Small scale substructure, collapse time and dynamical friction

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Abstract. We solve numerically the equations of motion for the collapse of a shell of baryonic matter, made of galaxies and substructure of $10^6 M_\odot \div 10^9 M_\odot$, taking into account the dynamical friction and the parameters on which it depends: the peaks' height ν_c, the number of peaks inside a protocluster N_{tot} multiplied by the correlation function $\xi(r)$ evaluated at $r = 0$, the filtering radius R_f and the core radius of the cluster of galaxies, r_0. We show how the collapse time of a shell of baryonic matter depends on those parameters.

1. Introduction

The problem of the formation and evolution of clusters of galaxies has been one of the crucial topic over the last years (see *e.g.* Colafrancesco et al. 1989, Kaiser 1993, Antonuccio-Delogu 1992, Colafrancesco et al. 1995 and Ryden & Gunn 1987). It is well known that the formation of cosmic structures is strictly related to the evolution of the density perturbations: in the present *paradigma* of structure formation, it is generally assumed that cosmic structures of size $\sim$ R form preferentially around the local maxima of the primordial density field, once it is smoothed on the filtering scale R_f. These linear density fluctuations eventually evolve towards the nonlinear regime under the action of gravitational instability; they detach from the Hubble flow at *turn around* epoch t_m, given by:

$$t_m = \left[\frac{3\pi}{32G\rho_b}(1 + \overline{\delta}) \right]^{1/2} (1 + z)^{3/2} \qquad (1)$$

where ρ_b is the mean background density, z is the redshift and $\overline{\delta}$ is the mean overdensity within the nonlinear region. After the *turn around* epoch, the fluctuations start to recollapse when their overdensity reaches the value $\overline{\delta} = 1$. Since

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the density field depends on the matter that dominates the universe, the mean characteristics of the cosmic structures depend on the assumed model. In this context the most succesful model is the biased Cold Dark Matter (hereafter CDM) (see *e.g.* Liddle & Lyth 1993) based on a scale invariant spectrum of density fluctuations growing under gravitational instability. In such scenario the formation of the structures occurs through a "*bottom up*" mechanism.

A simple model that describes the collapse of a perturbation of density is that of Gunn & Gott (1972, hereafther GG72). This is in contradiction with the predictions of CDM models. It is well known that in a CDM Universe, an abundant production of substructures during the evolution of the fluctuations is predicted. The presence of substructure is very important for the dynamics of collapsing shells of baryonic matter made of galaxies and substructure of $10^6 M_\odot \div 10^9 M_\odot$, falling into the central regions of a cluster of galaxies. In presence of substructure it is necessary to modify the equation of motion:

$$\frac{d^2 r}{dt^2} = -\frac{GM}{r^2(t)} \tag{2}$$

since the graininess of mass distribution in the system induces dynamical friction that introduces a frictional force term. In a material system, the gravitational field can be decomposed into an average field, $\mathbf{F}_0(r)$, generated from the smoothed out distribution of mass, and a stochastic component of the field, $\mathbf{F}_{stoch}(r)$, generated from the fluctuations in number of the neighbouring particles. The stochastic component of the gravitational field is specified assigning a probability density, $W(\mathbf{F})$, (Chandrasekhar & von Neumann 1942, hereafter CvN42). In an infinite homogeneous unclustered system $W(\mathbf{F})$ is given by the Holtsmark distribution (CvN42) whilst in inhomogeneous and clustered systems $W(\mathbf{F})$ is given by Kandrup (1980) and Antonuccio-Delogu & Atrio-Barandela (1992, hereafter AA92) and respectively. The stochastic force, $\mathbf{F}_{stoch}$, in a self-gravitating system modifies the motion of particles as if it was a frictional force. In fact, a particle moving faster than its neighbours produces a deflection of their orbits in such a way that the average density is greater in the direction opposite to that of motion, causing a slowing down in its motion. Adopting the notation of GG72 (see also their eqs. 6 and 8) and remembering that $T_{c0}/2$ is the collapse time in the absence of dynamical friction (GG72), one can write:

$$T_{c0} = \frac{\pi \bar{\rho}_i \rho_{ci}^{1/2}}{H_i (\bar{\rho}_i - \rho_{ci})^{3/2}} \tag{3}$$

where ρ_{ci} is the critical density at a time t_i and $\bar{\rho}_i$ is the *average density* inside r_i at t_i. The equation of motion of a shell of baryonic matter in presence of dynamical friction, using the dimensionless time variable $\tau = \frac{t}{T_{c0}}$, can be written in the form:

$$\frac{d^2 a}{d\tau^2} = -\frac{4\pi G \rho_{ci}(1 + \overline{\delta_i})}{a^2(t)} T_{co}^2 - \eta T_{c0} \frac{da}{d\tau} \tag{4}$$

(Antonuccio-Delogu & Colafrancesco 1994, hereafter AC94) where $\overline{\delta_i}$ is the overdensity within r_i and η is the coefficient of dynamical friction. Supposing that

there are no correlations among random force and their derivatives, we have:

$$\eta = \frac{\int d^3 F W(F) F^2 T(F)}{2\langle v^2 \rangle} \tag{5}$$

(Kandrup 1980), where $T(F)$ is the average *"duration"* of a random force impulse of magnitude F, $W(F)$ is the probability distribution of stochastic force (which for a clustered system is given in eq. 37 of AA92).

2. The collapse time

In a previous paper (Del Popolo & Gambera 1996, hereafter DPG96), one of us showed how the expansion parameter $a(\tau)$ depends on the dynamical friction, solving eq.(4) numerically. DPG96 assigned some values to η and then solved eq.(4) through a numerical method not taking into account the parameters on which η depends. In this paper, we examine how the dynamical friction coefficient η varies according to the parameters and how the collapse time depends on them. We solved eq.(5) numerically and the other equations on which it depends for a outskirts shell of baryonic matter with $\overline{\delta} = 0.01$ inside the spherical regions (protocluster), for different values of ν_c, R_f, r_0 and Ξ, where:

- ν_c is the peaks' height;

- R_f is the filtering radius;

- r_0 is the parameter of the power-law density profile; Theoretical work (Ryden 1988) suggests that the density profile inside a protogalactic dark matter halo, before relaxation and baryonic infall, can be approximated by a power-law:

$$\rho(r) = \frac{\rho_0 r_0^p}{r^p} \tag{6}$$

  where $p \approx 1.6$ on a protogalactic scale.

- Ξ is the product $N_{tot} \cdot \xi(0)$ where N_{tot} is the total number of peaks inside a protocluster and $\xi(0)$ is the correlation function calculated in $r = 0$. AA92 have demonstrated that in the hypothesis $m_{av} \gg 1 M_\odot$, where m_{av} is the average mass of the subpeaks, the dependence of the dynamical friction coefficient on N_{tot} and $\xi(r)$ may be expressed as a dependence on a single parameter that we define as:

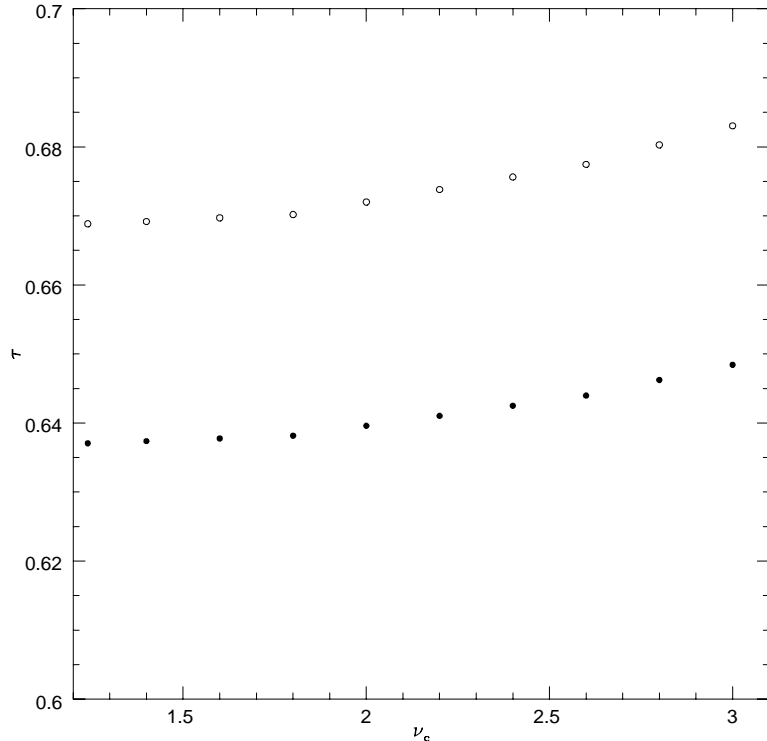$$\Xi \equiv N_{tot} \cdot \xi(0) \tag{7}$$

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Figure 1.   Collapse time τ of a shell of matter made of galaxies and substructure when dynamical friction is taken into account, versus ν_c We assume a core radius of $r_0 = 1h^{-1}Mpc$ and a filtering radius $R_f = 0.74h^{-1}Mpc$. *Open circles:* $\Xi = 10^3$; *filled circles:* $\Xi = 10^2$.

After having determinated η solving numerically eq. (5), we get τ as a function of ν_c, R_f, r_0 and Ξ solving eq.(4). We perform these calculations for different set of values of ν_c, R_f, r_0 and Ξ; the results are shown in figs. 1÷4. Before commenting upon the figures, we want to remark that the dependence of τ on $\overline{\delta}$ is qualitatively shown in fig. 5 of AC94. We observe that for $\overline{\delta} > 10^{-2}$ the collapse time in presence of dynamical friction is always larger than in the imperturbated case but the magnitude of the deviation is negligible for larger $\overline{\delta}$, whilst for $\overline{\delta} \leq 10^{-2}$ the deviations increase steeply with lower $\overline{\delta}$. Then, having considered $\overline{\delta} = 0.01$, the estimation we get for τ in §4 must be considered as a lower limit. In figure 1 we show the collapse time in presence of dynamical friction, versus the peaks' height, for different values of Ξ. In this picture, we show how τ grows for larger values of ν_c and for larger values of Ξ. Similarly, in fig. 2 we note how τ increases for larger values of ν_c and of R_f. The slope of the curves confirm our prevision on the behaviour of the collapse of a shell of baryonic matter falling into the central regions of a cluster of galaxies in the presence of dynamical friction: the dynamical friction slows down the collapse (as DPG96 had already shown) and the effect, as we are showing in figs. 1 and
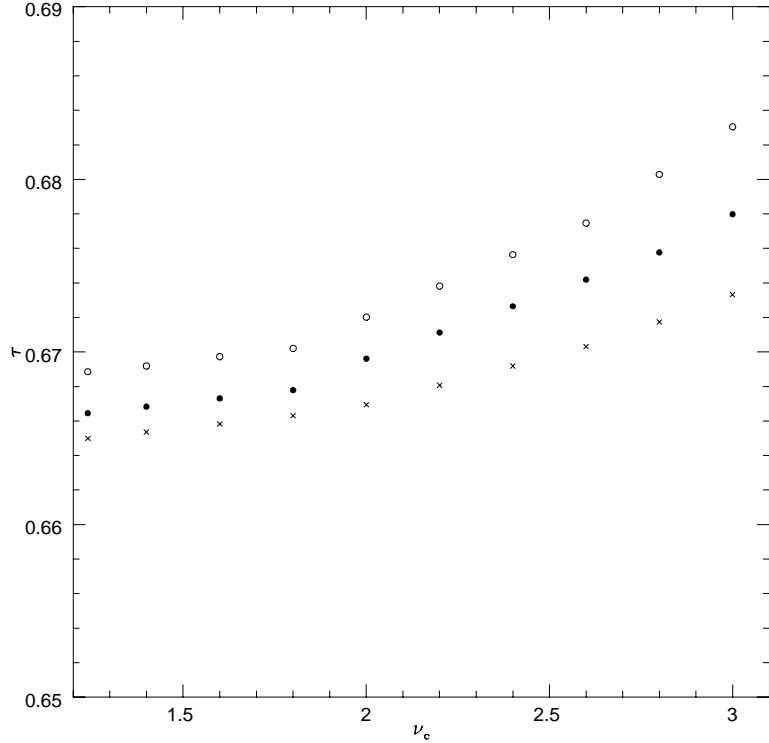
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Figure 2. Collapse time τ of a shell of matter made of galaxies and substructure when dynamical friction is taken into account, versus ν_c We assume a core radius of $r_0 = 1h^{-1}Mpc$ and a fixed correlation $\Xi = 10^3$. *Open circles: $R_f = 0.74h^{-1}Mpc$; filled circles: $R_f = 0.65h^{-1}Mpc$; crosses: $R_f = 0.55h^{-1}Mpc$.*

2, increases as Ξ, R_f, ν_c grow. Here we want to remind that we are considering only the peaks of the local density field with central height ν larger than a critical threshold ν_c. This latter quantity is chosen to be the threshold at which r_{peak} $(\nu \geq \nu_c) << l_{av}$ where r_{peak} is the typical size of the peaks and l_{av} is the average peak separation (see also Bardeen et al. 1986).

In figs. 3 and 4 we show how the collapse time varies with the core radius of the protocluster r_0. Note how τ grows as r_0 decreases: the smaller the core of the protocluster, the larger the time of collapse in the presence of dynamical friction; besides we show how this effect increases for larger values of both ν_c and Ξ.
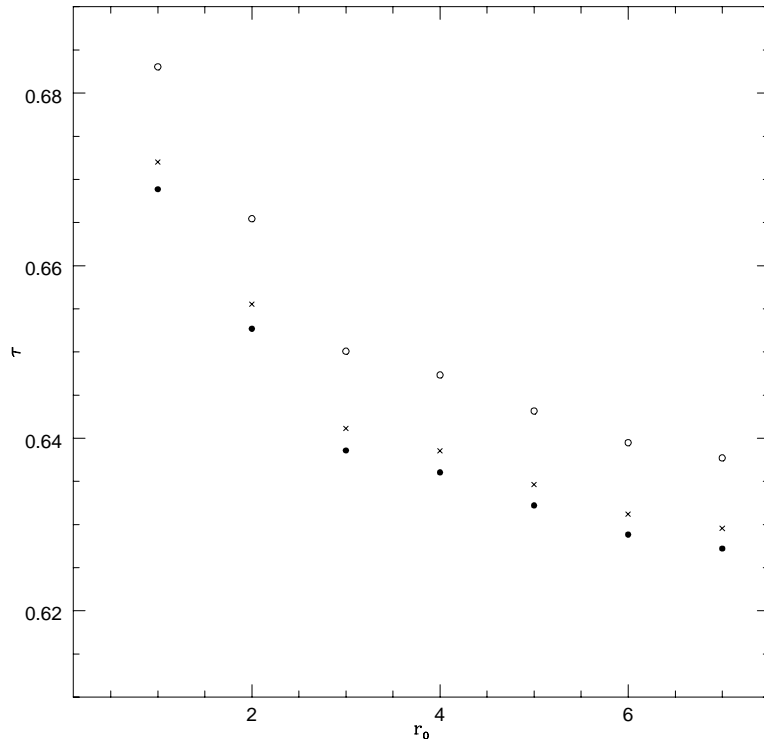
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Figure 3. Collapse time τ versus r_0 We assume a filtering radius $R_f = 0.74 h^{-1} Mpc$. and a total number of peaks of substructure $\Xi = 10^3$. *Open circles: $\nu_c = 3$; crosses: $\nu_c = 2$; filled circles: $\nu_c = 1.24$.*

3.   Conclusions and discussion

In the first part of this work we have shown how the collapse time τ of a shell of baryonic matter made of galaxies and substructure depends on some parameters. In figures 1 and 2 we can see how τ grows when Ξ or R_f or ν_c increases. It means that the effects of the presence of the dynamical friction should be more evident in the outer regions of rich clusters of galaxies. Besides, we show how the collapse time of an infalling shell increases with decreasing values of r_0, and becomes very large for $r_0 \leq 2 Mpc$ (see fig. 3). Then the slowing down of the collapse of an outer shell, of baryonic matter, within a cluster of galaxies owing to the dynamical friction is more remarkable in the clusters with core of little dimension.
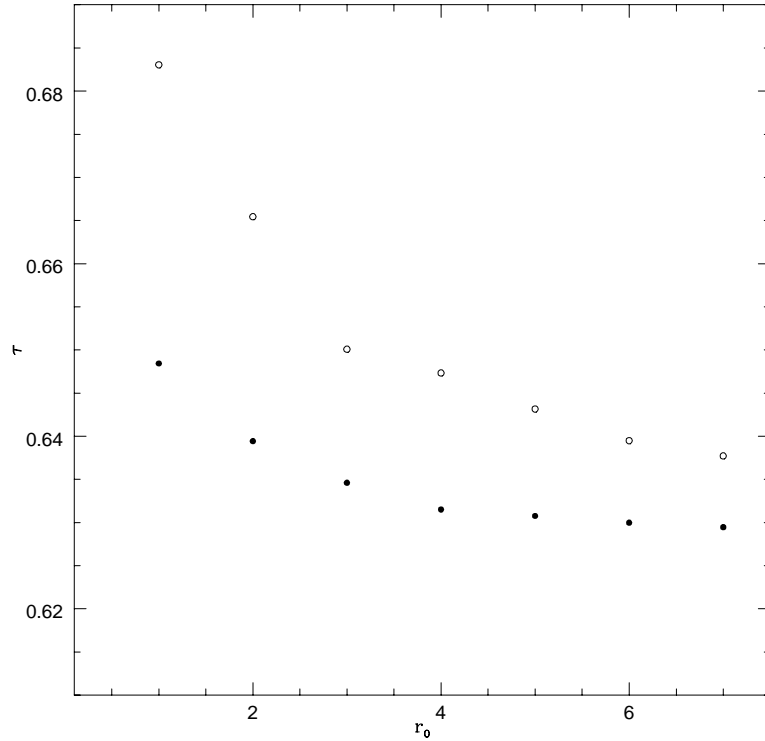
Figure 4.   Collapse time τ of a shell of matter made of galaxies and substructure when dynamical friction is taken into account, versus r_0 We assume a filtering radius $R_f = 0.74h^{-1}Mpc$ and a peaks' height $\nu_c = 3$. *Open circles:* $\Xi = 10^3$; *filled circles:* $\Xi = 10^2$.

References

Antonuccio-Delogu, V., 1992, Ph.D. dissertation, ISAS, Trieste

Antonuccio-Delogu, V., Atrio-Barandela, F., 1992, Ap.J 392, 403 (AA92)

Antonuccio-Delogu, V., Colafrancesco, S., 1994, Ap.J. 427, 72 (AC94)

Bardeen, J.M, Bond, J.R., Kaiser, N., Szalay, A.S., 1986, Ap.J 304, 15

Colafrancesco, S., Lucchin, F., Matarrese, S., 1989, Ap.J 345, 3

Colafrancesco, S., Antonuccio, V., Del Popolo, A., 1995, Ap. J

Chandrasekhar, S., von Neumann, J., 1942, Ap.J 95, 489 (CvN)

Chandrasekhar, S., von Neumann, J., 1943,Ap.J 97,1

Del Popolo, A., Gambera, M., accepted for the pubblication Astr. & Astrop. (DPG96)

Efstathiou, G., Rees, M.J., 1988, MNRAS, 230, 5p.

Gunn, J.E., Gott, J.R., 1972, Ap.J 176, 1 (GG72)

Kaiser, N., 1993, in Cosmic velocity fields, ed. F.R. Bouchet, M. Lachieze-Rey, (Gif-sur-Yvette: Editions Frontieres), 533

Kandrup, H.E., 1980, Phys. Rep. 63, n 1, 1

Kolb, E.W., Turner, M.S., 1990, The early Universe (Addison-Wesley)

Liddle, A.R., Lyth, D.H.,1993, Phys. Rep. 231, n 1, 2

Peacock, J.A., Heavens, A.F., 1990, MNRAS 243, 133

Ryden, B.S., 1988, Ap.J 329, 589

Ryden, B.S., & Gunn, J.E., 1987, Ap.J 318, 15