where we take T_* to be the effective temperature of the unperturbed star. Thus a fraction H_*/R_* of the star's luminosity goes into the disk, where H_* is the disk height at the stellar surface. In general, the star's luminosity is much smaller than L_{acc} , and in some cases $H_* \ll R_*$, so the luminosity entering the inner edge of the disk is insignificant, and at the outer edge the radial flux is negligible; thus we can neglect these radial radiation terms.

We can then use $TdS = dU - Pd\rho/\rho^2$ and the expression for L_{diss} given above to write an expression for the luminosity radiated by the disk

$$\begin{split} L &= L_{acc} \left(1 - j \frac{\Omega_*}{\Omega_K(R_*)} + \frac{1}{2} \frac{\Omega_*^2}{\Omega_K^2(R_*)} \right) + \dot{M} \int \frac{dP}{\rho} + \dot{M} \int dU - \dot{M} \int \frac{P}{\rho^2} d\rho \\ &= L_{acc} \left(1 - j \frac{\Omega_*}{\Omega_K(R_*)} + \frac{1}{2} \frac{\Omega_*^2}{\Omega_K^2(R_*)} \right) + \frac{5}{2} \dot{M} \int d\left(\frac{P}{\rho}\right), \\ &= L_{acc} \left(1 - j \frac{\Omega_*}{\Omega_K(R_*)} + \frac{1}{2} \frac{\Omega_*^2}{\Omega_K^2(R_*)} \right) - \dot{M} c_P T_c(R_*) \end{split}$$

where the internal energy $U = 3P/2\rho$, and $T_c(R_*) \gg T_c(R_{out})$. The first term is the total energy dissipated in the disk; $\dot{M}c_PT_c(R_*)$ of this is advected into the star, and the remainder is radiated away.

It is easy to see how the square of the disk thickness at R_* is a direct measure of the importance of advection. The rate of energy advection into the star is $\dot{M}c_PT_c(R_*) \simeq 2.5\dot{M}c_s^2(R_*)$, while the accretion luminosity is $GM_*\dot{M}R_* = \dot{M}\Omega_K^2(R_*)$. Thus a fraction $f = 2.5c_s^2(R_*)/\Omega_K^2(R_*) = 2.5(H_*/R_*)^2$ of the accretion luminosity is advected into the star.

4.2. Advective Heating in CVs and Pre-Main-Sequence Stars

Using boundary layer solutions, we can find the midplane temperature at R_* for various types of accretion disks. For a high- \dot{M} CV solution taken from Popham & Narayan (1995) (with $\dot{M} = 10^{-8} \,\mathrm{M_{\odot}} \,\mathrm{yr^{-1}}$, $M_* = 0.6 \,\mathrm{M_{\odot}}$, as discussed above), we find $T_c(R_*) \simeq 7 \times 10^5 \,\mathrm{K}$, and $H_*/R_* \simeq 3.3 \times 10^{-2}$. Thus thermal energy is advected into the white dwarf at the rate of $1.5 \times 10^{32} \mathrm{ergs} \,\mathrm{s^{-1}}$, or about $2.6 \times 10^{-3} L_{acc}$.

To calculate the energy advection into T Tauri stars, we use a boundary layer solution with $\dot{M} = 10^{-7} \,\mathrm{M_{\odot}} \,\mathrm{yr^{-1}}$ from Popham *et al.* (1993). This solution has $T_c(R_*) \simeq 2 \times 10^4$ K, and $H_*/R_* \simeq 5.7 \times 10^{-2}$. Therefore the rate of energy advection into the star is about $4 \times 10^{31} \mathrm{ergs} \,\mathrm{s^{-1}}$, which is $\sim 8 \times 10^{-3} L_{acc}$.

In FU Orionis objects, the situation is quite different. Here the disk is quite thick as a result of the high accretion rates $\dot{M} \simeq 10^{-4} \,\mathrm{M_{\odot}} \,\mathrm{yr^{-1}}$ reached during FU Orionis outbursts. The optical depth from the disk surface to the midplane is very large, so the midplane temperatures are high. In the solutions for FU Orionis and V1057 Cygni calculated by Popham *et al.* (1996), $T_c(R_*) \simeq 2 - 2.5 \times 10^5 \,\mathrm{K}$, and $H_*/R_* \simeq 0.39$, about 10 times as large as in the CV and T Tauri solutions. Since the fraction of the accretion luminosity advected into the star goes as the square of H_*/R_* , it is on the order of 100 times larger than in the thin disks. For $H_*/R_* = 0.39$, we have $2.5(H_*/R_*)^2 \simeq 0.38$, so 38% of the accretion luminosity is carried into the star. The accretion luminosities of these systems are quite large, generally around $1 - 3 \times 10^{36} \mathrm{ergs s^{-1}}$, so the rates of energy advection are also very large, around $4 - 12 \times 10^{35} \mathrm{ergs s^{-1}} \simeq 100 - 300 \,\mathrm{L_{\odot}}$. Of course, these rates are only reached during FU Orionis outbursts.

5. Discussion

5.1. The Relative Importance of Various Sources of Stellar Heating

In the preceding sections, we have calculated the rates of various types of stellar heating in different accreting systems. The types of stellar heating we have considered include heating by radiation emitted from the disk surface, direct radiative heating through the disk–star boundary, and advection of thermal energy. The accreting systems for which we analyze the stellar heating rates include steadily accreting thin disk systems such as cataclysmic variables and T Tauri stars, thick disk FU Orionis systems, and systems undergoing outbursts such as dwarf novae and FU Orionis systems.

We find that in steady thin disk systems, radiation emitted from the disk surface is the most important source of stellar heating. In CVs and T Tauri stars, we find that over 20% of the accretion luminosity reaches the star. Most of this comes from the boundary layer region. Advection of thermal energy is unimportant in these systems, since almost all of the accretion energy is radiated away, leaving very little thermal energy in the accreting material.

In steady thick disk systems, which may apply to some FU Orionis objects, the situation is quite different. The dominant source of stellar heating is advection of thermal energy into the star. A large fraction of the accretion luminosity is stored in the accreting material, resulting in a thick disk, and carried into the star. Heating by radiation from the disk surface is much less important than in the thin disk systems. Since the boundary layer luminosity is emitted over a wide range of radii, a much smaller fraction of it reaches the stellar surface.

We have argued that direct radiative heating through the disk-star boundary will be insignificant in steadily accreting systems, since any temperature gradient which would cause the boundary layer luminosity to flow inward will be reversed on a short timescale. In outbursting systems, direct radiative heating can become important if the timescale on which the boundary layer temperature rises is shorter than the heating timescale of the stellar envelope. Direct heating of the star will continue until the envelope is heated to the new boundary layer temperature or the outburst ends. Thus we expect that direct heating may be important during the early portion of both dwarf nova and FU Orionis outbursts.

In this paper, we have concentrated on external sources of stellar heating, which result from the transfer of radiation or hot material from an accretion disk onto the surface of the star. Additional heating may result from processes which occur within the star as a result of accretion, such as compressional and shear heating. Compressional heating of the star results from the change in the stellar structure due to the mass added by accretion. It should be particularly important in white dwarfs, which become much more centrally condensed as their masses increase. Sion (1995) has computed the effects of this heating in white dwarfs during dwarf nova outbursts with accretion rates of $10^{-8} - 10^{-7} \,\mathrm{M_{\odot} \ yr^{-1}}$. He found that compressional heating alone can produce variations of up to 5000 degrees K in the effective temperature of the white dwarf from outburst to quiescence. Shear heating will take place as the internal rotation of the star adjusts to the torque applied to the surface layers by accretion. Since both of these processes take place within the interior of the star, they depend less strongly on the details of the disk structure, and take place over a longer timescale than the external heating sources we have discussed in this paper.

5.2. Effects of Heating on the Star

The main focus of this paper is on determining the importance of various types of stellar heating in different types of accreting systems. Nonetheless, it seems appropriate to briefly consider the effects of this heating upon the accreting star. All of the types of stellar heating described in §2–4 above heat the star from outside, but do so in rather different ways.

The external illumination of the stellar surface by radiation from the disk, described in $\S2$, will heat the atmosphere of the star. The effects of external illumination of a stellar atmosphere have been studied by a number of authors, many of whom have focused on irradiation of the donor star in accreting X-ray binary systems. If the incident radiation is in the form of X-rays, the heating due to external illumination may drive a wind from the surface of the star (see, e.g., Tavani & London 1993 and references therein). In CVs, much of the boundary layer emission will be in X-rays, and as we discussed in §2, a significant fraction of the boundary layer emission will be intercepted by the equatorial regions of the star. Tavani & London (1993) found that incident X-ray fluxes $\simeq 10^{15}$ ergs cm⁻² s⁻¹ on a white dwarf drove winds with mass loss rates $\sim 10^{18}$ g s⁻¹. As Fig. 2b demonstrates, incident X-ray fluxes of this magnitude are produced by the boundary layer radiation in CVs, but only in the equatorial region of the accreting white dwarf. Since Tavani & London's (1993) mass loss rates were based on spherically symmetric winds, and because those winds arose from an extremely low-mass, large-radius $(M_{WD} = 0.1 \,\mathrm{M_{\odot}})$ $R_{WD} = 5 \times 10^9$ cm) white dwarf, the mass loss rates from CVs should be expected to be quite a bit smaller. Nonetheless, irradiation of the equatorial regions of the white dwarf could make an important contribution to driving winds from CVs.

In pre-main-sequence stars, the external illumination will be in the form of much softer optical and ultraviolet radiation. Most studies of irradiated atmospheres in these systems have focused on the illumination of the disk atmosphere by radiation from the star (Calvet *et al.* 1991; Malbet & Bertout 1991), the opposite situation to the one described here. However, these authors treated the disk atmosphere in a way that should be reasonably applicable to the irradiation of the star by the disk. These studies found that irradiation heats the outer layers of the atmosphere, producing a sort of "chromosphere". This heated upper layer can produce emission lines which can fill in the absorption lines of the unperturbed stellar atmosphere.

Direct radiative heating across the disk-star interface during disk outbursts will heat an equatorial region which sits at a larger optical depth from the surface. When the outbursts ends and the disk temperature drops, this heated equatorial region will cool. A decline in the ultraviolet flux during quiescence has been observed in several dwarf nova systems (Hassall, Pringle, & Verbunt 1985; Verbunt *et al.* 1987; Kiplinger, Sion, & Szkody 1991) and has been confirmed to be due to cooling of the white dwarf (Long *et al.* 1994; Gänsicke & Beuermann 1996).

Pringle (1988) modeled the heating and cooling of the white dwarf surface layers which should occur during a dwarf nova outburst and subsequent quiescent phase. He used a range of temperatures for the external heat source, and varied this external temperature with time to simulate a dwarf nova outburst. Pringle found that the assumed temperature of the heat source had a dramatic effect on the timescale for cooling, since higher temperatures heated the white dwarf to a greater depth. A fairly high external temperature $\sim 900,000$ K was required to make the cooling last long enough to agree with observations by Verbunt *et al.* (1987) of the decrease in the UV flux of VW Hydri during quiescence.

In the discussion above, we cited the example of a $0.6 \,\mathrm{M}_{\odot}$ white dwarf accreting at $10^{-8} \,\mathrm{M}_{\odot} \,\mathrm{yr}^{-1}$, for which our boundary layer solution gave $T_{BL} \simeq 350,000$ K. This temperature is substantially lower than the temperature of 900,000 K cited by Pringle (1988), and his results indicate that this boundary layer temperature would be far too low to heat the white dwarf deeply enough to explain the observed cooling timescale. However, other boundary layer solutions for different input parameters can reach substantially higher temperatures; for instance, the solutions with $\dot{M} = 10^{-7} \,\mathrm{M}_{\odot} \,\mathrm{yr}^{-1}$ reach $T_{BL} \sim 800,000 \,\mathrm{K}$, and those with $M_* = 1 \,\mathrm{M}_{\odot}$ reach $T_{BL} \sim 600,000 \,\mathrm{K}$ (Popham & Narayan 1995). A solution with increased \dot{M} and M_* would reach even higher temperatures. This suggests that direct radiative heating across the disk-star interface is likely to be most important in dwarf novae which contain massive white dwarfs, or which have very high accretion rates during outbursts.

Advective heating of the central star results from the deposition of heated material onto the star. The most detailed treatment of this process to date remains that of Prialnik & Livio (1985), who calculated the effects of accretion onto a 0.2 M_{\odot} main-sequence star. They varied the mass accretion rate and the fraction of the accretion luminosity retained by the accreting material. FU Orionis objects have high mass accretion rates $\dot{M} \simeq 10^{-4} \,\mathrm{M}_{\odot} \,\mathrm{yr}^{-1}$, and the accreting material retains a large fraction $\alpha \simeq 0.375$ of the accretion luminosity, as discussed in §4. For these values of \dot{M} and α , the 0.2 M_{\odot} main-sequence star studied by Prialnik & Livio undergoes dynamically unstable expansion. If these results can be applied to the accreting stars in FU Orionis objects, they suggest that advective heating will cause the star to expand rapidly udring outbursts. This may account for the rather large radii inferred by disk models for the stars in FU Orionis objects, as compared to the radii of T Tauri stars (e.g., Popham *et al.* 1996). In addition, the advected energy added to the star during FU Orionis outbursts may change the position of the stellar birthline in the H-R diagram, as emphasized by Hartmann, Cassen, & Kenyon (1996).

5.3. Problems with One-Dimensional, Steady-State Models

The heating of an accreting star by a luminous boundary layer is difficult to study with one-dimensional, steady-state models. The fundamental reasons for this are, first, although the mass accretion rate may be constant throughout the disk, when the accreting material reaches the star, it should build up on the stellar surface. To accurately study this requires a model where \dot{M} can vary with radius, which must be time-dependent. Second, the approximations used to treat the vertical structure of the disk must break down within the star.

Another problem with one-dimensional, steady-state boundary layer models is the difficulty in assigning the location of the disk-star boundary. Our model treats the disk, the boundary layer, and the "star" (the settling region which serves as an approximate version of the outer layers of the star) using the same set of equations. We have found that it can be quite difficult to tell where the disk ends and the star begins. In fact, in our previous papers, we have used several different criteria to define the disk-star boundary, where $R = R_*$. In our T Tauri solution (Popham *et al.* 1993), we assigned R_* to be the point where the angular velocity Ω drops to half of its peak value. In our CV solutions (Popham & Narayan 1995), we chose the point where the rate of energy dissipation by viscosity drops below the rate of energy advection. Finally, in our FU Orionis solutions (Popham 1996; Popham *et al.* 1996), we chose the point where the radial velocity of the accreting material drops below a set value, which we took to be 1000cm s⁻¹.

In general, the ambiguity in the stellar radius is fairly small in thin disk (CV and T Tauri) solutions. This is because the properties of the disk vary over a short radial scale; however, this rapid variation means that the small ambiguity in the location of R_* results in fairly large uncertainties in the values of quantities such as $T(R_*)$. In thick disk solutions like the ones for FU Orionis objects, the uncertainty in R_* is much larger, because the variations in the disk occur over quite large lengthscales, comparable to R_* itself. Of course, the slower variations of the disk properties keep the uncertainties in these quantities from becoming too large.

The ambiguity in the location of the stellar radius affects the values derived in this paper for the rates of stellar heating. For instance, if we selected a smaller stellar radius for our CV and T Tauri solutions, we would find a somewhat larger rate of heating by advection, since the temperature of the accreting material rises rapidly with decreasing R in this region. In general, although the uncertainty in the stellar radius may alter the heating rates we have computed here, it should not affect the qualitative conclusions we have drawn about which types of stellar heating will be important in various types of accreting systems.

Two-dimensional, time-dependent models of boundary layers which include the outer layers of the star should be able to resolve the uncertainties in the location of the stellar radius and the radiative flux across the disk-star interface. Such models have been constructed by Kley for boundary layers in CVs (Kley 1989, 1991) and pre-main-sequence stars (Kley & Lin 1996). However, thus far none of these models has been run long enough to reach a steady state; however, the most recent models of pre-main-sequence stars have been carried out for as long as 1000 orbital periods (Kley & Lin 1996). The initial conditions for these models take the temperature distribution in the star to be that of a star undisturbed by accretion. Thus initially there is a minimum in the temperature at R_* , similar to the temperature minimum assumed in the steady-state "cool" boundary layer models discussed in §3. This region should be heated by radiative diffusion from both sides, filling in the temperature minimum. Kley & Lin (1996) estimate that this should take place in ~ 10⁵ orbital periods, or about 100 years. Unfortunately this timescale is still too long for the heating of this region to be verified by current models. Nonetheless, if future models of this type can be extended over substantially longer timescales, they should be able to resolve this issue.

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Fig. 1.— (a) The effective temperature of the boundary layer and inner disk, for a cataclysmic variable (solid line), an FU Orionis object (short-dashed line), and a T Tauri star (long-dashed line). The thin disk (CV and T Tauri) solutions have strong peaks in the effective temperature due to the boundary layer, while the thick disk FU Orionis solution has the boundary layer luminosity spread out over a large region. (b) The fraction of the flux emitted at a given disk radius which is intercepted by the stellar surface.

Fig. 2.— (a) The incident flux on the stellar surface, in units of $F_* = 3GM_*\dot{M}/8\pi R_*^3$, for disks with a $T(r) \propto r^{-3/4}$ temperature distribution (solid line) and a Shakura-Sunyaev $T(r) \propto r^{-3/4}(1-r^{-1/2})^{1/4}$ distribution (dashed line). (b) The boundary layer and disk flux incident on the stellar surface given by boundary layer solutions for a cataclysmic variable (solid line), an FU Orionis object (long-dashed line), and a T Tauri star (short-dashed line). The dotted lines show the incident flux that would result from a Shakura-Sunyaev-type solution with the same parameters. For the thin disk (CV and T Tauri) solutions, the flux from the boundary layer produces a pronounced peak in the incident flux on the stellar surface, but this peak only extends to 15 or 20 degrees above the disk plane. For the thick disk FU Orionis solution, the boundary layer flux is spread more uniformly over most of the stellar surface.



