Comment on

## "Big Bang Nucleosynthesis and Active-Sterile Neutrino Mixing: Evidence for Maximal $\nu_{\mu} \leftrightarrow \nu_{\tau}$ Mixing in Super Kamiokande?"

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## Abstract

The paper "Big Bang Nucleosynthesis and Active-Sterile Neutrino Mixing: Evidence for Maximal  $\nu_{\mu} \leftrightarrow \nu_{\tau}$  Mixing in Super Kamiokande?" by X. Shi and G. M. Fuller (astro-ph/9810075) discusses the cosmological implications of the  $\nu_{\mu} \rightarrow \nu_s$  solution to the atmospheric neutrino anomaly. It incorrectly concludes that a lower bound on the  $\nu_{\tau}$  mass of 15 eV is needed in order for the  $\nu_{\mu} \rightarrow \nu_s$  oscillations to be consistent with a Big Bang Nucleosynthesis upper bound on the effective number of neutrino flavours of 3.3. Since such a large  $\nu_{\tau}$  mass is disfavoured from large scale structure formation considerations, the strong, but incorrect, conclusion is made that cosmology favours the  $\nu_{\mu} \rightarrow \nu_{\tau}$ solution to the atmospheric neutrino problem. We explain the nature of the error. We conclude that cosmology is, at present, consistent with both the  $\nu_{\mu} \rightarrow \nu_s$  and  $\nu_{\mu} \rightarrow \nu_{\tau}$  possibilities. The large up-down asymmetry observed by SuperKamiokande for atmospheric neutrino induced  $\mu$ -like events provides compelling evidence for the disappearence of muon-neutrinos [1]. The most natural explanation of this phenomenon is neutrino mass driven oscillations between  $\nu_{\mu}$  and another flavour  $\nu_x$ , with oscillation parameters in the approximate range [2]

$$10^{-3} < \Delta m_{\mu x}^2 / \text{eV}^2 < 10^{-2}, \qquad 0.8 < \sin^2 2\theta_{\mu x} < 1,$$
 (1)

where  $\Delta m_{\mu x}^2$  is the squared mass difference, and  $\theta_{\mu x}$  is the vacuum mixing angle. The SuperKamiokande results cannot at present distinguish between the competing  $\nu_x = \nu_s$  and  $\nu_x = \nu_\tau$  possibilities, where  $\nu_s$  is a sterile neutrino.

The  $\nu_s$  solution appears to provide a cosmological challenge: with the parameters as per Eq.(1) naive calculations [3] suggest that the  $\nu_s$  is thermally equilibrated prior to the Big Bang Nucleosynthesis (BBN) epoch, leading to an effective number of neutrino flavours  $N_{\text{eff}} = 4$ . It is at present unclear if  $N_{\text{eff}} = 4$  is inconsistent, but the low primordial deuterium abundance result reported in Ref. [4] favours  $N_{\text{eff}} \ll 4$ . The naive calculations referred to above ignore the creation of neutrino-antineutrino asymmetries by active-sterile neutrino oscillations [5]. Large neutrino-antineutrino asymmetries suppress  $\nu_{\mu} \rightarrow \nu_s$  oscillations, and generically invalidate conclusions drawn from the naive calculations.

Neutrino asymmetry evolution as driven by active-sterile oscillations goes through two distinct phases at temperatures immediately below the critical temperature  $T_c$  at which neutrino asymmetry growth begins. The first phase is explosive growth that initially is exponential in character. After a time, the non-linear evolution equation for the neutrino asymmetry turns the exponential growth into power law  $T^{-4}$  growth.

We have shown in Ref. [6] that small angle  $\nu_{\tau} \rightarrow \nu_s$  oscillations with  $m_{\nu_{\tau}} > m_{\nu_s}$  will, for a range of parameters, create a large  $\nu_{\tau}$  asymmetry which will then suppress  $\nu_{\mu} \rightarrow \nu_s$ oscillations. The situation is a little complicated because  $\nu_{\mu} \rightarrow \nu_s$  oscillations act to create a  $\nu_{\mu}$  asymmetry which can compensate for the effect of the large  $\nu_{\tau}$  asymmetry in the effective potential for  $\nu_{\mu} \rightarrow \nu_s$  oscillations. Indeed the Wolfenstein part of the effective potentials [7] for  $\nu_{\tau} \rightarrow \nu_s$  oscillations and  $\nu_{\mu} \rightarrow \nu_s$  oscillations are respectively proportional to the quantities

$$L^{(\tau)} \sim 2L_{\nu_{\tau}} + L_{\nu_{\mu}}, L^{(\mu)} \sim 2L_{\nu_{\mu}} + L_{\nu_{\tau}},$$
(2)

where  $L_f \equiv (n_f - n_{\bar{f}})/n_{\gamma}$  is the asymmetry for species f whose number density is denoted by  $n_f$ . The generation of  $L_{\nu_{\tau}}$  asymmetry by  $\nu_{\tau} \rightarrow \nu_s$  oscillations will also generate a large  $L^{(\mu)}$  asymmetry, which then suppresses  $\nu_{\mu} \rightarrow \nu_s$  oscillations provided that these oscillations do not generate a significant  $L_{\nu_{\mu}}$  asymmetry in such a way as to drive  $L^{(\mu)}$  to zero. Whether or not the  $\nu_{\mu} \rightarrow \nu_s$  oscillations can compensate for the effects of the large  $\nu_{\tau}$  asymmetry will depend on the rates at which  $L_{\nu_{\tau}}$  and  $L_{\nu_{\mu}}$  are generated. The rates at which  $L_{\nu_{\tau}}$  and  $L_{\nu_{\mu}}$ are generated depend on the oscillation parameters,  $\Delta m^2$  and  $\sin^2 2\theta$ , for each oscillation mode. If we fix  $\Delta m_{\mu s}^2$  and  $\sin^2 2\theta_{\mu s}$  to fit the atmospheric neutrino anomaly then this leaves two free parameters,  $\Delta m_{\tau s}^2$  and  $\sin^2 2\theta_{\tau s}$  for the  $\nu_{\tau} \rightarrow \nu_s$  oscillations. The issue therefore is to calculate the region of  $(\sin^2 2\theta_{\tau s}, \Delta m_{\tau s}^2)$  parameter space where the large  $\nu_{\tau} \rightarrow \nu_s$ oscillation generated  $\nu_{\tau}$  asymmetry is not compensated by a  $\nu_{\mu} \rightarrow \nu_s$  oscillation generated  $\nu_{\mu}$  asymmetry. For reasons explained in a moment, this issue is best handled numerically. The numerical integration of the coupled evolution equations for  $L_{\nu_{\tau}}$  and  $L_{\nu_{\mu}}$  for this system was first done in Ref. [6], and updated in Ref. [8]. See Figure 7 of Ref. [6] and Figure 2 of Ref. [8] for the pertinent results. These results show that  $m_{\nu_{\tau}} > few \ eV$  is required, with the precise value being a function of  $\sin^2 2\theta_{\tau s}$ . Thus, the interesting conclusion is that if the atmospheric neutrino anomaly is due to  $\nu_{\mu} \rightarrow \nu_s$  oscillations then the  $\nu_{\tau}$  must have a cosmologically interesting mass. Furthermore, the mass of the  $\nu_{\tau}$  should be in the range which can be probed by the Chorus and Nomad experiments.

Also contained in Ref. [6] (see section VI) was an approximate analytic computation of the allowed region of parameter space. The main difficulty in solving this problem analytically is that the evolution equations for  $L_{\nu_{\tau}}$  and  $L_{\nu_{\mu}}$  are complicated coupled non-linear integrodifferential equations. For the approximate analytic computation of Ref. [6] we made the simplifing assumption that the  $\nu_{\tau}$  asymmetry is in the  $T^{-4}$  phase when  $\nu_{\mu} \rightarrow \nu_s$  oscillations are most strongly acting to produce a compensating  $L_{\nu_{\mu}}$ . This simplify assumption makes things much easier since it means that

$$\frac{dL_{\nu_{\tau}}}{dT} \sim \frac{-4L_{\nu_{\tau}}}{T}.$$
(3)

With this simplifying assumption (plus some other assumptions) we obtained the allowed region of parameter space where the  $\nu_{\tau} \rightarrow \nu_s$  oscillation generated  $L_{\nu_{\tau}}$  asymmetry successfully suppressed maximal  $\nu_{\mu} \rightarrow \nu_s$  oscillations. The result was the following simple expression [see Eq.(144) of Ref. [6]]<sup>1</sup>,

$$|\Delta m_{\tau s}^2 / \mathrm{eV}^2| \stackrel{>}{\sim} 6 \times 10^5 \left( |\Delta m_{\mu s}^2 / \mathrm{eV}^2| \right)^{12/11}$$
 (4)

This result is independent of  $\sin^2 2\theta_{\tau s}$  just because Eq.(3) is independent of  $\sin^2 2\theta_{\tau s}$ . However, our numerical work (also in section VI of Ref. [6]) showed that the simplifying assumption that leads to Eq.(3) is not valid. The reason is that the  $\nu_{\tau}$  asymmetry is still in the explosive growth phase when  $\nu_{\mu} \rightarrow \nu_s$  oscillations are most strongly acting to produce a compensating  $L_{\nu_{\mu}}$  asymmetry. This means that the rate at which  $L_{\nu_{\tau}}$  is generated is actually much greater than assumed in the derivation of Eq.(4). This makes the allowed region *significantly* larger and it also depends on  $\sin^2 2\theta_{\tau s}$  (since the rate of  $L_{\nu_{\tau}}$  generation depends on  $\sin^2 2\theta_{\tau s}$  in the explosive growth phase). Thus, it turns out that the analytic approach is not very accurate because it underestimates the allowed region by up to several orders of magnitude in  $\Delta m_{\tau s}^2$ . However, it does provide useful qualitative insight, which is why we included it in Ref. [6].

This analytic approach has recently been re-examined by Shi and Fuller in Ref. [9]. Using a similar set of approximations, *including* Eq.(3), they find the slightly less stringent bound

$$|\Delta m_{\tau s}^2 / \mathrm{eV}^2| \gtrsim 3 \times 10^5 \left( |\Delta m_{\mu s}^2 / \mathrm{eV}^2| \right)^{12/11}$$
 (5)

<sup>&</sup>lt;sup>1</sup>Note that the meaning of this bound, assuming for the moment that it is exactly valid, is as follows: If it is obeyed, then negligible numbers of sterile neutrinos are produced by  $\nu_{\mu} \rightarrow \nu_{s}$  oscillations. If it is not obeyed, then the sterile neutrinos will eventually be brought into thermal equilibrium by  $\nu_{\mu} \rightarrow \nu_{s}$  oscillations.

Taking  $\Delta m_{\mu s}^2 = 10^{-3} \text{ eV}^2$ , Eq.(5) then implies a lower bound on  $m_{\nu_{\tau}}$  of about 15 eV. However, as we have explained above, and as was also pointed out in Ref. [6] (see pages 5167 and 5170), this analytic approach significantly underestimates the allowed region because the approximations upon which it is based are simply not valid. The main problem, as discussed above, is the assumption that the  $\nu_{\tau}$  asymmetry is in the  $T^{-4}$  phase when  $\nu_{\mu} \rightarrow \nu_s$  oscillations are most strongly acting to produce a compensating  $L_{\nu_{\mu}}$  asymmetry.

This point is easy to demonstrate. Consider, by way of concrete example,  $\nu_{\tau} \rightarrow \nu_s$  oscillations with the parameters  $\Delta m_{\tau s}^2 = -50 \text{ eV}^2$  and  $\sin^2 2\theta_{\tau s} = 10^{-8}$ . In the absence of other oscillation modes involving either  $\nu_{\tau}$  or  $\nu_s$ , the growth of the  $\nu_{\tau}$  asymmetry is illustrated in Fig.1 of Ref. [8] which we reproduce here for convenience. In this example the critical temperature is about 37 MeV, and the value of the tau-like asymmetry  $L_{\nu_{\tau}}$  is about  $10^{-5}$  when the explosive growth phase gives way to  $T^{-4}$  growth.

Consider now maximally-mixed  $\nu_{\mu} \rightarrow \nu_s$  oscillations. The MSW resonance condition for this mode is given by

$$V(p) = V_{\text{Wolfenstein}} + V_{\text{finite-T}} = 0 \tag{6}$$

where

$$V_{\text{Wolfenstein}} \simeq \pm \sqrt{2} G_F n_\gamma (2L_{\nu_\mu} + L_{\nu_\tau}) \tag{7}$$

and

$$V_{\text{finite-T}} = -\sqrt{2}G_F n_\gamma A \left(\frac{T}{m_W}\right)^2 \frac{p}{\langle p \rangle} \tag{8}$$

are the Wolfenstein (finite-density) and finite-temperature contributions to the effective matter potential [7], respectively [the + (-) sign refers to neutrino (anti-neutrino) oscillations]. The quantity  $G_F$  is the Fermi constant,  $A \simeq 15.3$ ,  $m_W$  is the W-boson mass, p is the neutrino momentum, and  $\langle p \rangle \simeq 3.15T$  is its thermal average. Taking for definiteness that  $L_{\nu_{\tau}} > 0$ , which means that the MSW resonance for  $\nu_{\mu} \rightarrow \nu_s$  oscillations occurs for the neutrinos rather than the anti-neutrinos, Eq.(6) can be solved for the resonance momentum  $p_{\rm res}$  to yield

$$\frac{p_{\rm res}}{\langle p \rangle} \simeq \frac{L_{\nu_\tau} m_W^2}{AT^2} \tag{9}$$

provided that  $L_{\nu_{\mu}} \ll L_{\nu_{\tau}}$ . Putting in  $L_{\nu_{\tau}} \sim 10^{-5}$  and  $T \sim 37$  MeV we find that

$$\frac{p_{\rm res}}{\langle p \rangle} \sim 3 \Rightarrow \frac{p_{\rm res}}{T} \sim 10.$$
 (10)

This means that as  $L_{\nu_{\tau}}$  develops from close to 0 to about  $10^{-5}$  during the explosive growth phase, the MSW resonance momentum for  $\nu_{\mu} \rightarrow \nu_s$  oscillation sweeps through most of the Fermi-Dirac distribution. This is what we mean when we say that the tau-like asymmetry is still in the explosive growth phase when  $\nu_{\mu} \rightarrow \nu_s$  oscillations are most strongly acting to produce a compensating  $L_{\nu_{\mu}}$  asymmetry. Of course, the above was derived assuming that  $\nu_{\mu} \rightarrow \nu_s$  oscillation are not strong enough to create a significant compensating  $L_{\nu_{\mu}}$ , so that  $L_{\nu_{\mu}} \ll L_{\nu_{\tau}}$  always holds. This is of course the main point. Are  $\nu_{\mu} \rightarrow \nu_s$  oscillations effective enough so that they create significant  $L_{\nu_{\mu}}$ so that  $p_{\rm res} \lesssim \langle p \rangle$  contrary to Eq.(10)? In order to work out exactly what happens, a detailed numerical calculation must be performed, which is one of the points we were emphasising above. However, the above argument clearly shows that  $L_{\nu_{\tau}}$  should not be taken to be in the  $T^{-4}$  phase. Note that if  $\sin^2 2\theta_{\tau s} > 10^{-8}$  then the explosive growth phase typically continues until even larger values of  $L_{\nu_{\tau}}$  are reached (i.e. larger than  $10^{-5}$ ). This obviously makes the  $T^{-4}$  growth assumption even less valid.

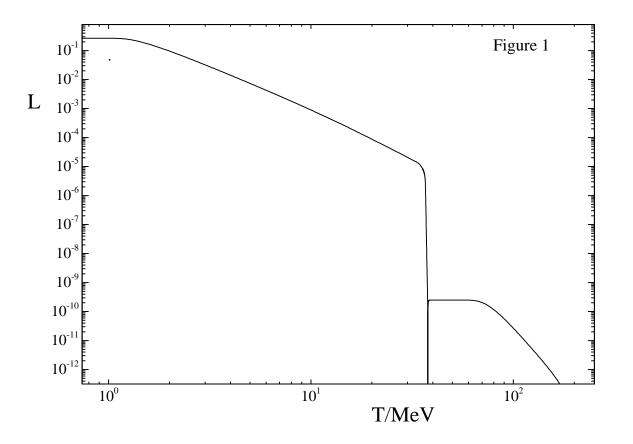
To summarise,  $\nu_{\mu} \rightarrow \nu_s$  oscillations can solve the atmosheric neutrino problem without leading to  $N_{\text{eff}} = 4$  neutrinos in the early Universe. This result occurs because  $\nu_{\tau} \rightarrow \nu_s$ oscillations generate a large  $L_{\nu_{\tau}}$  asymmetry (provided  $m_{\nu_{\tau}} > m_{\nu_s}$ ). The large  $L_{\nu_{\tau}}$  asymmetry will suppress the  $\nu_{\mu} \rightarrow \nu_s$  oscillations in the early Universe provided that the  $\nu_{\mu} \rightarrow \nu_s$ oscillations do not generate a large  $L_{\nu_{\mu}}$  which compensates for the effect of the large  $L_{\nu_{\tau}}$  in the matter term for  $\nu_{\mu} \rightarrow \nu_s$  oscillations. Calculating the 'allowed region' is best handled numerically and has already been done in Refs. [6,8]. The analytic approach of Shi and Fuller [9], which is based on our own analytic work, significantly underestimates the allowed region because it uses simplifying assumptions which are not valid. We conclude that *there is at present no cosmological objection to the*  $\nu_{\mu} \rightarrow \nu_s$  *solution to the atmospheric neutrino problem*.

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## Figure caption

Figure 1: The evolution of  $|L_{\nu_{\tau}}|$  for the example,  $\Delta m_{\tau s}^2 = -50 \ eV^2$ ,  $\sin^2 2\theta_{\tau s} = 10^{-8}$ .



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