Primordial Magnetic Fields that Last?*

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Abstract

The magnetic fields we observe in galaxies today may have their origins in the very early universe. While a number of mechanisms have been proposed which lead to an appreciable field amplitude at early times, the subsequent evolution of the field is of crucial importance, especially whether the correlation length of the field can grow as large as the size of a protogalaxy. This talk is a report on work in progress, in which we consider the fate of one specific primordial field scenario, driven by pseudoscalar effects near the electroweak phase transition. We argue that such a scenario has a number of attractive features, although it is still uncertain whether a field of appropriate size can survive until late times.

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1 Introduction

Observations clearly indicate that galaxies typically possess magnetic fields of strength $B \sim 10^{-6}$ Gauss which are coherent over length scales comparable to the size of the galaxies [1]. At this point it is unclear whether these fields originate in astrophysical processes operating during the epoch of galaxy formation and afterward, or can be traced back to a primordial mechanism in the early universe. If a primordial mechanism is responsible, it is necessary to generate a field of amplitude $B \sim 10^{-9}$ G over a comoving scale $\lambda \sim 10^6$ pc (the comoving size of a region which condenses to form a galaxy); such a field can be amplified during the process of condensation to the amplitudes observed today.

There are a number of requirements such a scenario must satisfy. First, it is necessary to generate a field of significant size in the early universe, as the field will tend to decay as $B \propto R^{-2}$ as the universe expands. (This simple scaling will be modified when we take into account plasma effects, but the need for a large initial field will only become more acute.) Second, the fields must not be significantly damped in between their formation and the condensation of protogalaxies. Damping can take different forms, including ordinary Ohmic dissipation (the exponential decay of electromagnetic fields in plasmas of finite conductivity) and "Silk" damping of MHD modes by photon and neutrino viscosity. Third, it is necessary to boost the coherence length of the fields, which are typically formed with much smaller length scales than those of galaxies. For example, the comoving horizon size at the electroweak scale (which sets an upper limit to the correlation length of any field generated by a causal mechanism at the electroweak phase transition) is smaller by a factor of 10^{-10} than the comoving length scale associated with a protogalaxy.

With these requirements in mind, we will examine the prospects of a particular scenario for primordial field generation. Similar arguments will apply to other possibilities, although we will see that this scenario has a number of attractive features.

2 Magnetic Fields from Pseudoscalars

Consider a pseudoscalar field ϕ which couples to electromagnetism via an interaction Lagrange density

$$\mathcal{L} = \phi F_{\mu\nu} \tilde{F}^{\mu\nu} , \qquad (1)$$

where the dual field strength tensor is defined by $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$. Such an interaction leads to a modified form of Maxwell's equations, given by

$$-\partial_t \mathbf{E} + \nabla \times \mathbf{B} = 4\pi \mathbf{J} - \dot{\phi} \mathbf{B} + \nabla \phi \times \mathbf{E}$$
(2)

$$\nabla \cdot \mathbf{E} = 4\pi \rho - \nabla \phi \cdot \mathbf{B} \tag{3}$$

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0 \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{5}$$

(For simplicity we are working in flat spacetime, but the generalization to Robertson-Walker universes is straightforward.) In a conducting plasma we will also have Ohm's Law, $\mathbf{J} = \sigma \mathbf{E}$, where σ is the conductivity.

We assume that the pseudoscalar is spatially homogeneous, so $\nabla \phi$ can be neglected. We also drop ρ , assuming there is no net charge density, and $\partial_t \mathbf{E}$, as time variations in the electric field will be small. Under these assumptions we derive an equation for the magnetic field in Fourier space,

$$\partial_t \mathbf{B} = -\frac{1}{4\pi\sigma} (k^2 \mathbf{B} + i\dot{\phi}\mathbf{k} \times \mathbf{B}) \ . \tag{6}$$

This equation can be analyzed by decomposing **B** into orthonormal modes perpendicular to the wavevector k, $\mathbf{B}(\mathbf{k}) = b_1\mathbf{u}_1 + b_2\mathbf{u}_2$. In fact, it is most convenient to work with circularly polarized modes, $b_{\pm} = b_1 \pm ib_2$. We can then solve explicitly for the time evolution:

$$b_{\pm}(t,\mathbf{k}) = b_{\pm}(0,\mathbf{k}) \exp\left[-\frac{k}{4\pi\sigma}\left(kt \pm \int \dot{\phi} \,dt\right)\right] \,. \tag{7}$$

We see that the b_{-} modes can grow exponentially if $\dot{\phi} > k$, with maximum growth for $k = \frac{1}{2}\dot{\phi}$.

The existence of such an exponentially growing mode suggests that such a pseudoscalar could generate large field strengths in the early universe. It remains to specify an identity and dynamics for the ϕ field itself. A scenario along these lines was proposed by Turner and Widrow [2], who noted that the axion was a particle with appropriate couplings, but did not analyze the possibility in detail. Garretson, Field and Carroll [3] considered a generic pseudo-Goldstone boson evolving during inflation, and found that it was not possible to generate fields which were both of sufficient strength and interestingly large length scales. More recently, Joyce and Shaposhnikov [4] note that a chemical potential for right-handed electron number, generated by processes at the grand unification scale, interacts with electromagnetism in an equivalent fashion, if we simply identify the chemical potential with $\dot{\phi}$. (See also the work of Cornwall [5] and Son [6].)

The Joyce and Shaposhnikov scenario, which involves only standard electroweak physics once the chemical potential is generated, is less flexible than a generic pseudoscalar boson and accordingly more predictive. For definiteness we will consider the fate of the magnetic fields generated by such a mechanism, although other pseudoscalars would have very similar effects. Joyce and Shaposhnikov estimate that their scenario can lead to magnetic fields of order $B \sim 10^{22}$ G on a length scale $\lambda \sim 10^{-8} H_{EW}^{-1}$, where H_{EW} is the Hubble parameter at the electroweak scale. As this is 18 orders of magnitude smaller than the desired comoving length scale, we must seek a mechanism for increasing the coherence length of the field.

3 The Inverse Cascade

A well-known property of ordinary hydrodynamical turbulence is the cascade of energy, injected at a certain length scale, down to smaller scales. In MHD, however, magnetic energy can both cascade to small scales and inverse cascade to large scales. This phenomenon was investigated numerically and analytically in the 1970's and 1980's by Pouquet and collaborators [7], and has been advocated as an important factor in the evolution of primordial magnetic fields by Brandenburg, Enqvist and Olesen [8].

In order for an inverse cascade to be operative, two requirements must be met: an injection of turbulence into the medium, and a nonvanishing expectation value for some pseudoscalar quantity. Turbulence, which is needed to transfer energy between disparate length scales, can arise from (for example) bubble collisions during a first-order electroweak phase transition. We will assume that this is the case, although little is known about the order of the electroweak transition in the real world. The necessity of a nonvanishing pseudoscalar can be seen by considering the time variation of the magnetic field, which satisfies $\partial_t \mathbf{B}(\mathbf{k}) \sim \mathbf{k} \times \mathbf{E}$. To obtain exponential growth in **B** at a wavenumber k, we require $\mathbf{E}(\mathbf{k}) = \alpha(\mathbf{k})\mathbf{B}(\mathbf{k})$, where α is a manifestly pseudoscalar coefficient. Numerical simulations by Pouquet et al. [7] have verified the existence of an inverse cascade if and only if the configuration possesses significant magnetic helicity, $H^M = \int \mathbf{A} \cdot \mathbf{B} d^3x$.

The pseudoscalar mechanism discussed in the previous section creates a field with maximal magnetic helicity. The circularly polarized modes b_{\pm} are of opposite helicity, and one will be suppressed while the other is amplified as ϕ evolves; the resulting field is maximally helical. Hence, such a field should have magnetic energy transferred to larger scales, while we would not expect such behavior in a generic situation.

The transfer of energy to large scales is typically very efficient; numerical simulations indicate [7] that the scale increases linearly by a factor $\Delta t/t_{\text{turb}}$,

where the turbulence timescale $t_{\rm turb}$ can be taken (somewhat optimistically) to be $T_{\rm EW}^{-1} \sim 10^{-15} H_{\rm EW}^{-1}$. Since the naive picture of linear growth would increase the coherence length beyond the Hubble distance, we can assume that the growth would saturate at that point. In the (once again optimistic, but not implausible) assumption that the field strength is undiminished by this process, we are therefore left with a field of amplitude $B \sim 10^{22}$ G and a length scale $\lambda \sim H_{\rm EW}^{-1}$ (a comoving scale of 10^{-4} pc).

4 Subsequent Evolution

A potentially important role in the evolution of the field from the electroweak scale to today can be played by the damping of MHD modes by photon and neutrino viscosity [9]. This damping can dramatically decrease the amplitude of primordial magnetic fields before they have a chance to form galactic fields. However, these modes do not necessarily represent oscillations around a zero-field configuration, but around a force-free field, for which $\mathbf{J} \times \mathbf{B} = 0$. For the situation under consideration, such damping will not be important, as numerical simulations [7] (as well as experiments in tokamaks [10]) indicate that the result of an inverse cascade is a force-free field configuration.

This leaves us with the question of whether the length scale of the magnetic field can be expanded to the dimensions of a protogalaxy. Presumably an inverse cascade mechanism cannot be very helpful, as there is likely to be no source of turbulence subsequent to the electroweak phase transition. A pessimistic scenario would imagine that the field is frozen in and expands with the universe, redshifting as R^{-2} . We would then be left with a field amplitude $B \sim 10^{-8}$ G on a scale of 10^{-4} pc today. To estimate the amplitude on galactic scales, we can consider the incoherent superposition of fields in uncorrelated domains. Since the resulting field goes like one over the square root of the number of domains, which in turn goes as the volume of the region under consideration, the field on megaparsec scales is

$$B(\text{Mpc}) \sim \left(\frac{1 \text{ Mpc}}{10^{-4} \text{ pc}}\right)^{-3/2} B(10^{-4} \text{ pc}) \sim 10^{-23} \text{ G} .$$
 (8)

This is much less than the sought-after amplitude 10^{-9} G, so it is necessary to imagine a more active mechanism for increasing the coherence length.

In fact, the coherence length will certainly grow faster, as uncorrelated domains come into causal contact and magnetic field lines smooth themselves out [11]. At this point we do not have a reliable estimate of the rate at which this happens, nor of the potential dilution of the field strength during this process. Instead, we can proceed under optimistic assumptions to see whether there is any prospect of generating the required field.

The optimistic expectation we consider is that the field rearranges itself at the Alfvén speed,

$$v_A = \left(\frac{\rho_B}{\rho_{\rm tot}}\right)^{1/2} \,, \tag{9}$$

which has the value $v_A \sim 10^{-2}$ during the radiation dominated era (for the parameters used above) and $10^{-2}T(\text{eV})$ during matter domination. The correlation length will then obey

$$\frac{d\lambda}{dt} = H\lambda + v_A , \qquad (10)$$

representing the separate effects of Hubble expansion and Alfvén rearrangement. Plugging in the appropriate numbers, we find that the Alfvén speed dominates until the time of matter-radiation equality, after which the Hubble expansion is most important (so that the comoving length remains constant). It so happens that the Hubble size at matter-radiation equality, which will characterize the correlation length of the magnetic field under these assumptions, corresponds to a comoving scale of 1 Mpc, nicely consistent with the size of a protogalaxy. Under our optimistic scenario, then, we are able to generate fields of the appropriate length scale, but no larger. Under the additional optimistic assumption that the amplitude of the field remains undiminished as it smooths out, we obtain $B(1 \text{ Mpc}) \sim 10^{-8} \text{ G}$, just as required for the primordial scenario.

The fact that we are only able to achieve this result under such optimistic conditions is somewhat discouraging. A more realistic calculation would include both the fact that viscosity will act to retard the Alfvén rearrangement, and that dissipation will act to diminish the amplitude of the field. At this point we are not confident in our understanding of the magnitude of these effects; work in this direction is in progress. While the prospects for primordial mechanisms for magnetic field generation do not seem hopeful in light of this analysis, there is still a chance that a more careful examination will reveal that our optimistic assumptions are actually warranted. The importance of this topic justifies a concerted effort to understand whether this possibility can be realized.

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