

Turbulence control by developing a spiral wave with a periodic signal injection in the complex Ginzburg-Landau equation

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Abstract

Turbulence control in the two-dimensional complex Ginzburg-Landau equation is investigated. A new approach is proposed for the control purpose. In the presence of a small spiral wave seed initiation, a fully developed turbulence can be completely annihilated by injecting a single periodic signal to a small fixed space area around the spiral wave tip. The control is achieved in a parameter region where the spiral wave of the uncontrolled system is absolutely unstable. The robustness, convenience and high control efficiency of this method is emphasized, and the mechanism underlying these practical advantages are intuitively understood.

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Turbulence exists popularly in nature, and it gives crucial influence to the behavior of various systems. Thus, the investigation of turbulence characteristics and turbulence control, as an extremely important issue, has attracted continual attention for more than one century in a large variety of fields of nature science, e.g. fluidic turbulence [1], chemical turbulence [2], and electrical turbulence in cardiac muscle [3]. The model of the complex Ginzburg-Landau equation (CGLE) has been extensively used for the study of turbulence and turbulence control problem. The CGLE model reads

$$\frac{\partial A}{\partial t} = A + (1 + iC_1)\nabla^2 A - (1 + iC_2)|A|^2 A \quad (1)$$

which can be derived universally in the vicinity of a homogeneous Hopf bifurcation in extended systems [4–7] with the complex variable $A(\mathbf{r}, t)$ being the order parameter at the bifurcation. Typical systems modeled by this equation include chemical oscillations, transversely extended lasers and electrohydrodynamic convection in liquid crystals.

In this paper we consider turbulence control by taking the CGLE as our model. Since the pioneering work of OGY, chaos control techniques have been well developed [8–10], and the ideas of chaos control has been applied extensively to spatiotemporal chaos control [11–17]. Recently, turbulence control in one-dimensional (1D) CGLE with gradient force has been investigated [18]. It was shown that for sufficiently strong gradient force one can successfully control violent defect turbulence by injecting control signals to few space points (even a single point) only. The reason for this high efficiency is that the control effects can be propagated from the injected space points to the points far away along the direction of the gradient force, and then the influence of the control can reach large space areas not directly controlled. In Ref. [19], Aranson *et al.* suggested a method of turbulence control in the CGLE without gradient force by developing a spiral wave with local feedback injection. The spiral wave propagation from the central defect along the radial direction plays the role of gradient force to transfer the control effect from the small controlled tip region to the uncontrolled space region, and to greatly enhance the control efficiency. Their method works when the spiral wave state of the uncontrolled system is convectively unstable while fails when it is

absolutely unstable [19]. In the present paper, we investigate the turbulence control in 2D CGLE without gradient force also by developing a regular spiral wave. Specifically, we focus on Eq. (1) with $\mathbf{r} = (x, y)$, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, and with no-flux boundary condition. The essential advance of this work from Ref. [19] is that we use a nonfeedback control approach which can successfully suppress turbulence even when the spiral wave of the uncontrolled system is absolutely unstable.

First, we fix $C_1 = -1.4$, slowly increase C_2 , and investigate the behavior of the CGLE without any control. For a give pair of C_1 and C_2 , the system has a spiral wave solution with a unique wave number k . The spiral wave is stable for $C_2 < C_2(C) \cong 0.37$ and convectively unstable in the region $C_2(C) < C_2 < C_2(A) \cong 0.79$ (see Fig. 1). A careful adiabatic procedure allow (see Fig. 1), in principle, to avoid breakup and to preserve on large spiral up to the absolute instability boundary. Not too far from the absolute instability limit one is left with a small spiral surrounded by turbulence whose radius does not depend on system size and/or distance to the boundaries but vanishes as one approaches the absolute instability threshold [20,4]. For $C_2 > C_2(A)$, for the given wave number of the spiral wave of the CGLE system, the perturbation growing rate becomes larger than the spiral wave moving rate and the spiral wave of the uncontrolled system become absolute unstable. For an arbitrary initial condition the system can quickly fall into a turbulence state. In Figs. 1(a)-(d), we slowly changed C_2 from 0.77 to 0.80. In Fig. 1(a) we take $C_2 = 0.77$ in the convectively unstable region and a stable spiral wave is observed. In Fig. 1(b), for $C_2 = 0.78$, near the absolutely unstable region, some turbulence appears far from the spiral tip, while in Fig. 1(c) and 1(d), turbulence definitely invade the spiral wave body and finally kill the spiral wave. In Fig. 1(e) we show the distributions of stable, convectively unstable and absolutely unstable region of the spiral wave solution.

In the convectively unstable regime of spiral waves, Aranson *et al.* showed [19] that they are able to suppress turbulence (bursts of turbulence separated by the nucleation of well defined spirals) by local feedback control in a small tip region, which can stabilize the tip defect and develop an entire spiral wave to annihilating all other defects together

with turbulence. Their method works in the region of convective instability and fails in the absolute instability $C_2 > C_2(A)$, because the feedback method is based on the existing spiral solution of the uncontrolled system, which can never be stabilized by the local control near the tip area in the absolutely unstable case [19]. In the following we focus on controlling turbulence in the region for $C_2 > C_2(A)$, *i.e.*, controlling turbulence like Figs. 1(d) by applying convenient local injection. In order to support and develop a spiral wave in the turbulent surrounding of Fig. 1(d), we need a well behaved tip serving as the seed. In experiments it is an easy matter to temporarily change the system parameters in a small space area (e.g., in chemical reaction systems it can be done by temporarily illuminating the given area by a light beam). With this change we can generate a small spiral wave island in the violent turbulent sea. In Fig. 2(a), we numerically generate this small spiral wave seed by changing C_2 parameter to $C_2 = 0.5$ and put a small spiral in the center 31×31 area for $t = -100$ t.u.. For $t \geq 0$, C_2 returns back to its normal value 0.8. Then our task is: to support the spiral wave seed of Fig. 2(a), and grow this seed into a large and entire spiral for killing turbulence.

Let us start from Fig. 2(a) (remember the parameters of Fig. 2(a) are the same as Fig. 1(d) in the whole space area, the small spiral seed serves only as an initial condition). Without control, turbulence can easily invade the center area of Fig. 2(a), and finally wipe out the spiral wave seed, leading the system quickly from Fig. 2(a) to the fully developed turbulent state Fig. 1(d), because the spiral wave of the system is absolutely unstable without control. Our main idea of turbulence control is to inject a periodic signal to a small fixed area around the initial spiral wave tip [*i.e.*, the center of Fig. 2(a)] to protect the spiral seed against the turbulence invasion, and even to develop the spiral wave to annihilate turbulence in the whole system. With control Eq. (1) is replaced by the following equation for $t > 0$

$$\frac{\partial A}{\partial t} = A + (1 + iC_1)\nabla^2 A - (1 + iC_2)|A|^2 A + \epsilon\delta_{i,\mu}\delta_{j,\nu}\exp(i\omega t) \quad (2)$$

where i, j are the integer numbers corresponding to the discretized x and y variables as

$x_i = (i - 1)$, $y_i = (j - 1)$, respectively; μ, ν are integer numbers. The control area is taken as a square in the space center with $n \times n$ sites (for $n = 1$, $\mu, \nu = 128$; for $n = 2$, $\mu, \nu = 127, 128$; for $n = 3$, $\mu, \nu = 127, 128, 129$ etc.). In Figs. 2(b)-(d) we take $n = 3$, $\epsilon = 0.5$, and $\omega = 1.2\omega_0$ ($\omega_0 = 0.3762$ is about the angular frequency of the spiral of the CGLE model (1) with $C_1 = -1.4$, $C_2 = 0.5$) in Eq. (2). We find that with the signal injection, the spiral seed can not only defeat the invasion of the surrounding turbulence, but also develop to a large spiral wave body by emitting waves into the turbulent region. In Fig. 2(d) the whole space is firmly controlled by the spiral wave while the system parameters remain in the absolutely unstable regime. The control efficiency in Fig. 2 is surprisingly high and the approach is rather simple. We use only a single signal injecting to 3×3 space sites area extremely small in comparison with the whole turbulent region of 256×256 sites to turn the violent turbulence to a perfect regular spiral wave. It is emphasized that in our control process the development of the spiral wave in the turbulent environment is not limited by the space area of Fig. 2. We have tried to do the control same as Fig. 2 by increasing the system size to 512×512 , the control signals (the same as in Fig. 2) can continually develop the spiral wave [with the seed same as Fig. 2(a)] until the original turbulence is annihilated in the entire space. It is then interesting to understand the mechanism underlying the above amusing control efficiency, and the facts affecting the control results.

The existence of a small spiral wave seed is necessary. We have tried to simulate the control equation (2) directly from the initial condition Fig. 2(a) without the spiral seed, turbulence could never be suppressed no matter how we adjusted ϵ , ω , and n . The reason for this failure is clear. There is no gradient force in Eq. (2), and usually the signal injection can influence a very small region around the controlled region only, and then no turbulence annihilation can be achieved [18]. With a spiral wave seed, the situation is dramatically changed. Spiral waves have a property to propagate the motion of tip region along the radial direction convectively. With this convective propagation the spiral wave state itself can transmit the effect of control signal from the tip region to far away along the radial direction, behaving like a spiral-wave-induced effective gradient force. Then, the mechanism

of turbulence control of Eq. (2) [or Fig. 2] in the presence of an initial spiral wave seed can be heuristically understood. The injected signal plays role of stimulating the motion of the spiral wave tip, and enhances its ability of generating and emitting waves. The control effect can propagate along the radial direction, together with these emitted waves. If the control signal can well match the tip motion, the latter can be so well stimulated that the emitted waves can be strong enough to stop the invasion of the surrounding turbulence and develop itself to finally suppress turbulence.

Up to now we have not yet understood the exact meaning of match between the injected signal and the spiral wave tip, but several facts related to this matter can be intuitive explained, based on the numerical manifestations. First, both too small and too large control forces are not favorable to the successful control. In Fig. 3 we plot the asymptotic states of Eq. (2) for different control strengths at $\omega = 0.8\omega_0$ and $n = 2$. The optimal force intensity is around $\epsilon = 1.5$. It is easy to accept the result of Fig. 3(a) because too weak control signal cannot inject enough "energy" to the spiral seed against the invasion of turbulence. The understanding of the failure of large ϵ in Fig. 3(d) is a bit nontrivial; the reason may be that too large forcing can modify the structure of the spiral wave and reduce its ability emitting strong waves for annihilating turbulence. Second, the frequency of the signal should be properly chosen. In Fig. 4 we plot the system states at $\epsilon = 0.5$, $n = 5$ and for different ω 's. The successful control can be achieved only for certain "resonant" frequency [Fig. 4(b)], too small [Fig. 4(a)] and too large [Fig. 4(c) and 4(d)] frequencies give no good results. Here we put quotation marks on the word "resonant" because the resonance is nonlinear and depends on the forcing amplitude and the control area. And we do not know precisely to which inner frequency the signal frequency is in resonance. Third, the control area should not be too large for the control purpose. In Fig. 5 we plot the full control region in the $\epsilon - n$ plane. For each pair of ϵ and n the controllability is determined by the optimal input frequency. For $n \geq 7$ we cannot achieve full control whatever ϵ and ω . This observation, which looks a bit against the intuition, is actually not surprising. In our case the periodic injection plays role in supporting the spiral tip. If the signal is injected to

too large area (note, $n = 7$ is about a half of wave length), the injection can cover both tip and arms of the spiral, and damage the tip structure, and then the mechanism of turbulence control described above no longer works.

In Fig. 2 the parameter C_2 is a bit above the absolute instability boundary. We can control turbulence, by developing spiral wave, for the parameter much deeply into the absolutely unstable region. For instance, in Fig. 6 we do the same as Fig. 2 by taking $C_2 = 0.95$, $n = 3$, $\epsilon = 1.18$, and $\omega = 2.3\omega_0$. A successful control is obvious achieved.

It is now interesting to ask why with the nonfeedback control we can suppress turbulence in the parameter region where the feedback control approach fails [19]. The reason is clearly shown in Fig. 7 where the solid line is the approximation of the separatrix between the absolutely unstable region and the convectively unstable region in $k - C_2$ plane with $C_1 = -1.4$ [20,4]. The circles in the figure show the wave numbers of spiral wave of uncontrolled system in the convectively unstable region. In this domain the feedback control can work well in turbulence suppression by stabilizing a given defect, and developing an entire spiral wave. Since the feedback control is based on these solutions, the local control effect in the spiral wave center can function only locally, and can not wipe out the turbulence in the region distant away from the spiral wave center, *i.e.*, the small spiral wave can not grow to eliminate the amplification of disturbances. With nonfeedback control we can modify the wave number of the resulting spiral waves shifted by the external periodic forcing from absolutely unstable regime to the corresponding squares (within the convectively unstable region). This wave number modification realizes the local stabilization of the spiral pattern and the annihilation of the system turbulence.

In conclusion we have proposed a new and practical method of turbulence control, based on the presence of an initial spiral wave seed. The approach is convenient and effective because we need only to inject a single periodic signal to a fixed and small space area. The method is effective even in a region where the spiral wave of uncontrolled system is absolutely unstable. We have tried different parameter regions for Eq. (2), and found similar control results of Figs. 2 and 6. We expect that our method may be effectively used in controlling

turbulence in oscillatory and excitable media which include a large variety of chemical and biological systems, but it may not be applicable to the fluidic turbulence control with large Reynolds number.

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FIGURE CAPTIONS

Fig. 1 $C_1 = -1.4$ for all figures in this paper. (a)-(d) The asymptotic spiral wave and turbulence solution of Eq. (1) by slowly increasing C_2 from 0.77 to 0.80. The spatial patterns are grey scale plot of $Re(A)$. (a) $C_2 = 0.77$; (b) $C_2 = 0.78$; (c) $C_2 = 0.79$; (d) $C_2 = 0.80$. (e) The distribution of stabilities of the spiral wave of Eq. (1). The corresponding wave is stable for $C_2 < C_2(C) \cong 0.37$ [20]; convectively unstable in the region $C_2(C) < C_2 < C_2(A) \cong 0.79$ which is obtained from our numerical results of Fig. 1(a)-(d); and absolutely unstable in the region $C_2 > C_2(A)$. The local feedback approach of Ref. [19] can control turbulence in the region $C_2 < C_2(A)$, our nonfeedback method can extend the effective control to $C_2 > C_2(A)$. In all simulations we discretize the space variables to 256×256 sites.

Fig. 2 (a) The spiral wave seed initiation for the control preparation. The state (a) will be used as the initial state for all the following figures. $C_2 = 0.80$, $n = 3$, $\epsilon = 0.5$, $\omega = 1.2\omega_0$, (b) $t = 150$ t.u., (c) $t = 450$ t.u., (d) $t = 750$ t.u.

Fig. 3 The asymptotic states ($t = 1350$ t.u.) of Eq. (2) for different control strengths. $C_2 = 0.80$, $n = 2$, $\omega = 0.8\omega_0$. (a) $\epsilon = 0.5$, (b) $\epsilon = 1.5$, (c) $\epsilon = 2.5$, (d) $\epsilon = 3.5$.

Fig. 4 The same as Fig. 3 with parameters changed to $C_2 = 0.80$, $n = 5$, $\epsilon = 0.5$, and (a) $\omega = 0.6\omega_0$, (b) $\omega = 0.8\omega_0$, (c) $\omega = 1.0\omega_0$, (d) $\omega = 1.2\omega_0$.

Fig. 5 The distributions of full control regions in the $n - \epsilon$ plane with ω taken as the optimal frequency for each pair of n and ϵ . By full control we mean that turbulence can be completely replaced by a perfect spiral wave in the asymptotic states ($t = 1350$ t.u.), like Fig. 2(d).

Fig. 6 The same as Fig. 2 with parameters changed to $C_2 = 0.95$, $n = 3$, $\epsilon = 1.18$, $\omega = 2.3\omega_0$. (a) $t = 800$ t.u., (b) $t = 1200$ t.u., (c) $t = 1600$ t.u., (d) $t = 3000$ t.u.

Fig. 7 The wave numbers of the spiral waves of the uncontrolled [circles, numerical results of Eq. (1)] and controlled [squares, numerical results of Eq. (2)] CGLE systems vs C_2 . The solid line is the approximation of the separatrix between the absolutely unstable region and the convectively unstable region in $k - C_2$ plane with $C_1 = -1.4$ [20,4]. From the

results of the uncontrolled spiral waves, one can see that the wave numbers of spiral waves are in the absolutely unstable regime when $C_2 > C_2(A)$. The local periodic driving shifts the spiral wave number from absolutely unstable regime to convectively unstable regime (the corresponding squares). The control parameters of Eq. (2) are: for $C_2 = 0.80$, $n = 1$, $\epsilon = 6.0$, $\omega = 1.0\omega_0$; for $C_2 = 0.85$, $n = 4$, $\epsilon = 0.5$, $\omega = 0.8\omega_0$; for $C_2 = 0.90$, $n = 5$, $\epsilon = 0.6$, $\omega = 0.8\omega_0$; for $C_2 = 0.95$, $n = 3$, $\epsilon = 1.18$, $\omega = 2.3\omega_0$.