

Emergent physics on vacuum energy and cosmological constant *

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May 1, 2019

Abstract

The phenomenon of emergent physics in condensed-matter many-body systems has become the paradigm of modern physics, and can probably also be applied to high-energy physics and cosmology. This encouraging fact comes from the universal properties of the ground state (the analog of the quantum vacuum) in fermionic many-body systems, described in terms of the momentum-space topology. In one of the two generic universality classes of fermionic quantum vacua the gauge fields, chiral fermions, Lorentz invariance, gravity, relativistic spin, and other features of the Standard Model gradually emerge at low energy. The condensed-matter experience provides us with some criteria for selecting the proper theories in particle physics and gravity, and even suggests specific solutions to different fundamental problems. In particular, it provides us with a plausible mechanism for the solution of the cosmological constant problem, which I will discuss in some detail.

*submitted to Proceedings of the 24th International Conference on Low Temperature Physics, the pdf file with the viewgraphs for oral presentation is in <http://ltdl.tkk.fi/personnel/THEORY/volovik/LT-volovik.pdf>

1 Introduction

In condensed matter physics we deal with many different strongly correlated and/or strongly interacting systems. There are no small parameters in such a system and we cannot treat it perturbatively. However, from our experience we know that at length scales much larger than the inter-atomic spacing, rather simple behavior emerges which is described by an effective theory. This theory is determined by the universality class to which the system belongs and does not depend on microscopic details of the system. There are several types of effective theories.

- A typical example of an effective theory is provided by the Ginzburg-Landau theory describing superconductivity in the vicinity of the transition temperature T_c . This theory, extended to multicomponent superfluids, superconductors and Bose condensates, as well as to the critical phenomena close to T_c , is determined by the symmetry of the system above T_c and describes the symmetry breaking below T_c .
- Effective theories of hydrodynamic type deal with the low-frequency collective modes away from the critical region. These are the two-fluid hydrodynamics of superfluid ^4He ; the London theory of superconductivity; their extension to spin and orbital dynamics of superfluid ^3He ; elasticity theory in crystals, etc. This type of effective theories also describes topologically non-trivial configurations (including the topological defects – singularities of the collective fields protected by topology, such as quantized vortices) and their dynamics (see the book [1] for review on the role of the topological quantum numbers in physics).
- In the limit $T \rightarrow 0$ an effective quantum field theory (QFT) emerges. It deals with the ground state of the system (the quantum vacuum), quasiparticle excitations above the vacuum (analog of elementary particles), and their interaction with low-energy collective modes (bosonic fields). The QFT kind of effective theories includes the Landau Fermi-liquid theory with its extension to non-Landau fermionic systems; the quantum Hall effect; the theory of superfluids and superconductors at $T \ll T_c$, etc. Here one encounters a phenomenon which is opposite to the symmetry breaking: the symmetry is enhanced in the limit

$T \rightarrow 0$ [2]. An example is provided by high-temperature superconductors with gap nodes: close to the nodes quasiparticles behave as 2+1 Dirac fermions, i.e. their spectrum acquires the Lorentz invariance. In superfluid $^3\text{He-A}$ other elements of the relativistic QFT (RQFT) emerge at $T \rightarrow 0$: chiral (Weyl) fermions, gauge invariance, and even some features of effective gravity [3].

In most cases effective theories cannot be derived from first principles, i.e. from the underlying microscopic theory [4]. If we want to check that our principles of construction of effective theories are correct and also to search for other possible universality classes, we use some very simple models, which either contain a small parameter, or are exactly solvable. Example is the BCS theory of a weakly interacting Fermi gas, from which all the types of the effective theories of superconductivity – Ginzburg-Landau, London and QFT – can be derived within their regions of applicability.

In particle physics effective theories are also major tools [5]. The Standard Model of quark and leptons and electroweak and strong interactions operating below 10^3GeV is considered as an effective low-energy RQFT emerging well below the "microscopic" Planck energy scale $E_P \sim 10^{19}\text{GeV}$. It is supplemented by the Ginzburg-Landau type theory of electroweak phase transition, and by the hydrodynamic type theory of gravity – the Einstein general relativity theory. The chiral symmetry and nuclear physics are the other examples of effective theories; they emerge in the low-energy limit of the quantum chromodynamics. In addition, the condensed matter examples ($^3\text{He-A}$ in particular) suggest that not only these effective theories, but even the fundamental physical laws on which they are based (relativistic invariance, gauge invariance, general relativity, relation between spin and statistics, etc.) can be emergent. According to this view the quantum vacuum – the modern ether – can be thought of as some kind of condensed-matter medium. This may or may not be true, but in any case it is always instructive to treat the elementary particle physics with the methods and experiences of the condensed matter physics.

2 Fermi point and Standard Model

The universality classes of QFT are based on the topology in momentum space. All the information is encoded in the low-energy asymptote of the

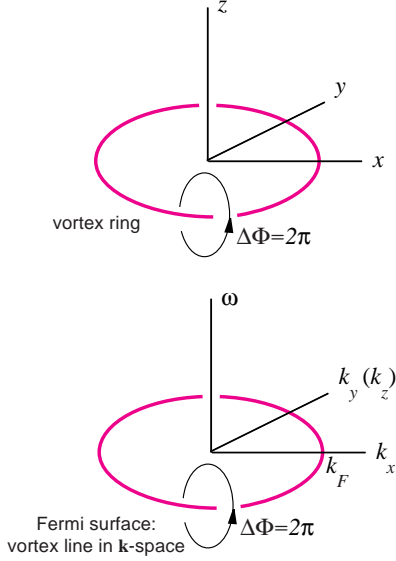


Figure 1: *Top*: vortex loop in superfluids and superconductors. The phase Φ of the order parameter $\Psi = |\Psi|e^{i\Phi}$ changes by 2π around the vortex line and is not determined at the line. *Bottom*: a Fermi surface is a vortex in momentum space. The Green's function near the Fermi surface is $G = (i\omega - v_F(k - k_F))^{-1}$. Let us consider the two-dimensional (2D) system, where $k^2 = k_x^2 + k_y^2$. The phase Φ of the Green's function $G = |G|e^{i\Phi}$ changes by 2π around the line situated at $\omega = 0$ and $k = k_F$ in the 3D momentum-frequency space (ω, k_x, k_y) . In the 3D system, where $k^2 = k_x^2 + k_y^2 + k_z^2$, the vortex line becomes the surface in the 4D momentum-frequency space (ω, k_x, k_y, k_z) with the same winding number.

Green's function for fermions $G(\mathbf{k}, i\omega)$. The singularities in the Green's function in momentum space remind the topological defects living in real space [3, 6]. Such a singularity in the \mathbf{k} -space as the Fermi surface is analogous to a quantized vortex in the \mathbf{r} -space. It is described by the same topological invariant – the winding number (Fig. 1). Protected by topology, the Fermi surface survives in spite of the interaction between fermions. On the emergence of a Fermi surface in string theory see Ref. [7].

Another generic behavior emerges in superfluid $^3\text{He-A}$. The energy spec-

trum of the Bogoliubov–Nambu fermionic quasiparticles in ${}^3\text{He-A}$ is

$$E^2(\mathbf{k}) = v_F^2(k - k_F)^2 + \Delta^2(\mathbf{k}) \quad , \quad \Delta^2(\mathbf{k}) = c_\perp^2 \left(\mathbf{k} \times \hat{\mathbf{I}} \right)^2, \quad (1)$$

where p_F is the Fermi momentum, v_F is the Fermi velocity, and $\hat{\mathbf{I}}$ is the direction of the angular momentum of the Cooper pairs.

As distinct from conventional superconductors with s -wave pairing, the gap Δ in this p -wave superfluid is anisotropic and vanishes for $\mathbf{k} \parallel \hat{\mathbf{I}}$ (Fig. 2). As a result the energy spectrum $E(\mathbf{k})$ has zeroes at two points $\mathbf{k} = \pm k_F \hat{\mathbf{I}}$. Such point nodes in the quasiparticle spectrum are equivalent to point defects in real space – the hedgehogs – and thus are protected by topology. Moreover the spectrum of elementary particles in the Standard Model has also the same kind of topologically protected zeroes (Fig. 3). The quarks and leptons above the electroweak transition are massless, and their spectrum $E^2(\mathbf{k}) = c^2 k^2$ has a zero at $\mathbf{k} = 0$ described by the same topological invariant as the point nodes in ${}^3\text{He-A}$. This is the reason why superfluid ${}^3\text{He-A}$ shares many properties of the vacuum of the Standard Model.

Close to the zeroes the spectrum (1) acquires the “relativistic” form:

$$E^2(\mathbf{k}) = c_\parallel^2 (k_z \pm k_F)^2 + c_\perp^2 k_x^2 + c_\perp^2 k_y^2 \quad , \quad c_\parallel \equiv v_F \quad , \quad (2)$$

where the z -axis is chosen along $\hat{\mathbf{I}}$. For an experimentalist working with ${}^3\text{He-A}$ at low temperature, quasiparticles in Eq. (1) look like one-dimensional: they move only along the direction of the nodes (along $\hat{\mathbf{I}}$); otherwise they are Andreev reflected [8]. A more accurate consideration in the vicinity of the node in Eq. (2) reveals that they can move in the transverse direction too but about thousand times slower: the velocity of propagation in the transverse direction $c_\perp \sim 10^{-3} c_\parallel$.

On the other hand, low-energy inner observers living in the ${}^3\text{He-A}$ vacuum would not notice this huge anisotropy. They would find that their massless elementary particles move in all directions with the same speed, which is also the speed of light. The reason for this is that for their measurements of distance they would use rods made of quasiparticles: this is their matter. Such rods are not rigid and their lengths depend on the orientation. Also, the inner observers would not notice the “ether drift”, i.e. the motion of the superfluid vacuum: Michelson–Morley-type measurements of the speed of “light” in moving “ether” would give a negative result. This resembles the physical Lorentz–Fitzgerald contraction of length rods and the physical Lorentz

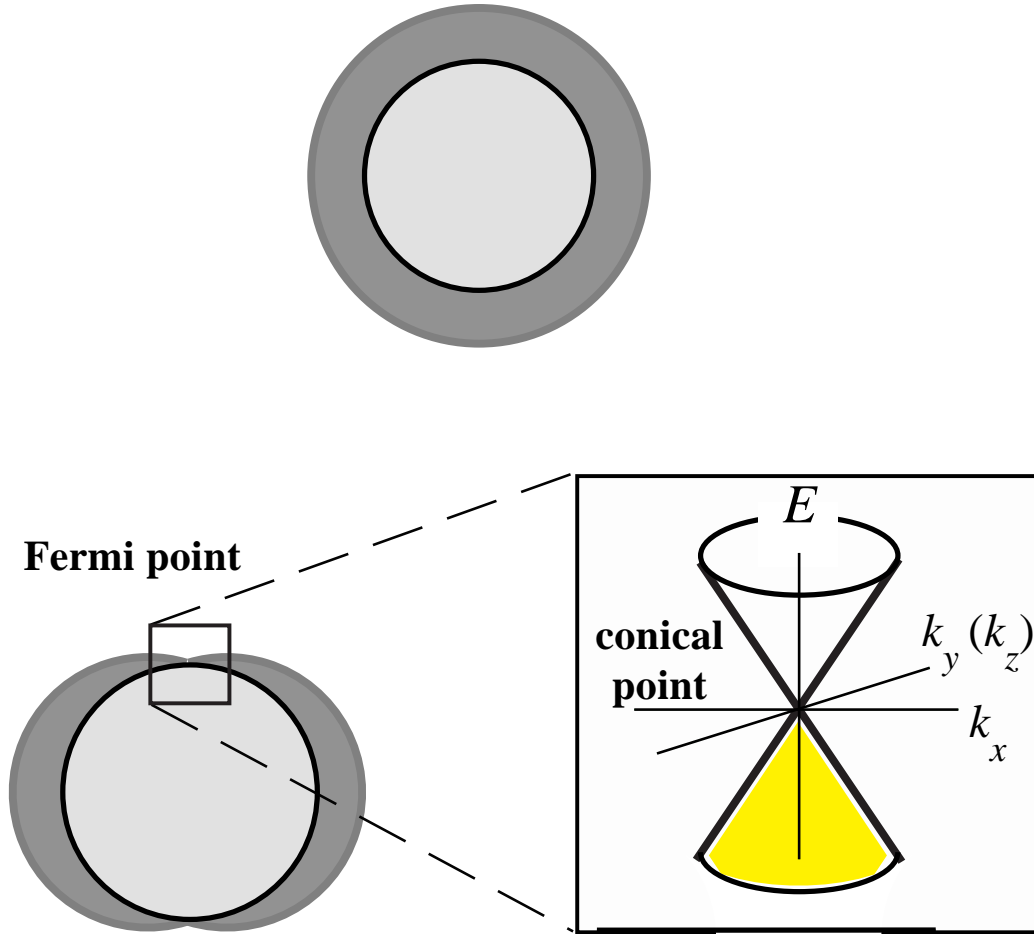


Figure 2: *Top*: isotropic gap in an s -wave superconductor. *Bottom left*: in p -wave superfluid ${}^3\text{He-A}$ the gap is anisotropic and vanishes for $\mathbf{k} \parallel \hat{\mathbf{1}}$. The energy spectrum (1) has two point nodes – Fermi points. *Bottom right*: close to the node the spectrum (2) is similar to the conical spectrum of right-handed or left-handed fermions of the Standard Model.

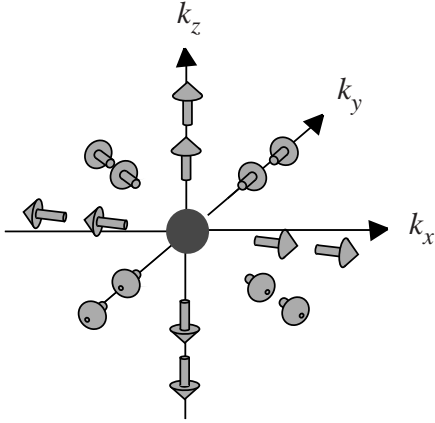


Figure 3: Fermi point is the hedgehog in momentum space. The Hamiltonian of the fermionic quadiparticles living close to the Fermi point is the same as either the Hamiltonian for right-handed particles $H = \hbar c \boldsymbol{\sigma} \cdot \mathbf{k}$ or that for the left-handed particles $H = -\hbar c \boldsymbol{\sigma} \cdot \mathbf{k}$. For each momentum \mathbf{k} we draw the direction of the particle spin $\boldsymbol{\sigma}$, which for right-handed particles is oriented along the momentum \mathbf{k} . The spin distribution in momentum space looks like a hedgehog, whose spines are represented by spins. The spines point outward for the right-handed particles, while for the left-handed particles for which spin is anti-parallel to momentum the spines of the hedgehog point inward. Direction of spin is not determined at singular point $\mathbf{k} = 0$ in the momentum space. The topological stability of the hedgehog singularity under deformations provides the generic behavior of the system with Fermi points in the limit of low energy. This is the reason why the chiral particles are protected in the Standard Model and why superfluid ${}^3\text{He-A}$ shares many properties of the vacuum of the Standard Model.

slowing down of clocks. Thus the inner observers would finally rediscover the fundamental Einstein principle of special relativity in their Universe, while we know that this Lorentz invariance is the phenomenon emerging at low energy only.

The physics emerging in the vicinity of the point nodes is remarkable. In addition to the Lorentz invariance, the other phenomena of the RQFT are reproduced. The collective motion of ${}^3\text{He-A}$ cannot destroy the topologically protected nodes, it can only shift the position of the nodes and the slopes of the “light cone”. The resulting general deformation of the energy spectrum near the nodes can be written in the form

$$g^{\mu\nu}(k_\mu - eA_\mu^{(a)})(k_\nu - eA_\nu^{(a)}) = 0 . \quad (3)$$

Here the four-vector A_μ describes the degrees of freedom of the ${}^3\text{He-A}$ vacuum which lead to the shift of the nodes. This is the dynamical “electromagnetic” field emerging at low energy, and $e = \pm 1$ is the “electric” charge of particles living in the vicinity of north and south poles correspondingly. The elements of the matrix $g^{\mu\nu}$ come from the other collective degrees of freedom which form the effective metric and thus play the role of emerging dynamical gravity. These emergent phenomena are background independent, if the system stays within the Fermi-point universality class. Background independence is the main criterion for the correct quantum theory of gravity. [10]

One may try to construct a condensed matter system with a large number of point nodes in the spectrum which would reproduce all the elements of the Standard Model: 16 chiral fermions per generation; $U(1)$, $SU(2)$ and $SU(3)$ gauge fields; and gravity. There are many open problems on this way especially with gravity: in ${}^3\text{He-A}$ the equations for the “gravitational field” (i.e. for the metric $g^{\mu\nu}$) only remotely resemble Einstein’s equations; while the equation for the “electromagnetic” field A_μ coincides with Maxwell’s equation only in a logarithmic approximation. However, even in the absence of exact correspondence between the condensed matter system and the Standard Model, there are many common points which allow us to make conclusions concerning some unsolved problems in particle physics and gravity. One of them is the problem of the weight of the vacuum – the cosmological constant problem [11, 12].

3 Vacuum energy and cosmological constant

3.1 Cosmological Term and Zero Point Energy

In 1917, Einstein proposed the model of our Universe with geometry of a three-dimensional sphere [9]. To obtain this perfect Universe, static and homogeneous, as a solution of equations of general relativity, he added the famous cosmological constant term – the λ -term. At that time the λ -term was somewhat strange, since it described the gravity of the empty space: the empty space gravitates as a medium with energy density $\epsilon = \lambda$ and pressure $p = -\lambda$, where λ is the cosmological constant. This medium has an equation of state

$$p = -\epsilon = -\lambda . \quad (4)$$

When it became clear that our Universe was not static, Einstein removed the λ -term from his equations.

However, later with development of quantum fields it was recognized that even in the absence of real particles the space is not empty: the vacuum is filled with zero point motion which has energy, and according to general relativity, the energy must gravitate. For example, each mode of electromagnetic field with momentum \mathbf{k} contributes to the vacuum energy the amount $\frac{1}{2}\hbar\omega(\mathbf{k}) = \frac{1}{2}\hbar ck$. Summing up all the photon modes and taking into account two polarizations of photons one obtains the following contribution to the energy density of the empty space and thus to λ :

$$\lambda = \epsilon_{\text{zero point}} = \int \frac{d^3k}{(2\pi)^3} \hbar ck . \quad (5)$$

Now it is non-zero, but it is too big, because it diverges at large k . The natural cut-off is provided by the Planck length scale a_P , since the effective theory of gravity – the Einstein general relativity – is only applicable at $k > 1/a_P$. Then the estimate of the cosmological constant, $\lambda \sim \hbar c/a_P^4$ exceeds by 120 orders of magnitude the upper limit posed by astronomical observations.

There are also contributions to the vacuum energy from the zero point motion of other bosonic fields, and a contribution from the occupied negative energy states of fermions (Fig. 4). If there is a supersymmetry – the symmetry between fermions and bosons – the contribution of bosons would

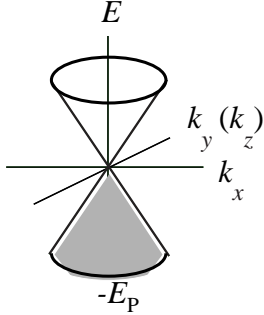


Figure 4: Occupied negative energy levels in the Dirac vacuum produce a huge negative contribution to the vacuum energy and thus to the cosmological constant. Summation of all negative energies $E(\mathbf{k}) = -\hbar ck$ in the interval $0 < E < -E_P$, where E_P is the Planck energy scale, gives the energy density of the Dirac vacuum: $\epsilon_{\text{Dirac vacuum}} = -\int (d^3k/(2\pi)^3) \hbar ck \sim -\hbar c/a_P^4$, where $a_P = \hbar c/E_P$ is the Planck length.

be canceled by the negative contribution of fermions. However, since the supersymmetry is not exact in our Universe, it can reduce the discrepancy between theory and experiment only by about 60 orders of magnitude. The physical vacuum remains too heavy, and this poses the main cosmological constant problem.

One may argue that there must exist some unknown but very simple principle, which leads to nullification of the cosmological constant. Indeed, in theories in which gravity emerges from the quantum matter fields, the flat space with $\lambda = 0$ appears as a classical equilibrium solution of the underlying microscopic equations [14]. But what to do with our estimation of the zero point energy of quantum fields and the energy of the Dirac vacuum, which are huge irrespective of whether the vacuum is in equilibrium or not?

Recently the experimental evidence for non-zero λ was established: it is on the order of magnitude of the energy density of matter, $\lambda \sim 2 - 3\epsilon_{\text{matter}}$ [13]. People find it easier to believe that the unknown mechanism of cancellation, if existed, would reduce λ to exactly zero rather than the observed very low value. So, why is λ non-zero? And also, why is it on the order of magnitude of the matter density? None of these questions has an answer within the effective quantum field theory, and that is why our condensed

matter experience is instructive, since we know both the effective theory and the underlying microscopic physics, and are able to connect them.

Since we are looking for the general principles governing the energy of the vacuum, it should not be of importance for us whether the QFT is fundamental or emergent. Moreover, we expect that these principles should not depend on whether or not the QFT obeys all the symmetries of the RQFT: these symmetries (Lorentz and gauge invariance, supersymmetry, etc.) still did not help us to nullify the vacuum energy. That is why, to find these principles, we can look at the quantum vacua whose microscopic structure is well known at least in principle. These are the ground states of the quantum condensed-matter systems such as superfluid liquids, Bose-Einstein condensates in ultra-cold gases, superconductors, insulators, systems experiencing the quantum Hall effect, etc. These systems provide us with a broad class of Quantum Field Theories which are not restricted by Lorentz invariance. This allows us to consider the cosmological constant problems from a more general perspective.

3.2 Zero Point Energy in Condensed Matter

The principle which leads to the cancellation of zero-point energy is more general; it comes from a thermodynamic analysis which is not constrained by symmetry or a universality class. To see it, let us consider two quantum vacua: the ground states of two quantum liquids, superfluid ^4He and one of the two superfluid phases of ^3He , the A-phase. We have chosen these two liquids because the spectrum of quasiparticles playing the major role at low energy is “relativistic”. This allows us to make the connection to the RQFT. In superfluid ^4He the relevant quasiparticles are phonons (the quanta of sound waves), and their spectrum is $E(k) = \hbar ck$, where c is the speed of sound. In superfluid $^3\text{He-A}$ the relevant quasiparticles are fermions. The corresponding “speed of light” c (the slope in the linear spectrum of these fermions in Eq. (2)) is anisotropic; it depends on the direction of their propagation.

Lets us start with superfluid ^4He and apply the same reasoning as we did in the case of the electromagnetic field, i.e. we assume that the energy of the ground state of the liquid comes from the zero point motion of the phonon

field. Then according to Eq. (5) one has for the energy density

$$\epsilon_{\text{zero point}} = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \hbar c k \sim \frac{\hbar c}{a_{\text{P}}^4} \sim \frac{E_{\text{P}}^4}{\hbar^3 c^3}, \quad (6)$$

where the role of the Planck length a_{P} is played by the interatomic spacing, and the role of the Planck energy scale $E_{\text{P}} = \hbar c/a_{\text{P}}$ is provided by the Debye temperature, $E_{\text{P}} = E_{\text{Debye}} \sim 1 \text{ K}$; $c \sim 10^4 \text{ cm/s}$.

The same reasoning for the fermionic liquid $^3\text{He-A}$ suggests that the vacuum energy comes from the Dirac sea of “elementary particles” with spectrum (2), i.e. from the occupied levels with negative energy (see Fig. 4):

$$\epsilon_{\text{Dirac vacuum}} = -2 \int \frac{d^3k}{(2\pi)^3} E(\mathbf{k}) \sim -\frac{E_{\text{P}}^4}{\hbar^3 c_{\parallel} c_{\perp}^2}. \quad (7)$$

Here the Planck energy cut-off is provided by the gap amplitude, $E_{\text{P}} = \Delta \sim c_{\perp} p_F \sim 1 \text{ mK}$; $c_{\parallel} \sim 10^4 \text{ cm/s}$; $c_{\perp} \sim 10 \text{ cm/s}$.

The above estimates were obtained by using the effective QFT for the “relativistic” fields in the two liquids in the same manner as we did for the quantum vacuum of the Standard Model. Now let us consider what the exact microscopic theory tells us about the vacuum energy.

3.3 Real Vacuum Energy in Condensed Matter

The underlying microscopic physics of these two liquids is the physics of a system of N atoms obeying the conventional quantum mechanics and described by the N -body Schrödinger wave function $\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_i, \dots, \mathbf{r}_N)$. The corresponding many-body Hamiltonian is

$$\mathcal{H} = -\frac{\hbar^2}{2m} \sum_{i=1}^N \frac{\partial^2}{\partial \mathbf{r}_i^2} + \sum_{i=1}^N \sum_{j=i+1}^N U(\mathbf{r}_i - \mathbf{r}_j), \quad (8)$$

where m is the bare mass of the atom, and $U(\mathbf{r}_i - \mathbf{r}_j)$ is the pair interaction of the bare atoms i and j . In the thermodynamic limit where the volume of the system $V \rightarrow \infty$ and N is macroscopically large, there emerges an equivalent description of the system in terms of quantum fields, in a procedure known as second quantization. The quantum field in the ^4He (^3He) system is presented by the bosonic (fermionic) annihilation operator $\psi(\mathbf{x})$. The

Schrödinger many-body Hamiltonian (8) becomes the Hamiltonian of the QFT [15]:

$$\begin{aligned} \hat{H}_{\text{QFT}} = \hat{H} - \mu \hat{N} = \int d\mathbf{x} \psi^\dagger(\mathbf{x}) \left[-\frac{\nabla^2}{2m} - \mu \right] \psi(\mathbf{x}) \\ + \frac{1}{2} \int d\mathbf{x} d\mathbf{y} U(\mathbf{x} - \mathbf{y}) \psi^\dagger(\mathbf{x}) \psi^\dagger(\mathbf{y}) \psi(\mathbf{y}) \psi(\mathbf{x}). \end{aligned} \quad (9)$$

Here $\hat{N} = \int d^3x \psi^\dagger(\mathbf{x}) \psi(\mathbf{x})$ is the operator of the particle number (number of atoms); μ is the chemical potential – the Lagrange multiplier introduced to take into account the conservation of the number of atoms. Putting aside the philosophical question of what is primary – quantum mechanics or quantum field theory – let us discuss the vacuum energy.

The energy density of the vacuum in the above QFT is given by the vacuum expectation value of the Hamiltonian \hat{H}_{QFT} in (9):

$$\epsilon = \frac{1}{V} \langle \hat{H}_{\text{QFT}} \rangle_{\text{vac}} . \quad (10)$$

In this thermodynamic limit one can apply the Gibbs-Duhem relation, $E - \mu N - TS = -pV$, which at $T = 0$ states:

$$\langle \hat{H} \rangle_{\text{vac}} - \mu \langle \hat{N} \rangle_{\text{vac}} = -pV , \quad (11)$$

where p is the pressure. Using Eqs. (9) and (10) one obtains the relation between the pressure and energy density in the vacuum state:

$$p = -\epsilon . \quad (12)$$

It is a general property, which follows from thermodynamics, that the vacuum behaves as a medium with the above equation of state. Thus it is not surprising that the equation of state (12) is applicable also to the particular case of the vacuum of the RQFT in Eq. (4). This demonstrates that the problem of the vacuum energy can be considered from a more general perspective not constrained by the relativistic Hamiltonians. Moreover, it is not important whether gravity emerges or not in the system.

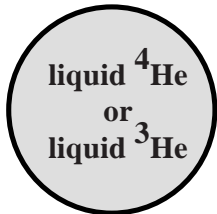


Figure 5: Droplet of quantum liquid. Naive estimation of the vacuum energy density in superfluid ${}^4\text{He}$ as the zero point energy of the phonon field gives $\epsilon_{\text{zero point}} \sim E_{\text{P}}^4/\hbar^3 c^3$, where E_{P} is the Debye energy. Naive estimation of the vacuum energy in superfluid ${}^3\text{He-A}$ as the energy of the Dirac vacuum gives $\epsilon_{\text{Dirac vacuum}} \sim -E_{\text{P}}^4/\hbar^3 c_{\parallel} c_{\perp}^2$, where E_{P} is the amplitude of the superfluid gap. But the real energy density of the vacuum in the droplets is much smaller: for both liquids it is $\epsilon_{\text{vac}} = -2\sigma/R$, where σ is the surface tension and R is the radius of the droplet. It vanishes in the thermodynamic limit: $\epsilon_{\text{vac}}(R \rightarrow \infty) = 0$. Inner observers living within the droplet would be surprised by the disparity of many orders of magnitude between their estimates and observations. For them it would be a great paradox, which is similar to our cosmological constant problem.

3.4 Nullification of Vacuum Energy

Let us consider a situation in which the quantum liquid is completely isolated from the environment. For example, we launch the liquid in space where it forms a droplet. The evaporation at $T = 0$ is absent in the liquid, that is why the ground state of the droplet exists. In the absence of external environment the external pressure is zero, and thus the pressure of the liquid in its vacuum state is $p = 2\sigma/R$, where σ is the surface tension and R the radius of the droplet. In the thermodynamic limit where $R \rightarrow \infty$, the pressure vanishes. Then according to the equation of state (12) for the vacuum, one has $\epsilon = -p = 0$. This nullification of the vacuum energy occurs irrespective of whether the liquid is made of fermionic or bosonic atoms.

If observers living within the droplet measure the vacuum energy (or the vacuum pressure) and compare it with their estimate, Eq. (6) or Eq. (7) depending on in which liquid they live, they will be surprised by the disparity of many orders of magnitude between the estimates and observations (see Fig. 5). But we can easily explain to these observers where their theory goes

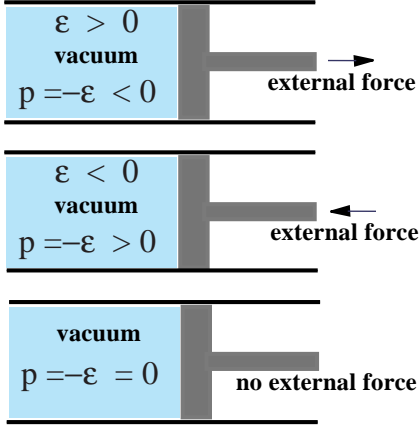


Figure 6: If the vacuum energy is positive, the vacuum tries to reduce its volume by moving the piston to the left. To reach an equilibrium, the external force must be applied which pulls the piston to the right and compensates for the negative vacuum pressure. In the same manner, if the vacuum energy is negative, the applied force must push the piston to the left to compensate for the positive vacuum pressure. If there is no external force from the environment, the self-sustained vacuum must have zero energy.

wrong. Equations (6) and (7) take into account only the degrees of freedom below the “Planck” cut-off energy, which are described by an effective theory. At higher energies, the microscopic energy of interacting atoms in Eq. (9) must be taken into account, which the low-energy observers are unable to do. When one sums up all the contributions to the vacuum energy, sub-Planckian and trans-Planckian, one obtains the zero result. The exact nullification occurs without any special fine-tuning, due to the thermodynamic relation applied to the whole equilibrium vacuum.

This thermodynamic analysis does not depend on the microscopic structure of the vacuum and thus can be applied to any quantum vacuum (Fig. 6), including the vacuum of the RQFT. The main lesson from condensed matter, which the particle physicists may or may not accept, is this: the energy density of the homogeneous equilibrium state of the quantum vacuum is zero in the absence of an external environment. The higher-energy (trans-Planckian) degrees of freedom of the quantum vacuum, whatever they are, perfectly cancel the huge positive contribution of the zero-point motion

of the quantum fields as well as the huge negative contribution of the Dirac vacuum.

This conclusion is supported by the relativistic model, in which our world represents the $(3+1)$ -dimensional membrane embedded in the $(4+1)$ -dimensional anti-de Sitter space. Huge contributions to the cosmological constant coming from different sources cancel each other without fine-tuning [16]. This is the consequence of the vacuum stability.

3.5 Why the Vacuum Energy is Non-Zero

Let us now try to answer the question why, in the present Universe, the energy density of the quantum vacuum is on the same order of magnitude as the energy density of matter. For that let us again exploit our quantum liquids as a guide. Till now we discussed the pure vacuum state, i.e. the state without a matter. In the QFT of quantum liquids the matter is represented by excitations above the vacuum – quasiparticles. We can introduce quasiparticles to the liquid droplets by raising their temperature T a non-zero value. The quasiparticles in both liquids are “relativistic” and massless. The pressure of the dilute gas of quasiparticles as a function of T has the same form as the pressure of ultra-relativistic matter (or radiation) in the hot Universe, if one uses the determinant of the effective (acoustic) metric:

$$p_{\text{matter}} = \gamma T^4 \sqrt{-g} . \quad (13)$$

For the quasiparticles in ${}^4\text{He}$, one has $\sqrt{-g} = c^{-3}$ and $\gamma = \pi^2/90$; for the fermionic quasiparticles in ${}^3\text{He-A}$, $\sqrt{-g} = c_{\perp}^{-2} c_{\parallel}^{-1}$ and $\gamma = 7\pi^2/360$. The gas of quasiparticles obeys the ultra-relativistic equation of state:

$$\epsilon_{\text{matter}} = 3p_{\text{matter}} . \quad (14)$$

Let us consider again the droplet of a quantum liquid which is isolated from the environment, but now at a finite T . In the absence of an environment and for a sufficiently big droplet, where we can neglect the surface tension, the total pressure in the droplet must be zero. This means, that in equilibrium, the partial pressure of the matter (quasiparticles) in Eq. (13) must be necessarily compensated by the negative pressure of the quantum vacuum (superfluid condensate):

$$p_{\text{matter}} + p_{\text{vac}} = 0 . \quad (15)$$

The induced negative vacuum pressure leads to the positive vacuum energy density according to the equation of state (12) for the vacuum, and one obtains the following relation between the energy density of the vacuum and that of the ultra-relativistic matter (or radiation) in thermodynamic equilibrium:

$$\epsilon_{\text{vac}} = -p_{\text{vac}} = p_{\text{matter}} = \frac{1}{3}\epsilon_{\text{matter}} . \quad (16)$$

This is actually what occurs in quantum liquids, but the resulting equation,

$$\epsilon_{\text{vac}} = \kappa\epsilon_{\text{matter}} , \quad (17)$$

with $\kappa = \frac{1}{3}$, does not depend on the details of the system. It is determined by the equation of state for the matter and is equally applicable to: (i) a superfluid condensate + quasiparticles with a linear “relativistic” spectrum; and (ii) the vacuum of relativistic quantum fields + an ultra-relativistic matter (but still in the absence of gravity).

What is the implication of this result to our Universe? It demonstrates that when the vacuum is disturbed, the vacuum pressure responds to the perturbation; as a result the vacuum energy density becomes non-zero. In the above quantum-liquid examples the vacuum is perturbed by a “relativistic matter”. The vacuum is also perturbed by the surface tension of the curved 2D surface of the droplet which adds its own partial pressure. The corresponding response of the vacuum pressure is $2\sigma/R$.

Applying this to the general relativity, we can conclude that the homogeneous equilibrium state of the quantum vacuum without a matter is not gravitating, but the disturbed quantum vacuum has a weight. In the Einstein Universe the vacuum is perturbed by the matter and also by the gravitational field (the 3D space curvature). These perturbations induce the non-zero cosmological constant, which was first calculated by Einstein who found that $\kappa = \frac{1}{2}$ for the cold static Universe [9] (for the hot static Universe filled with ultrarelativistic matter, $\kappa = 1$). In the expanding or rotating Universe the vacuum is perturbed by expansion or rotation, etc. In all these cases, the value of the vacuum energy density is proportional to the magnitude of perturbations. Since all the perturbations of the vacuum are small in the present Universe, the present cosmological constant must be small.

4 Conclusion

What is the condensed matter experience good for? It provides us with some criteria for selecting the proper theories in particle physics and gravity, For example, some scenarios of inflation are prohibited, since according to the Gibbs-Duhem relation the metastable false vacuum also has zero energy [17]. The condensed matter experience suggests its specific solutions to different fundamental problems, such as cosmological constant problem. It demonstrates how the symmetry and physical laws emerge in different corners of parameters, including the zero energy corner. It also provides us with a variety of universality classes and corresponding effective theories, which are not restricted by Lorentz invariance and by other imposed symmetries.

The effective field theory is the major tool in condensed matter and particle physics. But it is not appropriate for the calculation of the vacuum energy in terms of the zero-point energy of effective quantum fields. Both in condensed matter and particle physics, the contribution of the zero-point energy to the vacuum energy exceeds, by many orders of magnitude, the measured vacuum energy. The condensed matter, however, gives a clue to this apparent paradox: it demonstrates that this huge contribution is cancelled by the microscopic (trans-Planckian) degrees of freedom that are beyond the effective theory. We may know nothing about the trans-Planckian physics, but the cancellation does not depend on the microscopic details, being determined by the general laws of thermodynamics. This allows us to understand, in particular, what happens after the cosmological phase transition, when the vacuum energy decreases and thus becomes negative. The microscopic degrees of freedom will dynamically readjust themselves to the new vacuum state, relaxing the vacuum energy back to zero [17]. Actually, the observed compensation of zero-point energy suggests that there exists an underlying microscopic background and the general relativity is an effective theory rather than a fundamental one.

In the disturbed vacuum, the compensation is not complete, and this gives rise to the non-zero vacuum energy proportional to disturbances. The cosmological constant is small simply because in the present Universe all the disturbances are small: the matter is very dilute, and the expansion is very slow, i.e. the vacuum of the Universe is very close to its equilibrium state. One of the disturbing factors in our Universe is the gravitating matter, this is why it is natural that the measured cosmological constant is on the order

of the energy density of the matter: $\kappa \sim 3$ in Eq. (17).

Thus, from the condensed matter point of view, there are no great paradoxes related to the vacuum energy and cosmological constant. Instead we have the practical problem to be solved: how to calculate κ and its time dependence. Of course, this problem is not simple, since it requires the physics beyond the Einstein equations, and there are too many routes on the way back from the effective theory to the microscopic physics.

This work is supported in part by the Russian Ministry of Education and Science, through the Leading Scientific School grant #2338.2003.2, and by the European Science Foundation COSLAB Program.

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