

# Nonequilibrium cosmic neutrinos and nucleosynthesis.

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## Abstract

The neutrino role in primordial nucleosynthesis is reviewed. The importance of nonequilibrium effects is emphasized both for the standard massless and possibly massive neutrinos. The upper bound on tau neutrino mass is presented. A spatial variation of primordial abundances and a possibility of observing them by precise measurements of the CMB anisotropy are considered. The nucleosynthesis bounds on the parameters of neutrino oscillations into sterile neutrinos are discussed.

## 1 Introduction

It is well known that neutrinos have a very important impact on cosmology (for a recent review and a list of references see e.g. [1]). In particular, the comparison of primordial nucleosynthesis theory with observational data permits to put rather stringent bounds on neutrino properties, their mass, number of flavors, possible new interactions, etc. To this end a detailed study of neutrino kinetics in the primeval plasma, especially nonequilibrium corrections, which happen to be quite essential, is of primary importance and the proper treatment of the latter significantly changes the results of the simpler equilibrium calculations. Technically the problem is very complicated and demands an accurate numerical solution of a system of coupled integro-differential kinetic equations, however in many cases a rough order-of-magnitude estimate of non-equilibrium corrections can be done analytically.

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Surprisingly nonequilibrium corrections to the energy spectrum are not very small even for massless neutrinos in the standard model and of course they are very large in the case of possibly heavy  $\nu_\tau$  with the mass in MeV range. Account of nonequilibrium effects permits to considerably improve the nucleosynthesis bounds on the mass of  $\nu_\tau$ .

Another important effect, where deviations from the standard equilibrium non-degenerate Fermi-Dirac distribution

$$f(p, t) = \frac{1}{\exp[E/T(t)] + 1} \quad (1)$$

is essential for nucleosynthesis, is a possible lepton asymmetry. The latter could be either primordial, generated at a very early stage, or it may arise during nucleosynthesis epoch due to nonequilibrium neutrino oscillations. Even if lepton asymmetry is not generated by oscillations, they still might have a very important impact on nucleosynthesis and the study of nucleosynthesis leads to interesting bounds on oscillation parameters.

If there are some new particles, abundant in the plasma during nucleosynthesis, they would change the cooling rate of the plasma and thus change the standard abundances of the light elements. Normally the effect of such new particles is just to change the expansion/cooling rates but in some cases their interaction may also produce nonequilibrium neutrinos, especially  $\nu_e$ , and in this case the impact on nucleosynthesis would be significantly different.

In what follows I briefly review these subjects. In section 2 nonequilibrium corrections to the spectra of normal massless neutrinos and their possible observational manifestations are discussed. In section 3 the role of a possibly heavy  $\nu_\tau$  in nucleosynthesis is considered and an upper bound on its mass is presented. Neutrino degeneracy and especially a possible variation of the latter on the cosmological scales (a few 100 Mpc or even Gpc) is discussed in section 4. In section 5 neutrino oscillations are considered. In Conclusion the main results of this brief review is summarized.

## 2 Nonequilibrium massless neutrinos

It is usually assumed that thermal relics with  $m = 0$  are in perfect equilibrium state even after decoupling. For the photons in cosmic microwave background (CMB) it is known with a very high accuracy. The same assumption is made about neutrinos so that their distribution is given by eq. (1). Indeed when the interaction rate is high in comparison with the expansion rate,  $\Gamma_{int} \gg H$ , the equilibrium is evidently established. When interactions can be neglected the distribution function may have an arbitrary form but for massless particles the equilibrium distribution is preserved if it was established earlier at a dense and hot stage when the interaction was fast. One can see that from kinetic equation in the expanding universe:

$$(\partial_t - Hp\partial_p)f_j(p_j, t) = I_j^{coll} \quad (2)$$

where the collision integral in the r.h.s. vanishes for the equilibrium functions:

$$f^{(eq)} = \left(e^{E/T - \mu/T} \pm 1\right)^{-1} \quad (3)$$

The temperature  $T$  and chemical potential  $\mu$  may be functions of time.

The l.h.s. is annihilated by  $f = f^{(eq)}$  if the following condition is fulfilled for any value of particle energy  $E$  and momentum  $p = \sqrt{E^2 - m^2}$ :

$$\frac{\dot{T}}{T} + H\frac{p}{E}\frac{\partial E}{\partial p} - \frac{\mu}{E}\left(\frac{\dot{\mu}}{\mu} - \frac{\dot{T}}{T}\right) = 0 \quad (4)$$

This can only be true if  $p = E$  (i.e  $m = 0$ ),  $\dot{T}/T = -H$ , and  $\mu \sim T$ . It can be shown that for massless particles, which initially possessed equilibrium distribution, temperature and chemical potential indeed satisfy these requirements for  $I^{coll} = 0$ , so the equilibrium distribution is not destroyed even when the interaction is switched off.

It would be true for neutrinos if they instantly decoupled from the electromagnetic component of the plasma (electrons, positrons, and photons) at the moment when

neutrino interactions was strong enough so that at the moment of decoupling they were in thermally equilibrium state with the same temperature as photons and  $e^\pm$ . According to simple estimates the decoupling temperature,  $T_{dec}$ , for  $\nu_e$  is about 2 MeV and that for  $\nu_\mu$  and  $\nu_\tau$  is about 3 MeV. In reality the decoupling is not instantaneous and even below  $T_{dec}$  there is some residual interaction between  $e^\pm$  and neutrinos. An important point is that after neutrino decoupling the temperature of the electromagnetic component of the plasma became somewhat higher than the neutrino temperature. The electromagnetic part of the plasma is heated by the annihilation of *massive* electrons and positrons. This is the well known effect which ultimately results in the present day ratio of temperatures,  $T_\gamma/T_\nu = (11/4)^{1/3}$ . During primordial nucleosynthesis the temperature difference between electromagnetic and neutrino components of the plasma is small but still non-vanishing. Due to this temperature difference the annihilation of the hotter electrons/positrons,  $e^+e^- \rightarrow \bar{\nu}\nu$ , heats up the neutrino component of the plasma and distorts neutrino spectrum. The average neutrino heating under assumption of their equilibrium spectrum was estimated in refs. [2, 3]. Spectrum distortion in Boltzmann approximation was calculated numerically in ref. [4] and analytically in [5]. In accordance with the latter it takes the form:

$$\frac{\delta f_{\nu_e}}{f_{\nu_e}} \approx 3 \cdot 10^{-4} \frac{E}{T} \left( \frac{11E}{4T} - 3 \right) \quad (5)$$

The distortion of the spectra of  $\nu_\mu$  and  $\nu_\tau$  is approximately twice weaker. Here  $\delta f = f - f^{(eq)}$ .

An exact numerical treatment of the problem was first done in ref. [6] and later, with a better precision and a corrected expression for the matrix element of one of the participating reactions, in ref. [7]. The system of coupled kinetic equations (2) governing the neutrino distribution functions with the collision integral of the form

$$I_{coll} = \frac{1}{2E_1} \sum \int \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_3}{2E_3(2\pi)^3} \frac{d^3p_4}{2E_4(2\pi)^3} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) F(f_1, f_2, f_3, f_4) S |A|_{12 \rightarrow 34}^2 \quad (6)$$

was solved numerically with the precision about  $10^{-4}$ . In the expression (6)

$$F(f_1, f_2, f_3, f_4) = f_3 f_4 (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_3)(1 - f_4) \quad (7)$$

and  $|A|_{12 \rightarrow 34}^2$  is the matrix element squared of the 4-fermion weak interaction. The results of ref. [7] confirmed the shape of spectrum distortion (5). The total relative change in neutrino energy densities was found to be

$$\delta\rho_{\nu_e}/\rho_0 = 0.9\%, \quad \delta\rho_{\nu_\mu, \nu_\tau}/\rho_0 = 0.4\%, \quad (8)$$

where  $\rho_0$  is the unperturbed neutrino energy density.

Naively one would expect that distortion of neutrino energy density at a per cent level would result in the similar distortion in the primordial abundances of light elements. However this does not occur by the following reason. An excess of neutrinos at high energy tail of the spectrum results in excessive destruction of neutrons in the reaction:



and an excessive creation in the reaction:



This nonequilibrium contribution into the second process is more efficient because the number density of protons at nucleosynthesis (when  $T \approx 0.7$  MeV) is 6-7 times larger than that of neutrons. So an excess of high energy neutrinos results in an increase of the frozen neutron-to proton ratio,  $r$ , and in the corresponding increase of  ${}^4He$ . On the other hand an excess of neutrinos at low energies results in a decrease of  $r$  because reaction (10) is suppressed due to threshold effects. It happened that the discussed above nonequilibrium spectrum distortion took place in the middle between the two extremes and the net influence of these distortion on e.g.  ${}^4He$  is quite small, the change of the mass fraction of  ${}^4He$  is  $\sim 10^{-4}$ .

Though quite small, such extra heating of neutrinos may be in principle noticed in future high precision measurements of CMB anisotropies [8, 9]. A change in neutrino energy density with respect to the standard case would result in a shift of equilibrium epoch between matter and radiation, which is imprinted on the form of the angular spectrum of fluctuations of CMB. Because of potential observability of distortion of neutrino energy density it was recalculated in ref. [9] where a larger result, than found in the previous papers, was obtained. In this connection we [10] repeated our calculations with a larger number of integration points, and wider momentum range, checked the stability of our calculation procedure and confirmed our previous results [7] with the precision of about  $10^{-4}$ . One possible source of disagreement may be an incorrect probability of the reactions  $\nu_a + \nu_a \rightarrow \nu_a + \nu_a$  ( $a = e, \mu, \tau$ ) used in ref. [9] and a smaller number of integration points in the essential region.

### 3 Nonequilibrium massive $\nu_\tau$ and nucleosynthesis

It is well known that comparison of calculated primordial abundances of light elements with observations permits to put a constraint on the expansion/cooling rate at the primordial nucleosynthesis (NS) epoch. In particular such arguments allow to limit the number of possible neutrino species (or other particles abundant at NS) [11, 12, 13, 14]. The present day data seem to exclude one extra neutrino species and possibly even 0.3 (for a recent review and analysis see e.g. ref. [15]).

Similar arguments permit to put a stringent bound on the mass of  $\nu_\tau$ , considerably better than the existing direct experimental limit,  $m_{\nu_\tau} < 18$  MeV [16]. Using this result and nucleosynthesis data one can conclude that  $m_{\nu_\tau} < 0.5 - 1$  MeV, if such neutrino is stable at the nucleosynthesis time scale, i.e.  $\tau_{\nu_\tau} > 100$  sec.

Equilibrium energy density of massive particles is smaller than the energy density of massless ones. So if it was the case, then massive  $\nu_\tau$  would effectively correspond

to a smaller number of massless neutrinos. However when expansion rate  $H = \dot{a}/a$  becomes smaller than the rate of  $\nu_\tau$ -annihilation, equilibrium is no more maintained and the number and energy densities of  $\nu_\tau$  become larger than their equilibrium values. Because of that one massive  $\nu_\tau$  could correspond to several massless neutrino species. Original calculations of the bound on  $m_{\nu_\tau}$  [17] were made under the simplifying assumptions of Boltzmann statistics and kinetic equilibrium of all participating particles. In other words the distribution of massless  $\nu_e$  and  $\nu_\tau$  were taken as  $f = \exp(-E/T)$ , while the distribution of massive  $\nu_\tau$  were assumed to have the form:

$$f_{\nu_\tau} = \exp(-E/T + \xi) \quad (11)$$

where the dimensionless (pseudo)chemical potential  $\xi$  is a function of time only and does not depend on the particle momentum. (If lepton asymmetry is vanishingly small the values of  $\xi$  for particles and antiparticles are the same.) In this approximation the complicated integro-differential equations (2,6) are reduced to the well known ordinary differential equation [18]:

$$\dot{n}_\nu + 3Hn_\nu = \langle \sigma_{ann} v \rangle (n_\nu^{(eq)2} - n_\nu^2) \quad (12)$$

Here  $n^{(eq)}$  is the equilibrium number density,  $v$  is the velocity of annihilating particles, and angular brackets mean thermal averaging.

The assumption of kinetic equilibrium (11) is fulfilled if the rate of elastic scattering at the moment of annihilation freezing,  $\Gamma_{ann} \sim H$ , is much higher than both the expansion rate,  $H$ , and the rate of annihilation,  $\Gamma_{ann}$ . It is generally correct because the cross-sections of annihilation and elastic scattering are usually of similar magnitudes but the rate of annihilation,  $\Gamma_{ann} \sim \sigma_{ann} n_m$  is suppressed relative to the rate of elastic scattering,  $\Gamma_{el} \sim \sigma_{el} n_0$ , due to Boltzmann suppression of the number density of massive particles,  $n_m$ , with respect to that of massless ones,  $n_0$ . However in the case of MeV-neutrinos both rates  $\Gamma_{ann}$  and  $\Gamma_{el}$  at the moment of annihilation

freezing are of the same order of magnitude. Correspondingly assumption of kinetic equilibrium at annihilation freezing is strongly violated. A semi-analytic calculations of the deviations from kinetic equilibrium were done in ref. [19], where a perturbative approach was developed. In the case of a momentum-independent amplitude of elastic scattering the integro-differential kinetic equation in the Boltzmann limit can be reduced to the following differential equation:

$$JC'' + 2J'C' = -\frac{64\pi^3 H x^2}{|A_0|^2 m} e^{y/2} \partial_y \left\{ e^{-y} \partial_y \left[ e^{(u+y)/2} u y \partial_x \left( C e^{-u} \right) \right] \right\} \quad (13)$$

where  $x = m/T$ ,  $y = p/T$ , prime means differentiation with respect to  $y$ ,  $C(x, y) = \exp(\sqrt{x^2 + y^2}) f_m(x, y)$ , and  $f_m$  is the unknown distribution function of massive particles.

A direct application of perturbation theory (with respect to small deviation from equilibrium) to the integro-differential kinetic equation (2) is impossible or very difficult because the momentum dependence of the anzats for the first order approximation to  $f(p, t)$  is not known. Numerical solution of exact kinetic equations [22] shows a reasonable agreement with the semi-analytic approach based on eq. (13).

It can be easily shown that the spectrum of massive  $\nu_\tau$  is softer (colder) than the equilibrium one. Indeed if elastic scattering of  $\nu_\tau$ , which would maintain kinetic equilibrium is switched-off, the nonrelativistic  $\nu_\tau$  cool down as  $1/a^2$ , while relativistic particles cool as  $1/a$ , where  $a(t)$  is the cosmological scale factor. Since the cross-section of annihilation by the weak interactions is proportional to the energy squared of the annihilating particles, the annihilation of nonequilibrium  $\nu_\tau$  is less efficient and their number density becomes larger than in the equilibrium case.

After the pioneering paper [17] the frozen number density of  $\nu_\tau$  was calculated with increased accuracies in refs. [20, 21, 22]. The better was the accuracy the larger was the calculated frozen number density  $n_{\nu_\tau}^{(f)}$ . In the maximum of  $n_{\nu_\tau}^{(f)}$ , which occurs at  $m_{\nu_\tau} \approx 5$  MeV, the difference between the calculations of the papers [17] and [22] is



almost 50%. So the account of nonequilibrium effects in the distribution of massive  $\nu_\tau$  results in a larger frozen number density of  $\nu_\tau$  and in a stronger influence on primordial nucleosynthesis.

Another nonequilibrium effect is an extra cooling of massless  $\nu_e$  and  $\nu_\mu$  due to their elastic scattering on colder  $\nu_\tau$ ,  $\nu_{e,\mu} + \nu_\tau \rightarrow \nu_{e,\mu} + \nu_\tau$ . Because of that the inverse annihilation  $\nu_{e,\mu} + \bar{\nu}_{e,\mu} \rightarrow \nu_\tau + \nu_\tau$  is weaker and the frozen number density of  $\nu_\tau$  is smaller. But this is a second order effect and is relatively unimportant.

Considerably more important is an overall heating and modification of the spectrum of  $\nu_e$  (and of course of  $\bar{\nu}_e$ ) by the late annihilation  $\nu_\tau + \nu_\tau \rightarrow \nu_e + \bar{\nu}_e$  (the same is true for  $\nu_\mu$  but electronic neutrinos are more important for nucleosynthesis because they directly participate in the reactions (9,10) governing the frozen  $n/p$ -ratio. It is analogous to the similar effect originating from  $e^-e^+$ -annihilation, considered in the previous section, but significantly more profound. The overall heating and spectral distortion work in the opposite way for  $m_{\nu_\tau} > 1$  MeV. An overall increase of the number and energy densities of  $\nu_e$  and  $\bar{\nu}_e$  result in a smaller temperature of neutron freezing and in a decrease of the  $n/p$ -ratio. On the other hand a hotter spectrum of  $\nu_e$  shifts this ratio to a large value, as discussed in the previous section. The latter effect was estimated semi-analytically in ref. [23], where it was found that e.g. for  $m_{\nu_\tau} = 20$  MeV the spectral distortion is equivalent to 0.8 extra neutrino flavors for Dirac  $\nu_\tau$  and to 0.1 extra neutrino flavors for Majorana  $\nu_\tau$ . The effect of overall heating was found to be somewhat more significant [24, 22].

Though the frozen number density of  $\nu_\tau$  obtained in ref. [22] is the largest (in comparison to the results of refs. [17, 20, 21]), the influence of nonequilibrium corrections on nucleosynthesis found in [22] is somewhat weaker than that found in [24, 21] in the mass range above 15 MeV. It is possibly related to a larger momentum cut-off in numerical calculations of ref. [22], which gives rise to a smaller neutron freezing temperature. For the graphical presentation of the results and comparison with the

other papers one can address ref. [22]. The results of all nonequilibrium calculations are systematically and considerably larger than those of the equilibrium ones of ref. [17]. These newer and more accurate calculations permit to close the window in the mass range 10-20 MeV, which is not excluded by nucleosynthesis if the permitted number of extra neutrinos flavors is 1. Now even if 1 extra neutrino is permitted, the upper bound on  $m_{\nu_\tau}$  is about 1 MeV. If 0.3 extra neutrino flavors are allowed, the  $\nu_\tau$  mass is bounded from above by 0.3 MeV. These results are valid for the Majorana  $\nu_\tau$ . For the Dirac case the mass bound from SN1987 is much more restrictive and, moreover the calculations of nucleosynthesis limit on the mass of Dirac  $\nu_\tau$  are considerably more complicated because of a larger number of independent unknown distribution functions.

## 4 Lepton asymmetry and possible spatial variation of primordial abundances.

In the standard nucleosynthesis calculations is usually assumed that neutrinos are not degenerate, or in other words, that their chemical potentials are vanishing and their distributions are given by the expression (1). A justification for this assumption is a small value of the baryon asymmetry, but strictly speaking very little is known neither from observation nor theoretically about lepton asymmetry. The best observational bounds are found from primordial nucleosynthesis (for a recent reference see e.g. [25]). Theoretically lepton asymmetry could be as small as the baryon one, especially in the models with  $(B - L)$ -conservation, but it also may be as large as unity [26, 27, 28]. Moreover, the asymmetry could be not only large but also varying by unity at astronomically large scales [26].

The recent data [29, 30, 31, 32, 33, 34] though rather controversial, may possibly indicate that the abundance of primordial deuterium changes at the scales of the order

of a gigaparsec or several hundred megaparsecs. If the effect is real, there could be two possible explanations of it. First, baryon asymmetry of the universe may be not a universal constant but a varying function of space points [35, 36]. This possibility meets certain problems with the primordial  ${}^7\text{Li}$ -abundance [35] or with the isotropy of CMB [36]. Below we discuss another possible source of a possible variation of primordial abundances, namely spatially varying lepton asymmetries [37].

It is noteworthy that independently of the data and theory, there is a question what is known about light element abundances at large distances. For example what is the upper or lower limit on  $R_p$ , the mass fraction of primordial  ${}^4\text{He}$ , at the distances above 100 Mpc? Is  $R = 50 - 60\%$  or even close to 100% excluded? What is the characteristic scale where a large variation of primordial abundances are permitted? It is known from observations that the universe is (or better to say was at the early stage) very homogeneous energetically. From isotropy of CMB it follows that

$$\frac{\delta\rho}{\rho} < (\text{a few}) \times 10^{-5} \quad (14)$$

A natural implication of the energetical homogeneity is the chemical homogeneity but it is not necessarily so. It is interesting to consider a model which gives rise to a small cosmological energy variation but to a large chemical variation.

We assume that chemical potential of neutrinos, especially of  $\nu_e$ , are varying on the scales above a few hundred Mpc. To explain the possibly observed variation of deuterium, the dimensionless chemical potential of electronic neutrinos,  $\xi_{\nu_e}$  should vary by approximately unity. For example  $\xi_{\nu_e} = 0$  in our neighborhood and  $\xi_{\nu_e} = -1$  in deuterium rich regions. With such variation of electronic asymmetry we immediately obtain  $\delta\rho/\rho \approx (\text{a few}) \times 10^{-3}$ , much larger than the bound (8). To save the model one has to assume a kind of lepton conspiracy [37], namely if in some space region of the universe lepton asymmetry is given by the set of chemical potentials:

$$\{\xi_{\nu_e}, \xi_{\nu_\mu}, \xi_{\nu_\tau}\} = \{\alpha, \beta, \gamma\}, \quad (15)$$

then in another space region the asymmetry is given by a permutation of  $\alpha$ ,  $\beta$ , and  $\gamma$ . In this case the variation of the cosmological energy density would vanish in the first approximation. Though the assumption of lepton conspiracy looks as an a quite strong and artificial fine-tuning, it can be rather naturally realized due the flavor symmetry,  $e \leftrightarrow \mu \leftrightarrow \tau$ .

It can be easily checked that if the variation of  ${}^2H$  is created by the variation of  $\xi_{\nu_e}$  from 0 to (-1), the corresponding mass fraction of  ${}^4He$  in deuterium rich regions should be larger than 50% [37]. Such a large variation of helium mass fraction would result to a considerable density fluctuations due to different binding energies of helium and hydrogen. Rescaling the estimates of ref. [36] one can find [37] for the fluctuations of the CMB temperature :

$$\frac{\delta T}{T} \approx 10^{-5} \left( \frac{R_{hor}}{10\lambda} \right) \quad (16)$$

where  $\lambda$  is the wavelength of the fluctuation and  $R_{hor}$  is the present day horizon size. The restriction on the amplitude of temperature fluctuations would be satisfied if  $\lambda > 200 - 300 \text{Mpc}/h_{100}$  ( $h_{100} = H/100 \text{ km/sec/Mpc}$ ). Surprisingly direct astrophysical effects of such big fluctuations of  $R_p$  at distances above 100 Mpc cannot be observed presently, at least the evident simple ones.

Another possibly dangerous effect is the differential neutrino heating considered in section 2. If chemical potentials of neutrinos are different in different space points, their nonequilibrium heating by  $e^+e^-$ -annihilation would also be different. Correspondingly the photon temperature would also be different. This effect was estimated in ref. [37], where it was found that  $\delta T/T \approx 2 \times 10^{-5}$  for  $\delta \xi_\nu = 1$ .

A variation of mass fraction of primordial  ${}^4He$  could be observed in the future high precision measurements of CMB anisotropies at small angular scales [38]. There are two possible effects, first, a slight difference in recombination temperature which logarithmically depends on hydrogen-to-photon ratio, and second, a strong suppression

of high multipoles with an increase of  $R_p$ . The latter is related to the earlier helium recombination with respect to hydrogen and correspondingly to a smaller number of free electrons at the moment of hydrogen recombination. This in turn results in an increase of the mean free path of photons in the primeval plasma and in a stronger Silk damping. The position and the magnitude of the first acoustic peak remains practically unchanged [38].

This effect seems to be very promising for obtaining a bound on or an observation of a possible variation of primordial helium mass fraction. If this is the case then the amplitude of high multipoles at different directions on the sky would be quite different. The impact of the possible variation of primordial abundances on the angular spectrum of CMB anisotropy at low  $l$  is more model dependent. It may have a peak corresponding to the characteristic scale  $R > 200 - 300$  Mpc or a plateau, which would mimic the effect of the hot dark matter.

## 5 Neutrino oscillations and nucleosynthesis

An influence of oscillating neutrinos on nucleosynthesis depends on possible oscillation channels. If the oscillations do not create any new neutrino states, and if the initial (generated in the early universe) lepton asymmetry is small, the impact of the oscillations on nucleosynthesis is negligible. In the case of nonzero lepton asymmetry the oscillations between  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$  (and their antiparticles) would result in a mixing of the different lepton numbers. So that if the oscillations were fast enough at nucleosynthesis (NS) and the equilibrium was established, all chemical potentials would be equal. If the oscillations at NS were slow, then the asymmetries would not be equalized and due to different refraction indices for particles and antiparticles (see below) there might be even a significant amplification of asymmetries.

A more interesting for NS effect takes place if neutrino oscillations produce new

neutrino states, e.g. a sterile neutrino (or neutrinos). It may happen if the neutrino mass matrix contains both Dirac and Majorana mass terms [39]. In that case the sterile neutrino(s) is (are) just the usual neutrino(s) with a wrong (positive) helicity induced by the Dirac mass. If the characteristic time of oscillations is sufficiently small, so that thermal equilibrium with respect to formation of new states is fulfilled, there would be one or several new neutrino species in the plasma and the only effect on NS is the corresponding change in the expansion rate. It is mentioned above that one additional neutrino species is forbidden by NS. This condition permits to exclude a certain range of the oscillation parameters. In the original treatment of ref. [39] the influence of the medium on neutrino oscillations was neglected. In this case the characteristic time of oscillations is just the vacuum time:

$$\tau_{osc} = \frac{E}{\delta m^2} = 10^{-3} \text{ sec} \frac{E/\text{MeV}}{\delta m^2/10^{-6} \text{ eV}^2} \quad (17)$$

The rate of production of new neutrino species is  $\Gamma_{osc} = (\tau_{osc} \sin^2 2\theta)^{-1}$ . If  $\Gamma_{osc} \geq H$  then the extra neutrino species would be abundantly produced.

However for a large and interesting interval of masses and mixing angles the influence of the medium cannot be neglected and one should take into account that neutrino refraction index in the primeval plasma at NS epoch is not unity [40]:

$$n^\pm - 1 = \pm C_1 \eta_L \frac{G_F T^3}{E} + C_2 \frac{G_F^2 T^4}{\alpha} \quad (18)$$

where numerical coefficients  $C_j$  are of order unity,  $G_F$  is the Fermi coupling constant,  $\alpha = 1/137$  is the fine structure constant,  $E$  is the neutrino energy,  $T$  is their temperature, and  $\eta_L$  is the leptonic asymmetry of the plasma. There can be different asymmetries for different leptonic charges, then the expression above should be correspondingly changed.

Neutrino oscillations with the account of dispersion effect were considered in refs. [41, 42, 43, 44, 45, 46]. It was shown that the oscillation parameters are roughly

speaking bounded by

$$\sin^4 \theta |\delta m^2| < 10^{-2} \text{eV}^2, \text{ if } \sin^2 \theta < 0.1 \quad (19)$$

and

$$|\delta m|^2 < 10^{-6} \text{eV}^2, \text{ if } \sin^2 \theta \approx 1 \quad (20)$$

More recent calculations [47, 48, 49] led to further clarification of the bounds. It was shown in particular [48] that spectral distortion of oscillating neutrinos, neglected in earlier calculations, is quite essential for accurate determination of the changes in primordial abundances due to oscillations.

A very interesting effect may take place if the MSW-resonance condition is fulfilled for oscillations of neutrinos into sterile species. From the expression for the refraction index (18) one can see that the resonance condition is fulfilled either for neutrinos or anti-neutrinos depending on the sign of the mass difference. If for example the transition of neutrinos into sterile component is enhanced, then the leptonic asymmetry in the sector of the usual (not sterile) neutrinos would rise up and the oscillation would become more efficient, in turn producing more asymmetry. The equation for asymmetry generation has the form

$$\dot{L} = +AL \quad (21)$$

where  $L$  is the lepton asymmetry and  $A$  is a positive coefficient. When the back reaction of the oscillation on the initial state of the plasma can be neglected, the asymmetry rises up exponentially and can reach the values close to unity. This effect was noticed in refs. [41, 44] and the detailed calculations showing that the effect can be quite large was done in ref. [27]. If e.g. the asymmetry is generated in electronic charge, the impact on primordial nucleosynthesis would be quite significant and in particular the limits on oscillation parameters might be less restrictive. Still the

mixing angles close to one and relatively large mass differences,  $\delta m^2 > 10^{-3}$ , are forbidden if less than one extra neutrino species is allowed by NS.

## 6 Conclusion

We see that nonequilibrium neutrino kinetics is quite essential at nucleosynthesis. Even for the usual massless neutrinos the deviations from equilibrium are rather large, at a per cent level. Though it has very little effect on primordial abundances of light elements, about  $10^{-4}$ , the corresponding changes in neutrino energy density may be in principle observed in the future high precision measurement of angular variation of cosmic microwave background.

Much more significant are nonequilibrium corrections to the spectra of possibly massive tau-neutrinos if their mass lays in MeV region. Exact calculations with all nonequilibrium corrections differ from the simpler equilibrium ones as much as by 50%. The nonequilibrium results are more restrictive and permit to close a window for  $\nu_\tau$  mass near 15 MeV which existed in equilibrium calculations if primordial nucleosynthesis allowed for one extra neutrino species. This permits to put the upper bound on Majorana type mass of  $\nu_\tau$  down to approximately 1 MeV.

Despite the fact that the early universe was very smooth energetically, it is not excluded that chemically it is quite inhomogeneous. In particular there is absolutely no observational bounds on very large variations of primordial  ${}^4He$  at big distances. It is definitely worth to obtain from astronomical data any, even very crude limits on its mass fraction,  $R_p$ . Again CMB measurements could be very helpful in this respect. Varying  $R_p$  would give rise to a different amplitudes of high multipoles at different directions on the sky.

In the case of oscillations of the known neutrinos between themselves primordial nucleosynthesis does not permit to put any interesting bound on the parameters of the



oscillations. However if neutrino may oscillate into new (sterile) ones, large mixing angles and large mass differences are excluded.

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