COHERENCE AND SAKHAROV OSCILLATIONS IN THE MICROWAVE SKY[1]

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Abstract

I discuss the origin of the "Sakharov oscillations" (or "secondary Doppler peaks") in standard angular power spectra of the Cosmic Microwave Background anisotropies calculated for inflationary models. I argue that these oscillations appear because perturbations from inflation have a set of properties which makes them "passive perturbations". All passive perturbations undergo a period of linear "squeezing" resulting in a dramatic degree of (classical) phase coherence of pressure waves in the photon-baryon fluid. This phase coherence eventually is reflected in oscillatory features in the angular power spectrum of the temperature anisotropies observed today. Perturbations from cosmic defects are "active perturbations" which have sharply contrasting properties. Active perturbations are highly non-linear and the degree of phase coherence in a given model reflects the interplay between competing effects. A large class of active models have *non*-oscillatory angular power spectra, and only an extremely exotic class has the same degree of coherence as is found in all passive models. I discuss the significance of the search for these oscillations (which transcends the testing of any given model) and take a critical look at the degree to which the question of coherence has been treated so far in the literature.

1 Introduction

The Cosmic Microwave Background (CMB) provides us with perhaps the clearest window on the very early universe. Based just on our current understanding, the impact of the next generation of high resolution CMB experiments on theoretical cosmology is guaranteed to be enormous, and full implications of the new data have yet to be determined.

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The subject of this paper is the distinctive "Secondary Doppler Peaks" or "Sakharov Oscillations[1]" in the angular power spectra of the microwave anisotropies calculated for inflationary models. I discuss how these features reflect the very specific properties of "passive perturbations" of which the inflationary models are a subset. The very different nature of the defect based (or "active") models suppresses the tendency to generate Sakharov oscillations, although a different mechanism *can* produce Sakharov oscillations in certain active models.

Section 2 outlines the basic ingredients of a of CMB anisotropy calculation, emphasizing the differences between active and passive models. Section 3 spells out how the Sakharov oscillations appear in passive models. Section 4 describes the basic properties of active models make it hard to produce oscillations in the angular power spectrum, but also points out how some degree of oscillation is still possible. At the end of Section 4 I comment of the degree to which this issue has been addressed quantitatively in the literature. Concluding comments appear in Section 5.

Much of this paper is based on work with my collaborators P. Ferreira, J Magueijo, and D. Coulson, as reported in [2, 3, 4].

2 The evolution of the perturbations

Most models of structure formation consider perturbation which originate at an extremely early time (eg the GUT era or even the Planck era) and which have very small amplitudes (of order 10^{-6}) until well into the matter era. Perturbations of inflationary origin start as short wavelength quantum fluctuations which evolve (during the inflationary period) into classical perturbations on scales of astronomical interest. Defect based models undergo a phase transition (typically at around GUT temperatures, eg $T \approx 10^{16} GeV$) forming defects which generate inhomogeneities on all scales.

For all these models, once the inflationary period and/or phase transition is over, the Universe enters an epoch where all the matter components obey linear equations except for the defects (if they are present). This "Standard Big Bang" epoch can be divided into three distinct periods. The first of these is the "tight coupling" period where radiation and baryonic matter are tightly coupled and behave as a single perfect fluid. When the optical depth grows sufficiently the coupling becomes imperfect and the "damping period" is entered. Finally there is the "free streaming" period, where the CMB photons only interact with the other matter via gravity. While the second and third periods can have a significant impact on the overall shape of the angular power spectrum, all the physics which produces the Sakharov oscillations takes place in the tight coupling regime, which is the focus of the rest of this paper.

Working in in synchronous gauge, and following the conventions and definitions in references [5, 6, 2], the Fourier space perturbation equations are:

$$
\begin{aligned}
\dot{\tau}_{00} &= \Theta_D + \frac{1}{2\pi G}\left(\frac{\dot{a}}{a}\right)^2 \Omega_r \dot{s}\,[1+R] && (1)\\
\dot{\delta}_c &= 4\pi G \frac{a}{\dot{a}}\left(\tau_{00} - \Theta_{00}\right) - \frac{\dot{a}}{a}\left(\frac{3}{2}\Omega_c + 2\,[1+R]\,\Omega_r\right)\delta_c \\
&\quad - \frac{\dot{a}}{a} 2\,[1+R]\,\Omega_r s && (2)\\
\ddot{s} &= -\frac{\dot{R}}{1+R}\dot{s} - c_s^2 k^2 \left(s + \delta_c\right) && (3)
\end{aligned}
$$

Here $\tau_{\mu\nu}$ is the pseudo-stress tensor, $\Theta_D \equiv \partial_i \Theta_{0i}$, $\Theta_{\mu\nu}$ is the defect stress energy, a is the cosmic scale factor, G is Newton's constant, δ_X is the density contrast and Ω_X is the mean energy density over critical density of species X ($X = r$ for relativistic matter, c for cold matter, B for baryonic matter), $s \equiv \frac{3}{4}\delta_r - \delta_c$, $R = \frac{3}{4}\rho_B/\rho_r$, ρ_B and ρ_r are the mean densities in baryonic and relativistic matter respectively, c_s is the speed of sound and k is the comoving wavenumber. The dot denotes the conformal time derivative ∂_η.

In the inflationary case there are no defects and $\Theta_{\mu\nu} = 0$. With suitable initial conditions these linear equations completely describe the evolution of the perturbations. In the defect case $\Theta_{\mu\nu} \neq 0$, and certain components[2] of $\Theta_{\mu\nu}(\eta)$ are required as input. Cosmic defects are "stiff", which means $\Theta_{\mu\nu}(\eta)$ can be viewed as an external source for these equations. The additional equations from which one determines $\Theta_{\mu\nu}(\eta)$ are highly non-linear, although the solutions tend to have certain scaling properties which allow $\Theta_{\mu\nu}(\eta)$ to be modelled using a variety of techniques (see for example [2, 8, 7]).

3 The passive case: Squeezing and phase coherence

Quite generically, for wavelengths larger than the Hubble radius ($R_H \equiv a/\dot{a}$), Eqns [1-3] have one decaying and one growing solution. The growing solution reflects the gravitational instability, and, as required of any system which conserves phase space volume, there is a corresponding decaying solution. For example, in the radiation dominated epoch, the two long wavelength solutions for the radiation perturbation δ_r are $\delta_r \propto \eta^2$ and $\delta_r \propto \eta^{-2}$. Over time, R_H grows compared with a comoving wavelength so in the Standard Big Bang epoch a given mode starts with wavelength $\lambda >> R_H$ but eventually crosses into the $\lambda < R_H$ regime. In the period of tight coupling modes with $\lambda < R_H$ undergo oscillatory behavior since the radiation pressure stabilizes the fluid against gravitational collapse. This process of first undergoing unstable behavior which eventually converts to oscillatory behavior is the key to the formation of Sakharov oscillations.

I will now illustrate the process with a simple toy model. The simplest example of growing/decaying behavior is given by the upside down harmonic oscillator ($\ddot{q} = q$) which has the general solution $q = Ae^t + Be^{-t}$). Figure 1 (left panel) shows the trajectories in phase space for this system. As the "particle" rolls down the inverted parabolic potential both q and $\dot{q}$ increase arbitrarily and *any* starting point in phase space evolves arbitrarily close to the $\dot{q} = q$ axis. Any initial region in phase space will become squeezed against this axis and elongated along it, as illustrated by the circle in the figure. This "squeezing[3]" process is generic to any equation which has one growing and one decaying solution, although the simple shape of the squeezed region is specific to models with linear equations.

The right-side-up harmonic oscillator ($\ddot{q} = -q$) serves to illustrate the oscillating regime (when the wavelength is smaller than R_H). The right hand panel in Fig 1 shows the phase space trajectories for the right-side-up harmonic oscillator. The familiar oscillatory behavior describes circles in the $q - \dot{q}$ plane (in polar coordinates the angle corresponds to phase of the oscillation). The linearity ensures that the period of rotation is the same on all trajectories, thus preserving the shapes of any initial region as it rotates around.

The physical system in question (e.g. δ_r) undergoes first squeezing and then oscillatory behavior. During the unstable period the initial phase space distribution (dictated, in the case of an inflationary scenario, by the quantum zero-point fluctuations) is squeezed by many orders of magnitude. The distribution which enters the oscillatory period is thus highly squeezed

[2] Conservation of stress energy allows the equations to be manipulated so that different components of $\Theta_{\mu\nu}$ are required as input (a matter mainly of numerical convenience)[5, 6, 7, 8, 9, 10, 11].

[3] These dynamics are similar to those producing "squeezed states" of light in quantum optics, but the effect discussed here is completely classical. [12]

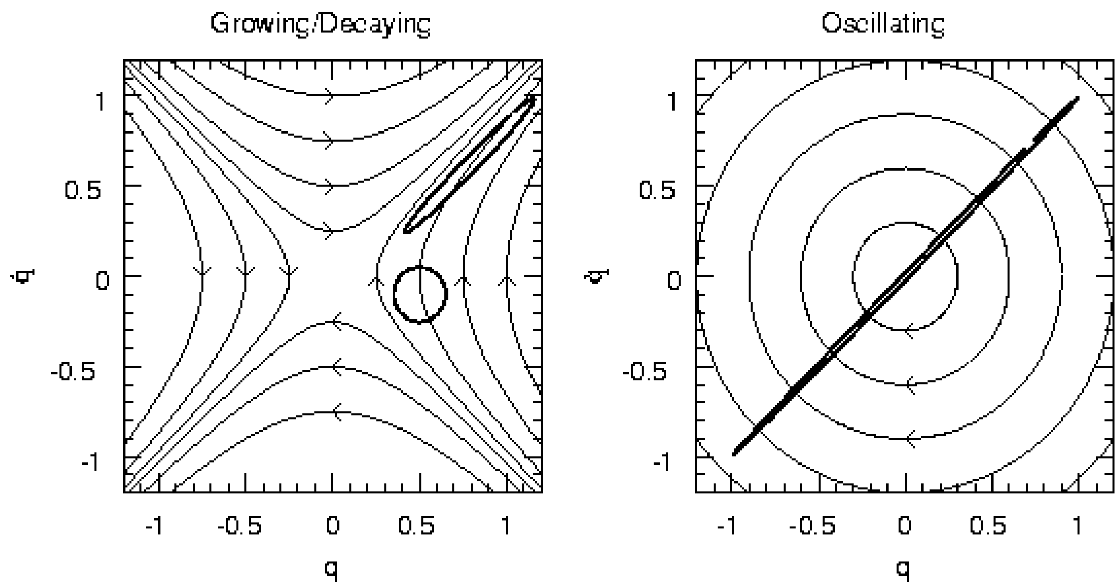

Figure 1: Phase space trajectories for the up-side-down harmonic oscillator (left panel) and right-side-up harmonic oscillator (right panel). These toy models illustrate the growing/decaying regime and oscillating regime (respectively). The growing/decaying regime causes "squeezing", which drives all solutions toward (and outward along) the $q = \ddot{q}$ axis. In a passive model the perturbations first encounter the squeezing regime and thus the phase space distribution which enters oscillatory regime is highly squeezed and a unique temporal phase (up to a shift by π) is specified for the oscillatory regime. The elongated curve on the right panel is the result of squeezing a circle (centered at the origin) by a factor of 100. Typically inflation models will have squeeze factors of 10^{20} or greater.

(much more so than the oblong shape depicted in the right panel of Fig 1). The end result it that the temporal phase of oscillation is rigorously dictated by the period of squeezing which went before.

This effect is illustrated more directly in Fig 2, where different solutions for δ_r are shown. Representative solutions are shown from across the entire ensemble, the growing solution domination guarantees that each one goes through zero at essentially the same time.

Figure 3 shows the ensemble averaged values of δ at a fixed time as a function of wavenumber (the power spectrum). The zeros correspond to modes which have been caught at the "zero-point" of their oscillations. The phase focusing across the entire ensemble guarantees that there will always be some wavenumbers where the power is zero. It is these zeros which are at the root of the oscillations in the angular power spectrum.

An important point is that these zeros are an absolutely fundamental feature in any passive theory. No amount of tinkering with the details can counteract that fact that an extended period of liner evolution will lead to growing mode domination, which in turn fixes the phase of the oscillations in the tight coupling era. If one were to require oscillation in a passive model to be out of phase from the prescribed value, one would imply domination by the decaying mode outside the Hubble radius – in other words a Universe which is not at all Robertson-Walker on scales greater than R_H.

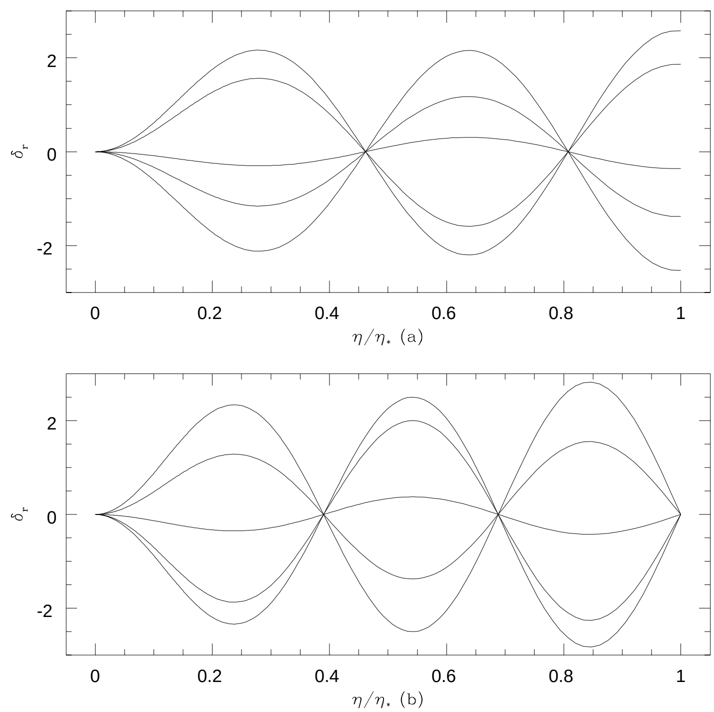

Figure 2: Passive perturbations: Evolution of two different modes during the tight coupling era. While in (a) elements of the ensemble have non-zero values at $\eta_\star$, in (b), *all* members of the ensemble will go to zero at the final time ($\eta_\star$), due to the fixed phase of oscillation set by the domination of the growing solution (or squeezing) which occurs before the onset of the oscillatory phase. The y-axis is in arbitrary units.

4 The active case: Coherence lost

4.1 The nature of the ensemble

As described in Section 2, the active case is very different from the passive case, due to the presence of what is effectively a source term in Eqns [1-3]. One consequence is that the whole notion of the ensemble average is changed. In the passive case any model with Gaussian initial conditions can be solved by solving Eqns [1-3] with the initial values for all quantities given by their initial RMS values. The properties of linearly evolved Gaussian distributions guarantee that this solution will always give the RMS values at any time. Thus the entire ensemble is represented by one solution.

In the active case this is not in general possible. One has to average over an ensemble of possible source histories, which is a much more involved calculation. In [3] we "square" the evolution equations to write the power spectrum as convolution of two-point functions of the sources, but there the added complexity requires the use of the full *un*equal time correlation functions.

4.2 Non-coherence

In general, the source term will "drive" the other matter components, and temporal phase coherence will be only as strong as it is within the ensemble of source terms. An illustration of this appears in Fig 4. In many active models the sources are sufficiently decoherent that no

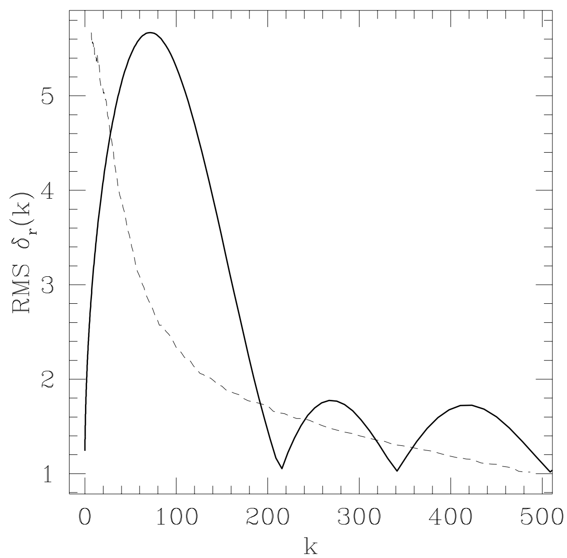

Figure 3: The r.m.s. value of δ_r evaluated at decoupling ($\eta_\star$) for a passive model (solid) and an active model (dashed). This is Fig 2 from ref [2] where the details are presented.

oscillations appear in the power spectrum (see for example the dashed curve in Fig 3).

4.3 Coherence regained

The source evolution is a highly non-linear process, so from the point view of a single wavenumber the source may be viewed as a "random" force term. At first glance it may seem impossible for such random force term could induce *any* temporal coherence, but here is how temporal coherence can occur: The source term only plays a significant role in Eqns [1-3] for a *finite* period of time. This is somewhat apparent in Fig 4. (The y-axis of Fig 4 shows a quantity specially chosen to indicate the significance of the source in Eqns [1-3].) In the limit where this period of significance is short compared to the natural oscillation time of δ_r (and happens at the same time across the entire ensemble) the ensemble of source histories *can* be phase coherent. The tendency for a given active model to produce oscillations in the power spectrum depends on the how "sharply peaked" the significance of the source term is in time.

*Note added:*It is also the case that on scales larger than R_H there are "squeezing" mechanisms at work, even for active perturbations. (The gravitational instability is, after all, still present.) In [**?**] a Green function method is developed which clearly illustrates how the active case involves a competition between squeezing effects, which tend to produce oscillations in the power spectrum, and the randomizing effect of the nonlinear source evolution. In extreme cases, where the "randomizing" effects are minimized it is even possible to have active models with mimic an inflationary signal[9, 10, 11].

4.4 Current Status

So far, we are just beginning to learn the degree of coherence which is present in the most popular active models. Much of the work makes use of the "high coherence limit" in which a single solution to Eqns [1-3] is meant to represent the RMS value. This only makes physical sense for the "sharply peaked" sources discussed in the previous subsection[3], but allows one to use code designed for passive perturbations with only minor changes.

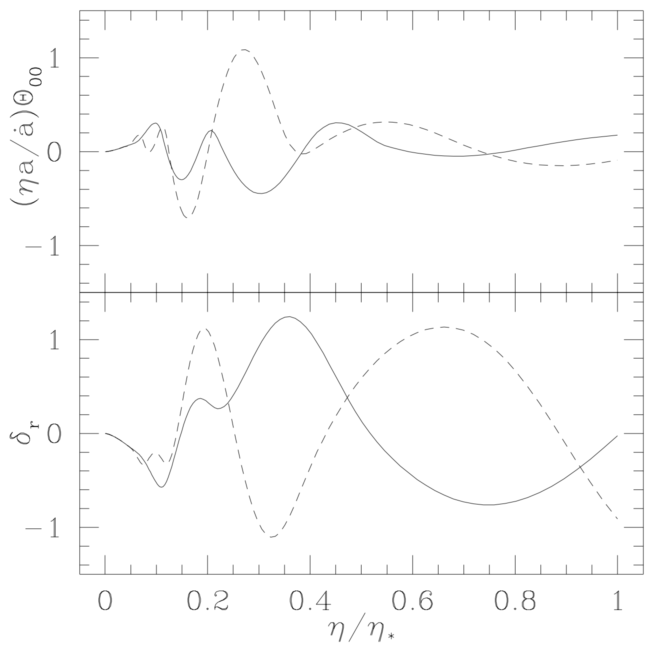

Figure 4: Active perturbations: Evolution of $\delta_r(k)$ and the corresponding source Θ_{00} during the tight coupling era (Θ_D is not shown). Two members of the ensemble are shown, with matching line types. Due to the randomness of the source, the ensemble includes solutions with a wide range of values at $\eta_\star$. Unlike the inflationary case (Figure 1) the phase of the temporal oscillations is not fixed. The y axis is in arbitrary units, and the source models are the same as for figure 2. The factor $\eta a/\dot{a}$ allows one to judge the relative importance (over time) of the Θ_{00} term in Eqn 2.

The question of coherence has been most aggressively pursued in the cosmic string case[2, 3, 4, 13], and every indication is that cosmic strings give a highly decoherent ensemble. However, there will still be room for some degree of skepticism until the production of gravity waves and small loops is incorporated in some realistic way ([14], see also the methods of[15]). Most of the work on cosmic texture models has not dealt quantitatively with the question of evidence for coherence in the microwave sky although it is pretty clear that textures should have more coherence than the cosmic strings. In [8] the "high coherence" limit was used for calculating the microwave sky, but numerical simulations were used to illustrate some coherent behaviour in the tight coupling era. In other calculations the high coherence limit has been used for convenience, and these papers have simply not claimed to treat the question of coherence. Recent simulations by Turok [15] are the sort which can in principle address the coherence question for a large number of active models, but the author is not willing to claim conclusive results until a large dynamic range is achieved.

5 Conclusions

The Sakharov oscillations (or secondary Doppler peaks) in the angular power spectrum of CMB anisotropies signify a high degree of coherence in the primordial perturbations. These oscillations are a certain prediction of all passive models (which includes all inflation based models) and can not be adjusted away[4]. As such, this prediction represents probably the most clear-cut opportunity to falsify *all* scenarios in which the perturbations have an inflationary origin. On the other hand the observation of substantial Sakharov oscillations in the data would have an enormous impact on the active models, ruling out all be a very special subset of these. I conclude that the search for Sakharov oscillations in the CMB sky represents an opportunity to gain very deep insights into the origin of the primordial perturbations. Every effort should be made to make sure that the experiments are able to achieve conclusive results on this matter[16].

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[4] I suppose it might be possible to construct some undulating inflaton potential which gives a non-oscillatory temperature anisotropy spectrum, but then the oscillations would turn up in other observations (such as those of the density field or CMB polarization).