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Triality between Inflation, Cyclic and Phantom Cosmologies

James E. Lidsey
Astronomy Unit, School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, E1 4NS, UK

It is shown that any spatially flat and isotropic universe undergoing accelerated expansion driven by a self-interacting scalar field can be directly related to a contracting, decelerating cosmology. The duality is made manifest by expressing the scale factor and Hubble parameter as functions of the scalar field and simultaneously interchanging these two quantities. The decelerating universe can be twinned with a cosmology sourced by a phantom scalar field by inverting the scale factor and leaving the Hubble parameter invariant. The accelerating model can be related to the same phantom universe by identifying the scale factor with the inverse of the Hubble parameter. The duality between accelerating and decelerating backgrounds can be extended to spatially curved cosmologies and models containing perfect fluids. A similar triality and associated scale factor duality is found in the Randall-Sundrum type II braneworld scenario.

Spatially isotropic and homogeneous Friedmann–Robertson–Walker (FRW) universes containing a scalar field, ϕ, that is minimally coupled to Einstein gravity and self–interacting through a potential, $V(\phi)$, provide an important framework for modern cosmology. If the potential is positive and sufficiently flat, such a field can drive an epoch of rapid, accelerated, inflationary expansion. (For a review, see, e.g., Ref. [1]). On the other hand, the universe undergoes a phase of slow, decelerated contraction if the potential is steep and negative. This latter type of potential arises in the ekpyrotic/cyclic cosmological scenario [2, 3] based on brane collisions in heterotic M–theory [4], where the scalar field is the moduli field parametrizing the separation between the branes.

In a collapsing, spatially flat FRW cosmology, the null energy condition $\rho+p \geq 0$ must be violated if the bounce between the collapsing and expanding phases is to be non–singular [3]. Violation of this condition is possible in cosmologies sourced by 'phantom' matter with an equation of state $w = p/\rho < -1$. A scalar field with negative kinetic energy is one example of this type of matter [5, 6]. Although phantom matter leads to instabilities in any low energy theory [7], there has recently been considerable interest in phantom cosmologies. This development has been motivated by observations of type Ia supernovae [8]: dark energy that violates the null energy condition is consistent with the data [9]. String theory provides further motivation for considering phantom matter [10]. Moreover, it was recently shown that a scale–invariant perturbation spectrum can be generated in a collapsing model sourced by both a standard and a phantom scalar field [11]. Finally, a universe dominated by a phantom scalar field undergoes superinflationary expansion, where the Hubble parameter increases with time and early universe models driven by such a field have been discussed [12].

The comoving Hubble scale decreases during a phase of accelerated expansion or decelerated contraction. This implies that inflation and cyclic cosmologies can, in principle, generate density perturbations on scales larger than the Hubble radius at the epoch of decoupling [13]. Evidence for the existence of large–scale perturbations is found in the anti–correlation of the temperature anisotropy and polarization maps detected by the WMAP satellite [14, 15]. Although the mechanisms that generate the perturbations in the two scenarios are radically different, a surprising duality between the spectra has recently been uncovered [16, 17]: for a given density perturbation spectrum generated from inflationary expansion with a spectral index n_S, there exists a contracting model that produces a spectrum with the same value of n_S. More specifically, the spectral index is given by

$$n_S - 1 \approx -\frac{2}{(1-\epsilon)^2}\left[\epsilon - \frac{(1-\epsilon^2)}{2}\left(\frac{d\ln\epsilon}{d\mathcal{N}}\right)\right], \qquad (1)$$

where $\epsilon \equiv -\dot{H}/H^2$, $\mathcal{N} = \ln[a_e H_e/aH]$, H is the Hubble parameter, a is the scale factor, a subscript 'e' denotes values at the end of the inflationary or contracting phase and higher–order derivatives of the form $(d\ln\epsilon/d\mathcal{N})^2$ and $d^2\ln\epsilon/d\mathcal{N}^2$ have been neglected. Eq. (1) is invariant under the duality $\epsilon \to 1/\epsilon$ and, consequently, identical, nearly scale–invariant perturbation spectra are generated by phases of rapid, accelerated expansion ($\epsilon \ll 1$) and slow, decelerated contraction ($\epsilon \gg 1$) [16, 17]. In the limit where $\epsilon =$ constant, it can be further shown that this duality is exact [17].

The duality between inflation and cyclic cosmology suggests there may be some formal correspondence between the two scenarios at the level of the effective, four–dimensional field equations. One method of generating inflating cosmologies from non–inflating backgrounds is to employ the form–invariance properties of the FRW field equations [18, 19]. Motivated by the above developments, the purpose of the present work is to show that inflationary, cyclic and phantom FRW cosmologies can be related through a series of symmetry transformations. This triality of transformations maps a given cosmological solution parametrized by a scale factor and scalar field potential onto other solutions sourced by fields with different interaction potentials.

We first consider the spatially flat FRW background.

The Friedmann and scalar field equations are given by

$$3H^2 = \frac{\ell}{2}\dot{\phi}^2 + V \tag{2}$$

$$\ddot{\phi} + 3H\dot{\phi} + \ell V' = 0, \tag{3}$$

where $\ell = \pm 1$ for conventional and phantom matter, respectively, a prime denotes differentiation with respect to the scalar field and units are chosen such that $c = 8\pi G = 1$. The energy density is given by $\rho = \ell\dot{\phi}^2/2 + V$. Eq. (3) can be expressed in the form

$$\dot{\rho} = -3\ell H\dot{\phi}^2 \tag{4}$$

or, equivalently, as

$$\dot{H} = -\frac{\ell}{2}\dot{\phi}^2 \tag{5}$$

and Eq. (5) can be rewritten in the form of a time–dependent harmonic oscillator:

$$\frac{d^2 b}{d\tau^2} - \frac{\ell}{2}\left(\frac{d\phi}{d\tau}\right)^2 b = 0, \tag{6}$$

where $b \equiv a^{-1}$ and $\tau \equiv \int^t dt/a(t)$ defines conformal time.

If a particular solution, $b_1(\tau)$, to Eq. (6) is known, a second, linearly independent solution is given in terms of a quadrature of the first: $b_2 = b_1 \int^\tau d\tau/b_1^2$. This indicates that for any given expanding, accelerating universe, there exists a dual cosmology where the functional form of the scalar field (when expressed in terms of conformal time) is invariant. In particular, for the *ansatz* $\phi = [\sqrt{2q}/(1-q)]\ln\tau$, where q is a constant and $\ell = 1$, Eq. (6) simplifies to

$$\frac{d^2 b}{d\tau^2} - \frac{q}{(1-q)^2}\frac{1}{\tau^2} b = 0 \tag{7}$$

and admits the two linearly independent solutions, $b_1 \propto \tau^{q/(q-1)}$ and $b_2 \propto \tau^{1/(1-q)}$. In terms of proper time, these correspond to $a \propto t^q$ and $a \propto t^{1/q}$, respectively, and the two branches are related by the transformation $q \to 1/q$. Indeed, Eq. (7) is invariant under this transformation. For $q > 1$, the time reversal of the second branch represents the decelerating, contracting solution of the cyclic scenario when the equation of state $\epsilon = 3(1+w)/2 = 1/q$ is constant [17].

Further insight may be gained by expressing Eqs. (2) and (3) in the 'Hamilton–Jacobi' form [20, 21, 22]:

$$V = 3H^2 - 2\ell H'^2 \tag{8}$$

$$H' = -\frac{\ell}{2}\dot{\phi}, \tag{9}$$

where the Hubble parameter is viewed as a function of the scalar field and it is assumed that the field is a monotonically varying function of proper time. It follows from the definition of the Hubble parameter that

$$a'H' = -\frac{\ell}{2}aH \tag{10}$$

and integrating Eq. (10) implies that

$$a(\phi) = \exp\left[-\frac{\ell}{2}\int^\phi d\phi \frac{H}{H'}\right]. \tag{11}$$

For a particular functional form of the Hubble parameter, $H(\phi)$, the potential is given directly by Eq. (8) and the scale factor is determined in terms of the single quadrature (11). The time–dependence of the scalar field can be deduced by integrating Eq. (9):

$$t = -\frac{\ell}{2}\int^\phi \frac{d\phi}{H'}. \tag{12}$$

We begin by discussing the duality that maps a conventional expanding, accelerating universe onto a decelerating background ($\ell = 1$). It is immediately apparent that Eq. (10) is invariant under the simultaneous interchange $H(\phi) \leftrightarrow a(\phi)$. We therefore consider the solution $\{H(\phi), V(\phi), a(\phi)\}$ for a standard scalar field cosmology and a new *ansatz* for the Hubble parameter of the form $\tilde{H}(\phi) = a(\phi)$. It follows from Eq. (11) that the new scale factor is given by

$$\tilde{a}(\phi) = \exp\left[-\frac{1}{2}\int^\phi d\phi \frac{a}{a'}\right]. \tag{13}$$

However, since $a(\phi)$ is itself a solution to the field equations, it satisfies Eq. (10). Hence, modulo an irrelevant constant of proportionality, the new scale factor is given by

$$\tilde{a}(\phi) = H(\phi), \qquad \tilde{H}(\phi) = a(\phi) \tag{14}$$

and the new potential is given in terms of the old Hubble parameter by

$$\tilde{V} = \left(3 - \frac{1}{2}\frac{H^2}{H'^2}\right)\exp\left[-\int^\phi d\phi \frac{H}{H'}\right]. \tag{15}$$

The duality transformation (14) inverts the Hubble–flow parameter, $\epsilon \equiv -\dot{H}/H^2$:

$$\epsilon = 2\frac{H'^2}{H^2}, \qquad \tilde{\epsilon} = \frac{1}{\epsilon} \tag{16}$$

and an accelerating, expanding universe is therefore mapped onto a decelerating solution. The self–dual solution is the coasting cosmology, $a = t$. The dual cosmology is itself an expanding model since $\tilde{H}(\phi) = a(\phi) \geq 0$. However, a time reversal leads immediately to a contracting solution. Moreover, an accelerating, expanding cosmology with $\epsilon < 1/3$ is dual to a decelerating, contracting model with a negative potential. Indeed, it can be shown that the expanding solutions are stable to small perturbations if $\epsilon < 1$ (corresponding to inflation driven by a positive potential) and contracting cosmologies are stable if $\tilde{\epsilon} > 3$ (corresponding to the cyclic scenario driven by a negative potential) [17, 23]. We therefore refer to the dual solution $\tilde{a}(\phi)$ as the cyclic cosmology.

To summarize thus far, inflating and cyclic cosmologies can be related by a simultaneous interchange of the Hubble parameter and the scale factor *when both are expressed as functions of the scalar field*. This generalizes the result of Ref. [17] for constant ϵ to that of arbitrary scalar field potentials, where ϵ is time–dependent. The dual potentials are related by a single quadrature involving the Hubble parameter. The comoving Hubble radius $[a(\phi)H(\phi)]^{-1}$ remains invariant under the duality and this implies that the derivative operator $d/d\mathcal{N} = -d/d\ln(aH)$ arising in expression (1) for the spectral index is also invariant.

In the slow–roll inflationary limit, $3H^2(\phi) \approx V(\phi)$ and $\epsilon \approx V'^2/(2V^2) \ll 1$. It follows from Eq. (1) that the spectral index of the perturbation spectrum generated during inflation is then given in terms of the potential by

$$n_S - 1 \approx -3\frac{V'^2}{V^2} + 2\frac{V''}{V}. \tag{17}$$

Eq. (17) can be expressed in terms of the dual cyclic potential. In the slow–roll limit of inflation, Eq. (15) simplifies to

$$\tilde{V} \approx -2\frac{V^2}{V'^2}\exp\left[-2\int^{\phi} d\phi \frac{V}{V'}\right] \tag{18}$$

and differentiating Eq. (18) with respect to the scalar field implies that

$$\frac{\tilde{V}'}{\tilde{V}} \approx 2\left(\frac{V'}{V} - \frac{V''}{V'} - \frac{V}{V'}\right). \tag{19}$$

Since the third term on the right hand side of Eq. (19) dominates, it follows that $V'/V \approx -2\tilde{V}/\tilde{V}'$. (This approximation is equivalent to the assumption that both potentials are approximately exponential in form). Differentiation of this relation with respect to the scalar field and substitution of the result back into Eq. (17) then results in the spectral index of density perturbations for the cyclic scenario [13]:

$$n_S - 1 \approx -4\left(1 + \frac{\tilde{V}^2}{\tilde{V}'^2} - \frac{\tilde{V}\tilde{V}''}{\tilde{V}'^2}\right). \tag{20}$$

We now consider the map between a standard scalar field cosmology $\{H(\phi), V(\phi), a(\phi)\}$ and a phantom cosmology with $\ell = -1$. If the dual Hubble parameter is given by $\hat{H} = a(\phi)$, the phantom cosmology is determined by the quadrature

$$\hat{a}(\phi) = \exp\left[\frac{1}{2}\int^{\phi} d\phi \frac{a}{a'}\right]. \tag{21}$$

Since $a(\phi)$ represents a solution to the standard scalar field equations, it satisfies Eq. (10) with $l = 1$. Substituting Eq. (10) into Eq. (21) and integrating then implies that

$$\hat{a}(\phi) = 1/H(\phi), \qquad \hat{H}(\phi) = a(\phi) \tag{22}$$

and the phantom potential is given by

$$\hat{V} = \left(3 + \frac{1}{2}\frac{H^2}{H'^2}\right)\exp\left[-\int^{\phi} d\phi \frac{H}{H'}\right]. \tag{23}$$

Thus, a positive or negative potential is mapped onto a positive potential, whereas the sign of the field's kinetic energy is reversed. The transformation inverts and simultaneously changes the sign of the Hubble–flow parameter, $\hat{\epsilon} = -1/\epsilon$.

The triality is completed by mapping the two dual cosmologies (14) and (22) directly onto one another. Comparison of Eqs. (14) and (22) immediately implies that the duality inverts the scale factors when each is expressed as a function of the scalar field:

$$\tilde{a}(\phi) = \frac{1}{\hat{a}(\phi)}, \qquad \tilde{H}(\phi) = \hat{H}(\phi). \tag{24}$$

This is similar to the scale factor duality of string cosmology [24], although in that case the field equations remain invariant whereas the transformation (24) relates solutions to field equations derived from different actions. (For reviews, see Refs. [25, 26].) Scale factor dualities between standard and phantom cosmologies have been found previously in the case where the phantom matter is a perfect fluid [27] and a self–interacting scalar field [28]. In the latter case, however, the transformation involved a Wick rotation of the scalar field together with a change in sign of the Hubble parameter. The Hubble parameter is invariant in Eq. (24). Moreover, Eqs. (15) and (23) imply that the two dual potentials generated from the same inflationary background where $\epsilon \ll 1$ differ only by a sign, $\tilde{V}(\phi) \approx -\hat{V}(\phi)$.

As an illustrative example of the triality, consider the power law solution [29]:

$$a = t^{2/\lambda^2}, \qquad \phi = \frac{2}{\lambda}\ln t, \qquad V = \frac{2(6-\lambda^2)}{\lambda^4}e^{-\lambda\phi}, \tag{25}$$

where $\lambda^2 < 6$ is a constant. In terms of the scalar field, the Hubble parameter and scale factor are given by $H(\phi) = (2/\lambda^2)e^{-\lambda\phi/2}$ and $a(\phi) = e^{\phi/\lambda}$, respectively. Employing the duality transformation (14) for $l = 1$ and integrating Eq. (12) implies that

$$\begin{aligned} \tilde{a} &= t^{\lambda^2/2}, \quad \tilde{\phi} = -\lambda\ln t \\ \tilde{V} &= \frac{\lambda^2}{4}\left(3\lambda^2 - 2\right)e^{2\phi/\lambda}, \end{aligned} \tag{26}$$

where we have rescaled the time variable $t \to 2t/\lambda^2$ without loss of generality. The time reversal of solution (26) is the canonical cyclic cosmology and represents a slowly collapsing universe for $\lambda \ll 1$ [17].

The duality (22) maps the power law solution (25) onto the corresponding phantom cosmology:

$$\hat{a} = (-t)^{-\lambda^2/2}, \quad \hat{\phi} = -\lambda \ln(-t)$$
$$\hat{V} = \frac{\lambda^2}{4}\left(3\lambda^2+2\right) e^{2\phi/\lambda}, \tag{27}$$

where the time variable is rescaled such that $t \to 2t/\lambda^2$. Eq. (27) is the power law superinflationary model considered recently in Ref. [12]. The scale factors in Eqs. (26) and (27) are indeed the inverse of each other.

Thus far, we have restricted the discussion to the spatially flat FRW models sourced by a single, self–interacting scalar field. It is of interest to investigate whether the dualities discussed above can be extended to more general backgrounds and matter sources. Of particular interest is the class of models containing both a scalar field and a perfect fluid with barotropic equation of state $p_{\rm mat} = (\gamma - 1)\rho_{\rm mat}$, where $\gamma \le 2$ is a constant and $p_{\rm mat}$ and $\rho_{\rm mat}$ represent the pressure and energy density of the fluid, respectively. The matter can be interpreted as a phantom fluid for $\gamma < 0$. If the fields are uncoupled, energy–momentum conservation implies that $\rho_{\rm mat} = Da^{-3\gamma}$, where D is a constant. The Friedmann equation is then given by

$$3H^2 = \rho + \frac{D}{a^{3\gamma}} \tag{28}$$

and the case of $\gamma = 2/3$ may be interpreted as a positively (negatively) curved FRW universe if $D = -1$ ($D = +1$).

The scalar field equation, Eq. (4), can be expressed in the form $\rho' = -3\ell H\dot{\phi}$ if the field is a monotonically varying function of proper time. The definition of the Hubble parameter then implies that $9\gamma H^2 = -\ell\chi'\rho'/\chi$, where $\chi \equiv a^{3\gamma}$. Substituting this expression into Eq. (28) allows the Friedmann equation to be expressed in the form [21]

$$\rho'(\phi)\chi'(\phi) + 3\ell\gamma\rho(\phi)\chi(\phi) = -3\ell\gamma D. \tag{29}$$

The general solution to Eq. (29) can be expressed in terms of quadratures:

$$\chi(\phi) = \exp\left[-3\ell\gamma\int^{\phi} d\phi \frac{\rho}{\rho'}\right]$$
$$\times \left[\Pi - 3\ell\gamma D\int^{\phi} d\phi\frac{1}{\rho'}\exp\left(3\ell\gamma\int^{\varphi} d\varphi\frac{\rho}{\rho'}\right)\right], \tag{30}$$

where Π is an arbitrary integration constant. However, since Eq. (29) is invariant under the simultaneous interchange $\rho(\phi) \leftrightarrow \chi(\phi)$, the general solution to Eq. (29) can also be expressed in the form

$$\rho(\phi) = \exp\left[-3\ell\gamma\int^{\phi} d\phi \frac{\chi}{\chi'}\right]$$
$$\times \left[\Pi - 3\ell\gamma D\int^{\phi} d\phi\frac{1}{\chi'}\exp\left(3\ell\gamma\int^{\varphi} d\varphi\frac{\chi}{\chi'}\right)\right]. \tag{31}$$

If we now consider a solution $\{\rho(\phi), \chi(\phi)\}$ for a standard scalar field cosmology ($\ell = 1$) and invoke a new *ansatz* $\tilde{\rho}(\phi) = \chi(\phi)$, comparison of Eqs. (30) and (31) implies that the dual cosmology is given by $\tilde{\chi}(\phi) = \rho(\phi)$ if the equation of state of the fluid, γ, remains invariant. It follows that

$$\tilde{\rho}(\phi) = a^{3\gamma}(\phi), \qquad \tilde{a}(\phi) = [\rho(\phi)]^{1/3\gamma} \tag{32}$$

and it may be verified directly that for $\gamma = 2/3$ the coasting solution, $a = t$, is self–dual under the transformation (32). We conclude, therefore, that the spatially flat duality between expanding, accelerating cosmologies and contracting, decelerating models may be extended to include spatial curvature and perfect fluid sources.

In the spatially flat model, the phantom duality (22) arises because Eq. (10) is invariant under the simultaneous interchange $H \to a$, $a \to 1/H$ and $\ell \to -\ell$. However, Eq. (29) is not invariant under $\rho \to \chi$, $\chi \to 1/\rho$ and $\ell \to -\ell$ when $D \neq 0$. On the other hand, a change in the sign of ℓ does leave Eq. (29) invariant if the sign of the barotropic index also changes, $\gamma \to -\gamma$. As a result, a standard scalar field/perfect fluid cosmology can be mapped onto a phantom model where both the scalar field and fluid are phantoms. The transformation is $\rho(\phi) \leftrightarrow \chi(\phi)$, $\gamma \to -\gamma$ and $\ell \to -\ell$. The necessary change in γ indicates that the phantom duality (22) can not be extended to spatially curved FRW backgrounds.

A further question of importance is whether similar dualities can be found in other cosmological scenarios developed from gravitational physics different to that of four–dimensional Einstein gravity. A much studied model is the Randall–Sundrum type II (RSII) braneworld, where a co–dimension one brane is embedded in five–dimensional Anti–de Sitter (AdS) space [30]. In this model, the Friedmann equation acquires a quadratic dependence on the energy density of matter confined to the brane [31]:

$$3H^2 = \rho + \frac{\rho^2}{2\lambda}, \tag{33}$$

where the brane tension, λ, has been tuned so that the effective four–dimensional cosmological constant vanishes. The equation of motion for a single scalar field confined to the brane is given by Eq. (4).

Eqs. (33) and (4) can be written in an alternative form by defining the new variable [32]:

$$y = \left(\frac{\rho}{\rho + 2\lambda}\right)^{1/2}. \tag{34}$$

Substituting Eq. (34) into Eqs. (33) and (4) then implies that

$$H = \left(\frac{2\lambda}{3}\right)^{1/2}\frac{y}{1-y^2} \tag{35}$$
$$\dot{\phi} = -\ell\left(\frac{8\lambda}{3}\right)^{1/2}\frac{y'}{1-y^2} \tag{36}$$

and it follows from the definition of the Hubble parameter that [32]

$$y'a' = -\frac{\ell}{2} ya. \tag{37}$$

Eq. (37) is invariant under the transformation $y(\phi) \leftrightarrow a(\phi)$ and has an identical form to that of Eq. (10). Consequently, the above triality for spatially flat FRW models based on Einstein gravity can be directly extended to the RSII braneworld scenario by simply replacing the Hubble parameter with the variable $y(\phi)$. We conclude, therefore, that there exists a triality in the RSII braneworld of the form

$$\tilde{\rho}(\phi) = \frac{2\lambda a^2(\phi)}{1 - a^2(\phi)} \qquad \tilde{a}(\phi) = \left(\frac{\rho}{\rho + 2\lambda}\right)^{\ell/2}, \tag{38}$$

where standard scalar field braneworlds are mapped onto one another if $\ell = 1$ and a standard brane cosmology is dual to a phantom model for $\ell = -1$. A further consequence of Eq. (38) is that the standard and phantom models generated from the same braneworld are related by a scale factor duality, $\tilde{a}(\phi) \leftrightarrow 1/\hat{a}(\phi)$.

In conclusion, a triality has been established between accelerating and decelerating cosmologies sourced by conventional and phantom scalar fields. The correspondence becomes apparent within the Hamilton–Jacobi framework of scalar field dynamics and arises in cosmological models based on Einstein gravity as well as the RSII braneworld.

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