General covariance and the objectivity of space-time point-events.

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Abstract

"The last remnant of physical objectivity of space-time" is disclosed in the case of a continuous family of spatially non-compact models of general relativity (GR). The physical individuation of point-events is furnished by the intrinsic degrees of freedom of the gravitational field, (viz, the Dirac observables) that represent - as it were - the ontic part of the metric field. The physical role of the epistemic part (viz. the gauge variables) is likewise clarified. At the end the philosophical import of the Hole Argument is substantially weakened and in fact the Argument itself dis-solved, while a peculiar four-dimensional holistic and structuralist view of space-time, (called point-structuralism), emerges, including elements common to the tradition of both substantivalism and relationism. The observables of our models undergo real temporal change: this gives new evidence to the fact that statements like the frozen-time character of evolution, as other ontological claims about GR, are model dependent.

I. INTRODUCTION

The fact that the requirement of general covariance might involve a threat to the physical objectivity of the points of space-time as represented by the theory of gravitation was becoming clear to Einstein even before the theory he was trying to construct was completed. It was during the years 1913-1915 that the threat took form with the famous Hole Argument (Lochbetrachtung) (Einstein, 1914) ¹. In the literature about classical field theories space-time points are usually taken to play the role of individuals, but it is often implicit that they can be distinguished only by the physical fields they carry. Yet, the Hole Argument apparently forbids precisely this kind of individuation, and since the Argument is a direct consequence of the general covariance of general relativity (GR), this conflict eventually led Einstein to state (our emphasis):

That this requirement of general covariance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion... (Einstein, 1916, p.117)

Although Einstein quickly bypassed the seeming cogency of the Hole Argument against the implementation of general covariance on the purely pragmatic grounds of the so-called *Point-Coincidence Argument*², the issue remained in the background of the theory until the Hole Argument received new life in recent years with a seminal paper by John Stachel (1980). This paper, followed seven years later by Earman and Norton's philosophical argument against the so-called space-time *manifold substantivalism* (Earman & Norton, 1987), opened a rich philosophical debate that is still alive today. The Hole Argument was immediately regarded by virtually all participants in the debate (Bartels, 1984; Butterfield, 1984, 1987, 1988, 1989; Maudlin, 1988; Rynasiewicz, 1994, 1996) as being intimately tied to the deep nature of space and time, at least as they are represented by the mathematical models of GR. It must be acknowledged that until now the debate had a purely philosophical relevance. From the physicists' point of view, GR has indeed been immunized against the Hole Argument - leaving aside any underlying philosophical issue - by simply embodying the Argument in the statement that *mathematically different* solutions of the Einstein equations related by *passive* - as well as *active* (see later) - diffeomorphisms are *physically equivalent*.

The main scope of this paper is to show that the immunization statement quoted above cannot be regarded as the last word on this matter both from the physical and the philosophical point of view. From the physical point of view, we will show that physical equivalence of solutions means much more than mere difference in mathematical description since it entails equivalence of the descriptions of phenomena made in different global, non-inertial frames, which are extended space-time laboratories with their (dynamically determined) chronogeometrical conventions and inertial potentials. On these same grounds, we will argue from the philosophical point of view that, first of all, the equivalence statement cannot have the implications that the Hole Argument has traditionally attributed to it and, second, that the equivalence does not compels us to take a definite position in front of the crude dichotomy substantivalism versus relationism; rather we shall show that, for specific models, it leads

¹ For a beautiful historical critique see Norton (1987, 1992, 1993).

² The assertion that the reality of the world-occurrence (in opposition to that dependent on the choice of reference system) subsists in space-time coincidences.

naturally to advance a *tertium quid* between these two positions that tries in some sense to overcome the crudeness of the debate by including elements common to the traditions of both *substantivalism* and *relationism*³. Third, we shall go beyond the simple statement of equivalence showing that space-time point-events can be *individuated physically* by the intrinsic degrees of freedom of the gravitational field.

It must be clear from the start, however, that, given the enormous mathematical variety of possible solutions of Einstein's equations, one should not expect that a clarification of the possible meaning of a would-be *objectivity* of space-time points could be obtained in general. More precisely, we shall indeed conclude that at least three of the main questions we discuss can be clarified for a definite continuous class of generic solutions corresponding to spatially non-compact space-times⁴, but not for the spatially compact ones. The former class is also privileged from the point of view of the inclusion of elementary particles. Consistently, we do not claim to draw general conclusions about the ontology of Einstein's space-times. While allowing that the ontology of general relativistic space-times can be different for different models, we only claim that a particular class of solutions exist which naturally leads to a concept of the space-time ontology consisting in a peculiar form of structural space-time realism. Our thesis holds that space-time point-events (the relata) do exist and we quantify over them; their properties are relational being conferred on them in a holistic way by the whole structure of the metric field and the extrinsic curvature on a simultaneity hyper-surface. At the same time, they are literally identifiable with the local values of the intrinsic degrees of freedom of the gravitational field (Dirac observables), and we will claim that, in a definite abstract sense, they also possess a special kind of *intrinsic* properties. In this way both the metric field and the point-events maintain - to paraphrase Newton their own manner of existence and we qualify our conception as "point-structuralism". Our view does not dissolve physical entities into mathematical structures, so that it supports a moderate *entity-realist* attitude towards both the metric field and its point-events, as well as a theory-realist attitude towards Einstein's field equations. In conclusion this work should be considered as a case study for the defence of a thesis about the nature of the identity of spacetime points characterized by a peculiar form of objectivity. Technically, this philosophical conclusion is made possible thanks to a specific methodology of gauge-fixing based on the notion of intrinsic pseudo-coordinates introduced by Bergmann and Komar. It should be

The substantivalist position is a form of realism about certain spatiotemporal structures, being committed to believing in the existence of those entities that are quantified over by our space-time theories, in particular space-time points. It conceives space-time, more or less, as a substance, that is as something that exists independently of any of the things in it. Accordingly, the extreme form of substantivalism identifies space-time with the bare mathematical manifold of events, perhaps as mereological fusion of all the points. On the other hand, the strong relationist position is the view that space-time arises as a mere abstraction from the spatiotemporal properties of other things, so that spatio-temporal relations are derivative and supervenient on physical relations obtaining among events and physical objects. Note that a simple anti-substantivalist position does not deny the reality of space-time (it is not merely anti-realist) but asserts that space-time has no reality independently of the bodies of fields it contains. The crucial question for our notion of spatiotemporal structuralism, is therefore the specification of the nature of fields that are indispensable for the very definition of physical space-time (e.g. the gravitational field with its causal structure) as distinguished from other physical fields.

⁴ The Christodoulou-Klainermann space-times (Christodoulou & Klainermann, 1993).

stressed that the uniqueness of its mathematical basis (the way in which the four scalar eigenvalues of the Weyl tensor can be equated to four scalar radar pseudo-coordinates by means of a definite class of gauge-fixing procedures) shows that this methodology constitutes the only possible way of disclosing the proper point-events ontology of the class of spacetimes we are referring to. For given initial data of the Dirac observables (which identify an Einstein's "universe"), any other kind of gauge-fixing procedure, would lead to gauge-equivalent solutions in which the underlying point-events ontology simply would not be shown.

At the technical level, we aim to show that some capabilities peculiar to the Hamiltonian approach to GR can be exploited for the purpose of better understanding important interpretive issues surrounding the theory. The Hamiltonian approach guarantees first of all that the initial value problem of Einstein's equations is mathematically well-posed, a circumstance that does not occur in a natural way within the configurational Lagrangian framework (Friedrich & Rendall, 2000; Rendall, 1998). Furthermore, on the basis of the Shanmugadhasan canonical transformation (Shanmugadhasan, 1973; Lusanna, 1993), this framework provides a net distinction between physical observables (the four so-called Dirac observables) connected to the (two) intrinsic degrees of freedom of the gravitational field, on one hand, and qauge variables, on the other. The latter - which express the typical arbitrariness of the theory and must be fixed (gauge-fixing) before solving the Einstein equations for the intrinsic degrees of freedom - turn out to play a fundamental role, no less than the Dirac observables, in clarifying the real import of the Hole Argument. It will be seen indeed that the resulting gauge character of GR is a crucial factor not only for clarifying the issue of the objectivity of space-time points, but also for a true dis-solution of the Hole Argument as to its philosophical implications⁵. We stress that reaching these conclusions within the Lagrangian formulation would be technically quite awkward if not impossible, since the Legendre pull-back of the non-point canonical transformations of the Hamiltonian formulation requires tools like the infinite-jet bundle formalism.

For the above reasons, the discussion in the following sections will be substantially grounded upon the fact that GR is a gauge theory, i.e., a theory in which the physical system being dealt with is described by more variables than there are physically independent degrees of freedom. Such extra variables are introduced to make the description more transparent and mathematically handy but require correspondingly a gauge symmetry having the role of extracting the physically relevant content. The important fact in our case is that while, from the mathematical point of view of the constrained Hamiltonian formalism, GR is a theory like any other (e.g., electromagnetism and Yang-Mills theory), from the physical point of view it is radically different, just because of its gauge invariance under a group of diffeomorphisms acting on space-time itself, instead of invariance under the action of a local inner Lie group. Furthermore, in GR we cannot rely from the beginning on empirically validated, gauge-invariant dynamical equations for the local fields, as it happens with electro-magnetism, where Maxwell equations can be written in terms of the gauge invariant electric and magnetic fields. On the contrary, Einstein's general covariance (viz. the gauge

⁵ Our stance about the content and the implications of the original Hole Argument contrasts with the manifestly covariant and generalized attitude towards the Hole phenomenology expounded by John Stachel in many papers (see e.g. Stachel & Iftime, 2005, and references therein). We shall defend our approach in Sections III and VI.

freedom of GR) is such that the introduction of extra (qauqe) variables does indeed make the mathematical description of general relativity more transparent (through manifest general covariance instead of manifest Lorentz covariance) but, at least prima facie, by ruling out any background structure at the outset, it also makes its physical interpretation more intriguing, and conceals at the same time the intrinsic properties of point-events. In GR the distinction between what is observable and what is not, is unavoidably entangled with the constitution of the very stage, space—time, where the play of physics is enacted: a stage, however, which also takes an active part in the play. In other words, the gauge-fixing mechanism plays the dual role of making the dynamics unique (as in all gauge theories), and of fixing the appearance of the spatio-temporal dynamical background. At the same time, this mechanism highlights a characteristic functional split of the metric tensor that can be briefly described as follows. First of all, any gauge-fixing is equivalent to the constitution of a global, non-inertial, extended, space-time laboratory with its coordinates, for every τ , as well as to a dynamical determination of the conventions about distant simultaneity. In particular, different *conventions* within the same space-time (the same "universe"), turn out to be simply quage-related options. Therefore, on one hand, the Dirac observables specify as it were - the *ontic* structure of space-time connected to the intrinsic degrees of freedom of the gravitational field (and - physically - to tidal-like effects). On the other, the gauge variables specify the built-in epistemic component⁶ of the metric tensor (physically related to the generalized inertial effects accessible in each extended laboratory)⁷

Summarizing, the gauge variables play a multiple role in completing the structural properties of the general-relativistic space-time: their fixing is necessary for solving Einstein's equations, for reconstructing the four-dimensional chrono-geometry emerging from the initial values of the four Dirac observables and for allowing empirical access to the theory through the definition of a spatiotemporal laboratory. It's important to stress in this connection that in GR, unlike in ordinary gauge theories, the reduced phase space $\tilde{\Omega}_4$ of the abstract observables (a realization of which is created by every gauge fixing) plays an abstract role, since the inertial effects associated to each NIF are lost because of the quotient procedure (see Sections IV,V,VI).

Apart from the dis-solution of the Hole Argument, which is expounded in Section III, the main result of our analysis is given in Section V where we show how the *ontic* part of the metric (the intrinsic degrees of freedom of the gravitational field) may confer a *physical*

⁶ In this paper *ontic* refers to the essential content of the gravitational field *qua* physical entity with two independent degrees of freedom, *epistemic* to that part of the information encoded in the metric that must be freely specified in order to get empirical access to the essential part (it refers therefore to the it appearance of the gravitational phenomena). We are perfectly aware that we are here overstating the philosophical import of terms like *ontic* and *epistemic* and their relationships. Nothing, however, hinges on these nuances in what follows.

⁷ Such a global, extended laboratory is a non-rigid, non-inertial frame (the only existing in GR) centered on the (in general) accelerated observer whose world-line is the origin of the 3-coordinates (Lusanna & Pauri, 2004a): hereafter it will be denoted by the acronym NIF. Any NIF is the result of a complete gauge-fixing (see Section IV). The gauge-fixing procedure determines the appearance of phenomena by determining uniquely the form of the inertial forces (Coriolis, Jacobi, centrifugal,...) in each point of a NIF. A crucial difference of this mechanism in GR with respect to the Newtonian case is the fact that the inertial potentials depend upon tidal effects, besides the coordinates of the non-inertial frame.

individuation onto space-time points⁸. Since - as will be seen - such degrees of freedom depend in a highly non-local way upon the values of the metric and the extrinsic curvature over a whole space-like surface of distant simultaneity, point-events receive a peculiar sort of properties that are conferred on them holistically by the whole simultaneous metrical structure. Admittedly, the distinction between ontic and epistemic parts, as well as the form of the space-like surfaces of distant simultaneity, are NIF-dependent⁹.

Finally, an additional important feature of the solutions of GR dealt with in our discussion is the following. The ADM formalism (Arnowitt & Deser & Misner, 1962) for spatially compact space-times without boundary implies that the Dirac Hamiltonian generates purely harmless gauge transformations, so that, being zero on the reduced phase space, it cannot engender any real temporal change. This is the origin of the so-called frozen evolution description; in this connection see Earman (2002), Belot & Earman (1999, 2001). However, in the case of the Christodoulou - Klainermann continuous family of spatially non-compact space-times, internal mathematical consistency (requiring the addition of the DeWitt surface term to the Hamiltonian (DeWitt, 1967), see later) entails that the generator of temporal evolution, namely the (now non-weakly vanishing) Dirac Hamiltonian, be instead the so-called weak ADM energy. This quantity does generate real temporal modifications of the canonical variables. In conclusion, this shows again that also statements like the assertion of the frozen-time picture of evolution, as other ontological claims about GR, are model dependent.

The main part of the technical developments underlying this work have already been introduced elsewhere (Pauri & Vallisneri, 2002, Lusanna & Pauri, 2004a, 2004b;) where additional properties of the Christodoulou-Klainermann family of space-times are also discussed. For a more general philosophical presentation, see Dorato & Pauri (2004).

II. NOETHER AND DYNAMICAL SYMMETRIES

Standard general covariance, which essentially amounts to the statement that the Einstein equations for the metric field ${}^4g(x)$ have a tensor character, implies first of all that the basic equations are form invariant under general coordinate transformations (passive diffeomorphisms), so that the Lagrangian density in the Einstein-Hilbert action is singular. Namely, passive diffeomorphisms are local Noether symmetries of the action, so that Dirac constraints appear correspondingly in the Hamiltonian formulation. The singular nature of the variational principle of the action entails in turn that four of the Einstein equations be in fact Lagrangian constraints, namely restrictions on the Cauchy data, while four combinations of Einstein's equations and their gradients vanish identically (contracted Bianchi identities). Thus, the ten components of the solution ${}^4g_{\mu\nu}(x)$ are in fact functionals of only two "de-

⁸ There is an unfortunate ambiguity in the usage of the term *space-time points* in the literature: sometimes it refers to elements of the mathematical structure that is the first layer of the space-time model, and sometimes to the points interpreted as *physical* events. We will adopt the term *point-event* in the latter sense and simply *point* in the former.

⁹ Yet, according to a *main conjecture* we have advanced elsewhere (see Lusanna & Pauri, 2004a, 2004b), a canonical basis of *scalars* (coordinate-independent quantities), or at least a Poisson algebra of them, should exist, making the above distinction between Dirac observables and gauge variables *fully invariant*, see Section VI and footnote 32.

terministic" dynamical degrees of freedom and eight further degrees of freedom which are left completely undetermined by Einstein's equations even once the Lagrangian constraints are satisfied. This state of affairs makes the treatment of both the Cauchy problem of the non-hyperbolic system of Einstein's equations and the definition of observables within the Lagrangian context (Friedrich & Rendall, 2000; Rendall, 1998) extremely complicated.

For the above reasons, standard general covariance is then interpreted, in modern terminology, as the statement that a physical solution of Einstein's equations properly corresponds to a 4-geometry, namely the equivalence class of all the 4-metric tensors, solutions of the equations, written in all possible 4-coordinate systems. This equivalence class is usually represented by the quotient ${}^4Geom = {}^4Riem/{}_PDiff M^4$, where 4Riem denotes the space of metric tensor solutions of Einstein's equations and ${}_PDiff$ is the infinite group of passive diffeomorphisms (general coordinate transformations). On the other hand, any two inequivalent Einstein space-times are different 4-geometries or "universes".

Consider now the abstract differential-geometric concept of active diffeomorphism D_A and its consequent action on the tensor fields defined on the differentiable manifold M^4 [see, for example, (Wald, 1984, pp.438-439)]. An active diffeomorphism D_A maps points of M^4 to points of M^4 : $D_A: p \to p' = D_A \cdot p$. Its tangent map D_A^* maps tensor fields $T \to D_A * \cdot T$ in such a way that $[T](p) \to [D_A^* \cdot T](p) \equiv [T'](p)$. Then $[D_A^* \cdot T](p) = [T](D_A^{-1} \cdot p)$. It is seen that the transformed tensor field $D_A^* \cdot T$ is a new tensor field whose components in general will have at p values that are different from those of the components of T. On the other hand, the components of $D_A^* \cdot T$ have at p' - by construction - the same values that the components of the original tensor field T have at p: $T'(D_A \cdot p) = T(p)$ or $T'(p) = T(D_A^{-1} \cdot p)$. The new tensor field $D_A^* \cdot T$ is called the drag-along (or push-forward) of T. There is another, nongeometrical - so-called dual - way of looking at the active diffeomorphisms (Norton, 1987). This duality is based on the circumstance that in each region of M^4 covered by two or more charts there is a one-to-one correspondence between an active diffeomorphism and a specific coordinate transformation. The coordinate transformation $\mathcal{T}_{D_A}: x(p) \to x'(p) = [\mathcal{T}_{D_A}x](p)$ which is dual to the active diffeomorphism D_A is defined so that $[\mathcal{T}_{D_A}x](D_A \cdot p) = x(p)$. Essentially, this duality transfers the functional dependence of the new tensor field in the new coordinate system to the old system of coordinates. By analogy, the coordinates of the new system [x'] are said to have been dragged-along with the active diffeomorphism D_A . It is important to note here, however, that the above dual view of active diffeomorphisms, as particular coordinate-transformations, is defined for the moment only implicitly.

In the abstract coordinate-independent language of differential geometry, Einstein's equations for the vacuum

$${}^{4}G_{\mu\nu}(x) \stackrel{def}{=} {}^{4}R_{\mu\nu}(x) - \frac{1}{2} {}^{4}R(x) {}^{4}g_{\mu\nu}(x) = 0. \tag{2.1}$$

can be written as G = 0, where G is the Einstein 2-tensor $(G = G_{\mu\nu}(x) dx^{\mu} \bigotimes dx^{\nu})$ in the coordinate chart x^{μ} . Under an active diffeomorphism $D_A : M^4 \mapsto M^4$, $D_A \in {}_ADiff M^4$, we have $G = 0 \mapsto D_A^*G = 0$, which shows that active diffeomorphisms are dynamical symmetries of the Einstein's tensor equations, i.e., they map solutions into solutions.

We have clarified elsewhere (Lusanna & Pauri, 2004a) the explicit relationships¹⁰ existing between passive and active diffeomorphisms on the basis of an important paper by Bergmann

¹⁰ At least for the infinitesimal active transformations.

and Komar (1972) in which it is shown that the largest group of passive dynamical symmetries of Einstein's equations is not $_PDiff\ M^4\ [x^{'\mu}=f^{\mu}(x^{\nu})]$ but instead a larger group of transformations of the form

$$Q: x'^{\mu} = f^{\mu}(x^{\nu}, {}^{4}g_{\alpha\beta}(x)),$$

$${}^{4}g'_{\mu\nu}(x'(x)) = \frac{\partial h^{\alpha}(x', {}^{4}g'(x'))}{\partial x'^{\mu}} \frac{\partial h^{\beta}(x', {}^{4}g'(x'))}{\partial x'^{\nu}} {}^{4}g_{\alpha\beta}(x).$$
(2.2)

In the case of completely Liouville-integrable systems, dynamical symmetries can be reinterpreted as maps of the space of Cauchy data onto itself. Although we don't have a general proof of the integrability of Einstein's equations, we know that if the initial value problem is well-posed and defined¹¹, as it is in the ADM Hamiltonian description, the space of Cauchy data is partitioned in gauge-equivalent classes of data: all of the Cauchy data in a given class identify a single 4-geometry or "universe". Therefore, under the given hypothesis, the dynamical symmetries of Einstein's equations fall in two classes only: a) those mapping different "universes" among themselves, and b) those acting within a single Einstein "universe", mapping gauge-equivalent Cauchy data among themselves. It is remarkable that, at least for the subset $Q' \subset Q$ (passive counterpart of a subset $_ADiff' M^4 \subset _ADiff M^4$) that corresponds to mappings among gauge-equivalent Cauchy data, the transformed metrics do indeed

¹¹ It is important to stress that in looking for global solutions of Einstein's equations as a system of partial differential equations, a number of preliminary specifications must be given. Among other things: a) the topology of space-time; b) whether the space-time is spatially-compact or asymptotically flat at spatial infinity; c) whether or not in the spatially-compact case there is a spatial boundary; d) the nature of the function space and the class of boundary conditions, either at spatial infinity or on the spatial boundary, for the 4-metric and its derivatives (only in the spatially-compact case without boundary there is no need of boundary conditions, replaced by periodicity conditions, so that these models of GR show the well-known Machian aspects which influenced Einstein and Wheeler). After these specifications have been made, a model of GR is identified. What remains to be worked out is the characterization of a well-posed initial value problem. Modulo technicalities, this requires choosing a 4-coordinate system and finding which combinations of the equations are of elliptic type (restrictions on the Cauchy data) and which are of hyperbolic type (evolution equations), namely the only ones requiring an initial value problem. At the Hamiltonian level, the elliptic equations are the first-class constraints identifying the constraint submanifold of phase space (see Section IV), while the hyperbolic equations are the Hamilton equations in a fixed gauge (a completely fixed Hamiltonian gauge corresponds on-shell to a 4-coordinate system, see Section IV). When the gauge variables can be separated from the Dirac observables, only the latter need an initial value problem (the gauge variables are arbitrary, modulo restrictions upon their range coming from the structure of the gauge orbits inside the constraint sub-manifold). Finally, given a space-like Cauchy surface in a 4-coordinate system (or in a fixed Hamiltonian gauge), each admissible set of Cauchy data gives rise to a different "universe" with the given boundary conditions. Clearly, each universe is defined modulo passive diffeomorphisms changing both the 4-coordinate system and the Cauchy surface (or modulo the Hamiltonian gauge transformations changing the gauge and the Cauchy surface) and also modulo the (on-shell) active diffeomorphisms.

belong to the same 4-geometry, i.e. the same equivalence class generated by applying all passive diffeomorphisms to the original 4-metrics: ${}^{4}Geom = {}^{4}Riem/_{P}Diff M^{4} = {}^{4}Riem/_{Q}'^{12}$.

Note finally that: a) an explicit passive representation of the infinite group of ${}_{A}Diff\,M^4$ is necessary anyway for our Hamiltonian treatment of the Hole Argument as well as for any comparison of the various viewpoints existing in the literature concerning the solutions of Einstein's equations; b) the group Q' describes the dynamical symmetries of Einstein's equations which are also local Noether symmetries of the Einstein-Hilbert action. The 4-metrics reached by using passive diffeomorphisms are, as it were, only a dense sub-set of the metrics obtainable by means of the group Q.

In conclusion, what is known as a 4-geometry, is also an equivalence class of solutions of Einstein's equations modulo the subset of dynamical symmetry transformations ${}_{A}Diff'M^4$, whose passive counterpart is Q'. Therefore, following Bergmann & Komar (1972), Wald (1984), we can state 13

$$^{4}Geom = {}^{4}Riem/_{P}Diff M^{4} = {}^{4}Riem/_{Q}' = {}^{4}Riem/_{A}Diff' M^{4}.$$
 (2.3)

III. THE HOLE ARGUMENT AND ITS DIS-SOLUTION

Although the issue could not be completely clear to Einstein in 1916, as shown by Norton (1987, 1992, 1993), it is precisely the nature of dynamical symmetry of the *active diffeomorphisms* that has been considered as expressing the *physically relevant* content of *general covariance*, as we shall presently see.

Remember, first of all that a mathematical model of GR is specified by a four-dimensional mathematical manifold M^4 and by a metrical tensor field g, where the latter represents both the chrono-geometrical structure of space-time and the potential for the inertial-gravitational field. Non-gravitational physical fields, when they are present, are also described by dynamical tensor fields, which appear to be sources of the Einstein equations. Assume now that M^4 contains a hole \mathcal{H} : that is, an open region where all the non-gravitational fields vanish so that the metric obeys the homogeneous Einstein equations. On M^4 we can define an active diffeomorphism D_A^* that re-maps the points inside \mathcal{H} , but blends smoothly into the identity map outside \mathcal{H} and on the boundary. By construction, for any point $x \in \mathcal{H}$ we have (in the

¹² Note, incidentally, that this circumstance is mathematically possible only because $_{P}Diff\,M^{4}$ is a non-normal dense sub-group of $Q^{'}$.

Eqs.(2.3) are usually taken for granted in mathematical physics, at least at the heuristic level. Since, however, the control in large of the group manifold of infinite-dimensional groups like $_PDiff\,M^4$ and Q' is, as yet, an open mathematical issue, one cannot be more rigorous on this point: see also the end of Section III. For more details about these issues, the interested reader should see Lusanna & Pauri (2004a, 2004b) where a new subset Q_{can} of Q is introduced, namely the Legendre pullback of the onshell Hamiltonian canonical transformations. We distinguish off-shell considerations, made within the variational framework before restricting to the dynamical solutions, from on-shell considerations, made after such a restriction. In (Lusanna & Pauri, 2004a and 2004b) it is shown, for instance, that we have also $^4Geom = ^4Riem/Q_{can}$, since, modulo technicalities, we have $Q_{can} = Q'$. Note that $_PDiff\,M^4 \cap Q_{can}$ are the passive diffeomorphisms which are re-interpretable as Hamiltonian gauge transformations.

abstract tensor notation) $g'(D_A x) = g(x)$, but of course $g'(x) \neq g(x)$ (in the same notation). The crucial fact is that from the general covariance of Einstein's equations it follows that if g is one of their solutions, so is the drag-along field $g' \equiv D_A^* g$.

What is the correct interpretation of the new field g'? Clearly, the transformation involves an active redistribution of the metric over the points of the manifold in \mathcal{H} , so the critical question is whether and how the points of the manifold are primarily individuated. Now, if we think of the points of \mathcal{H} as intrinsically individuated physical events, where intrinsic means that their identity is autonomous and independent of any physical field, the metric in particular - a claim that is associated with any kind of manifold substantivalism - then g and g' must be regarded as physically distinct solutions of the Einstein equations (after all, $g'(x) \neq g(x)$ at the same point x). This appears as a devastating conclusion for the causality, or better, the determinateness¹⁴ of the theory, because it implies that, even after we specify a physical solution for the gravitational and non-gravitational fields outside the hole - in particular, on a Cauchy surface for the initial value problem - we are still unable to predict a unique physical solution within the hole.

According to Earman and Norton (1987), the way out of the Hole Argument lies in abandoning manifold substantivalism: they claim that if diffeomorphically-related metric fields were to represent different physically possible "universes", then GR would turn into an indeterministic theory. And since the issue of whether determinism holds or not at the physical level cannot be decided by opting for a metaphysical doctrine like manifold substantivalism, they conclude that one should refute any kind of such substantivalism. Since, however, relationism does not amount to the mere negation of substantivalism, and since the literature contains so many conflicting usages of the term "relationism", they do not simply conclude that space-time is relational. They state the more general assumption (which - they claim - is applicable to all space-time theories) that "diffeomorphic models in a space-time theory represent the same physical situation". i.e. must be interpreted as describing the same "universe" (Leibniz equivalence).

The fact that the Leibniz equivalence seems here no more than a sophisticated re-phrasing of what physicists consider a foregone conclusion for general relativity, should not be taken at face value, for the real question for the opposing "sensible substantivalist" is whether or not space-time should be simply identified with the bare manifold deprived of any physical field, and of the metric field in particular, as Earman and Norton do, instead of with a set of points each endowed with its own metrical fingerprint¹⁵. Actually, this substantivalist could sustain the conviction - as we ourselves do - that the metric field, because of its basic causal structure, has ontological priority (Pauri, 1996) over all other fields and, therefore, it is not like any other field, as Earman and Norton would have it. And, according to this view, as we shall presently see, Leibniz equivalence is not the last word on the issue ¹⁶.

¹⁴ We prefer to avoid the term *determinism*, because we believe that its metaphysical flavor tends to overstate the issue at stake. This is especially true if *determinism* is taken in opposition to *indeterminism*, which is not mere absence of *determinism*. We are concerned here with the question of being *determinate* or *under-determinate*, referred to solutions of Einstein's equations.

¹⁵ See, for example, Bartels (1994) and Maudlin (1988).

¹⁶ Let us recall that already in 1984 Michael Friedman was lucidly aware of the unsatisfactory status of the understanding of the relation between diffeomorphic models in terms of Leibniz equivalence, when he wrote (our emphasis) "Further, if the above models are indeed equivalent representations of the same situation (as it would seem they must do) then how do we describe this physical situation intrinsically?

We do believe that the bare manifold of points, deprived of the infinitesimal pythagorean structure defining the basic distinction between temporal and spatial directions, let alone the causal structure which teach all the other fields how to move, can hardly be seen as space-time. Consequently, and in agreement with Stachel (1993), we believe that asserting that g and D_A^*g represent one and the same gravitational field implies that the mathematical individuation of the points of the differentiable manifold by their coordinates has no physical content until a metric tensor is specified ¹⁷. Stachel stresses that if g and D_A^*g must represent the same gravitational field, they cannot be physically distinguished in any way. Accordingly, when we act on g with D_A^* to create the drag-along field D_A^*g , no element of physical significance can be left behind: in particular, nothing that could identify a point x of the manifold itself as the same point of space-time for both g and D_A^*g . Instead, when x is mapped onto $x' = D_A^*x$, it carries over its identity, as specified by g'(x') = g(x). This means, for one thing, that "the last remnant of physical objectivity" of space-time points, if any, should be sought for in the physical content of the metric field itself.

These remarks led Stachel to the important conclusion that vis á vis the physical pointevents, the metric actually plays the role of individuating field. Precisely, Stachel suggested that this individuating role could be implemented by four invariant functionals of the metric, which Komar and Bergmann (Komar 1958; Bergmann & Komar, 1960) had already considered. Stachel, however, did not follow up on this proposal by providing a concrete realization in terms of solutions of Einstein's equations, something that we instead will presently do. At the same time we will show in Section VI that Stachel's suggestion, as it stands, remains at a too abstract level and fails to exploit the crucial distinction between ontic and arbitrary epistemic content of the Bergmann-Komar invariant functionals of the metric, that is necessary to specify the solutions they apply to.

We conclude this Section by summarizing the implications of our analysis about the meaning and the philosophical import of the Hole Argument. The force of the *indeterminacy* argument apparently rests on the following basic facts: (i) a solution of Einstein's equations must be preliminarily individuated outside (and, of course, inside) the Hole, otherwise there would be no meat for the Argument itself. Although the original formulation of the Hole Argument, as well as many subsequent expositions of it, are silent on this point, we will see that the Hole Argument is unavoidably entangled with the initial value problem 18 ; (ii) the active diffeomorphism D_A^* , which is purportedly chosen to be the identity outside the hole \mathcal{H} , is a dynamical symmetry of Einstein's equations, so that it maps solutions into solutions, equivalent (as 4-geometries or Einstein "universes") or not; (iii) since D_A^* is, by hypothesis, the identity on the Cauchy hyper-surface, it cannot map a solution defining a given Einstein

Finding such an *intrinsic* characterization (avoiding quantification over *bare* points) appears to be a non-trivial, and so far unsolved mathematical problem. (Note that it will not do simply to replace points with equivalence classes of points: for, in many cases, the equivalence class in question will contain all points of the manifold)", see Friedman, (1984).

¹⁷ Coordinatization is the only way to individuate the points mathematically since, as stressed by Hermann Weyl: "There is no distinguishing objective property by which one could tell apart one point from all others in a homogeneous space: at this level, fixation of a point is possible only by a demonstrative act as indicated by terms like this and there." (Weyl. 1946, p. 13). See also Schlick, (1917), quoted in M.Friedman, (2003), p.165.

¹⁸ It is interesting to find that David Hilbert stressed this point already in 1917 (Hilbert, 1917).

"universe" into a different "universe", which would necessarily correspond to inequivalent Cauchy data; but (iv) nevertheless, we are told by the Hole Argument that D_A^* engenders a "different" solution inside the Hole.

Actually, in spite of the *prima facie* geometric obviousness of the identity condition required for D_A^* outside the Hole, it is quite illusory trying to explain all the facets of the relations of the Argument with the initial value problem in the purely abstract way of differential geometry. The point is that the differential-geometric 4-D formulation cannot exploit the advantage that the Hamiltonian formulation possess of working off-shell (i.e. before going onto solutions of the Hamilton equations: see footnote 13). This is crucial since in the present context the 4-D active diffeomorphisms - qua dynamical symmetries of Einstein's equations - must be directly applied on *solutions* of GR. These solutions, however, cannot be exaustively managed in the 4-D configurational approach in terms of initial data because of the non-hyperbolicity of Einstein's equations. The Hole Argument needs that the Cauchy problem be formulated outside the Hole explicitly and in advance, a fact that requires abandoning the Lagrangian way in favor of Hamiltonian methods. At this point, the results of the previous Section (the passive counterpart of D_A^* must belong to Q', or belong to Q but not to Q') leave us with the sole option that, once rephrased in the passive Hamiltonian language, the active diffeomorphism D_A^* exploited by the Argument must lie in the subset Q' ($_ADiff'$ M^4). But, then, it must necessarily map Cauchy data into gaugeequivalent Cauchy data, precisely those gauge-equivalent data that generate the allegedly "different" solution within the Hole. In the end, the "difference" turn out to correspond to a mere different choice of the gauge for the same solution. Thus Leibniz equivalence boils down to mere *qauge equivalence* in its strict sense¹⁹, an effect that - for what said above - cannot be transparently displayed in the configurational geometric description. On the other hand, were the active diffeomorphism D_A^* , once passively rephrased, to belong to the group Q but not to the subset Q' (i.e., were it originally lying in ${}_{A}Diff M^4$, but not in $_ADiff'M^4$), then it would not correspond to a mere gauge equivalence and it would necessarily modify the Cauchy data outside the Hole. Therefore it would lead to a really different Einstein "universe" but it would violate the assumption of the Hole Argument that D_A^* be the identity on the Cauchy hyper-surface. In any case, it is seen that the disappearance of the "indeterminacy" rests upon the necessity of formulating the Cauchy problem before talking about the relevant properties of the solutions.

We conclude that - to the extent that the Cauchy problem is well-posed, i.e. in every globally hyperbolic space-time and not necessarily in our class only - exploiting the original Hole Argument to the effect of asking ontological questions about the general relativistic space-time is an enterprise devoid of real philosophical impact, in particular concerning the menace of indeterminism. There is clearly no room left for upholding manifold substantivalism, "different worlds", "metric essentialism" or any other metaphysical doctrine about space-time points in the face of the Hole Argument. Of course, such metaphysical doctrines can still be defended, yet independently of the Hole story. The Hole Argument maintains nevertheless an interesting open question regarding the issue of the physical (viz. dynamical) individuation of the point-events of M^4 (see Section V).

¹⁹ The physical meaning of this equivalence will be clarified in Section IV.

IV. THE CHRISTODOULOU-KLAINERMANN SPACE-TIMES, 3+1 SPLITTING, AND ADM CANONICAL REDUCTION

The Christodoulou-Klainermann space-times are a continuous family of space-times that are non-compact, globally hyperbolic, asymptotically flat at spatial infinity (asymptotic Minkowski metric, with asymptotic Poincaré symmetry group) and topologically trivial $(M^4 \equiv R^3 \times R)$, supporting global 4-coordinate systems.

The ADM Hamiltonian approach starts with a 3+1 splitting of the 4-dimensional manifold M^4 into constant-time hyper-surfaces $\Sigma_{\tau} \equiv R^3$, indexed by the parameter time τ , each equipped with coordinates σ^a (a = 1,2,3) and a three-metric 3g (in components $^3g_{ab}$). The parameter time τ and the coordinates σ^a (a = 1,2,3) are in fact Lorentz-scalar, radar coordinates adapted to the 3+1 splitting (Alba & Lusanna, 2003, 2005a). They are defined with respect to an arbitrary observer, a centroid $X^{\mu}(\tau)$, chosen as origin, whose proper time may be used as the parameter τ labelling the hyper-surfaces. On each hyper-surface all the clocks are conventionally synchronized to the value τ . Note that such coordinates are intrinsically frame-dependent since they parametrize a NIF centered on the arbitrary observer. The simultaneity (and Cauchy) hyper-surfaces Σ_{τ} are described by the embedding functions $x^{\mu} = z^{\mu}(\tau, \sigma^a) = X^{\mu}(\tau) + F^{\mu}(\tau, \sigma^a)$, $F^{\mu}(\tau, 0^a) = 0$.

An important point to be kept in mind is that the explicit functional form of embedding functions and - consequently - of the geometry of the 3 + 1 splitting of M^4 , thought to be implicitly given at the outset, remains arbitrary until the solution of Einstein's equations is worked out in a fixed gauge: see later.

Now, start at a point on Σ_{τ} , and displace it infinitesimally in a direction that is normal to Σ_{τ} . The resulting change in τ can be written as $\Delta \tau = N d\tau$, where N is the so-called lapse function. Moreover, the time displacement $d\tau$ will also shift the spatial coordinates: $\sigma^a(\tau + d\tau) = \sigma^a(\tau) + N^a d\tau$, where N^a is the shift vector. Then the interval between (τ, σ^a) and $(\tau + d\tau, \sigma^a + d\sigma^a)$ is: $ds^2 = N^2 d\tau^2 - {}^3g_{ab}(d\sigma^a + N^a d\tau)(d\sigma^b + N^b d\tau)$. The configurational variables N, N^a , ${}^3g_{ab}$ (replacing the 4-metric g) together with their 10 conjugate momenta, index a 20-dimensional phase space²¹. Expressed (modulo surface terms) in terms of the ADM variables, the Einstein-Hilbert action is a function of N, N^a , ${}^3g_{ab}$ and their first time-derivatives, or equivalently of N, N^a , ${}^3g_{ab}$ and the extrinsic curvature ${}^3K_{ab}$ of the hyper-surface Σ_{τ} , considered as an embedded manifold.

Since Einstein's original equations are not hyperbolic, it turns out that the canonical momenta are not all functionally independent, but satisfy four conditions known as primary constraints (they are given by the vanishing of the lapse and shift canonical momenta). Another four, secondary constraints, arise when we require that the primary constraints be preserved through evolution (the secondary constraints are called the super-hamiltonian $\mathcal{H}_0 \approx 0$, and the super-momentum $\mathcal{H}_a \approx 0$, (a = 1, 2, 3) constraints, respectively). The

Let us stress that the radar-coordinates are not ordinary coordinates x^{μ} in a chart of the Atlas \mathcal{A} of M^4 . They should be properly called pseudo-coordinates in a chart of the Atlas $\tilde{\mathcal{A}}$ defined by adding to M^4 the extra-structure of all its admissible 3+1 splittings: actually the new coordinates are adapted to this extra-structure. If the embedding of the constant-time hyper-surfaces Σ_{τ} of a 3+1 splitting into M^4 is described by the functions $z^{\mu}(\tau, \sigma^a)$, then the transition functions from the adapted radar-coordinates $\sigma^A = (\tau; \sigma^a)$ to the ordinary coordinates are $\frac{\partial z^{\mu}(\tau, \sigma^a)}{\partial \sigma^A}$.

²¹ Of course, all these *variables* are in fact *fields*.

eight constraints are given as functions of the canonical variables that vanish on the constraint surface. The existence of such constraints implies that not all the points of the 20-dimensional phase space represent physically meaningful states: rather, we are restricted to the constraint surface where all the constraints are satisfied, i.e., to a 12-dimensional (20-8) surface which, however, does not possess the geometrical structure of a true phase space. When used as generators of canonical transformations, the eight constraints map points on the constraint surface to points on the same surface; these transformations are known as gauge transformations.

To obtain the correct dynamics for the constrained system, we must consider the Dirac Hamiltonian, which is the sum of the DeWitt surface term (DeWitt, 1967) ²² [present only in spatially non-compact space-times and becoming the ADM energy after suitable manipulations (Lusanna, 2001; DePietri & Lusanna & Martucci & Russo, 2002)], of the secondary constraints multiplied by the lapse and shift functions, and of the primary constraints multiplied by arbitrary functions (the so-called Dirac multipliers). If, following Dirac, we make the reasonable demand that the evolution of all physical variables be unique - otherwise we would have real physical variables that are indeterminate and therefore neither observable nor measurable - then the points of the constraint surface lying on the same gauge orbit, i.e. linked by gauge transformations, must describe the same physical state. Conversely, only the functions in phase space that are invariant with respect to gauge transformations can describe physical quantities.

To eliminate this ambiguity and create a one-to-one mapping between points in the phase space and physical states, we must impose further constraints, known as gauge conditions or gauge-fixings. The gauge-fixings can be implemented by arbitrary functions of the canonical variables, except that they must define a reduced phase space that intersects each gauge orbit exactly once (orbit conditions). The number of independent gauge-fixing must be equal to the number of independent constraints (i.e. 8 in our case). The canonical reduction follows a cascade procedure²³. Precisely, the gauge-fixings to the super-hamiltonian and super-momentum come first (call it Γ_4): they determine the 3-coordinate system and the off-shell shape of Σ_{τ} ; then the requirement of their time constancy fixes the gauges with respect to the primary constraints: they determine the lapse and shift functions. Finally the requirement of time constancy for these latter gauge-fixings determines the Dirac multipliers. Therefore, the first level of gauge-fixing gives rise to a complete gauge-fixing, say Γ_8 , and is

The DeWitt surface term is uniquely determined as the sum of two parts: a) the surface integral to be extracted from the Einstein-Hilbert action to get the ADM action; b) a surface integral due to an integration by parts required by the Legendre transformation from the ADM action to phase space [see (Lusanna, 2001) after Eq.(5.5) and (Hawking & Horowitz, 1996)]. By adding a surface term different from the ADM one, we would get another action with the same equations of motion but an a-priori different canonical formulation. Still another option is to consider the metric and the Christoffel connection as independent configuration variables: this is the first-order Palatini formalism, which has a much larger gauge freedom including also second class constraints. All these canonical formulations must lead anyway to the same number of physical degrees of freedom.

²³ This procedure is the natural one for systems with first-class constraints, because it avoids mathematical inconsistencies like instabilities in the pre-symplectic geometric structure. Usually one adds the gauge fixings to all the constraints without taking this point into consideration, so that, especially in GR, there is the possibility to get coordinate singularities in a finite time.

sufficient to remove all the gauge arbitrariness. This is equivalent to the choice of a NIF, namely, as said in footnote 7, to the determination of the appearances of phenomena in a global extended laboratory (see later).

The Γ_8 procedure reduces the original 20-dimensional phase space to a copy Ω_4 of the abstract reduced phase-space $\tilde{\Omega}_4$ having 4 degrees of freedom per point (12 - 8 gauge-fixings). Abstractly, the reduced phase-space with its symplectic structure is defined by the quotient of the constraint surface with respect to the 8-dimensional group of gauge transformations and represents the space of the abstract gauge-invariant observables of GR: two configurational and two momentum variables. These observables carry the physical content of the theory in that they represent the intrinsic degrees of freedom of the gravitational field (remember that at this stage we are dealing with a pure gravitational field without matter).

For any complete gauge Γ_8 , we get a Γ_8 -dependent $copy \Omega_4$ of the abstract Ω_4 in terms of the symplectic structure (Dirac brackets) defined by the given gauge-fixings and coordinatized by four Dirac observables [call such field observables q^r , p_s (r,s = 1,2)]. The functional form of these Dirac observables (concrete realization of the gauge-invariant abstract observables in the given complete gauge Γ_8) in terms of the original canonical variables depends upon the chosen gauge, so that such observables - a priori - are neither tensors nor invariant under pDiff. In each gauge Γ_8 , the original 8 gauge variables are now uniquely determined functions of the Dirac observables. Yet, off shell, barring sophisticated mathematical complications, any two copies of Ω_4 are diffeomorphic images of one-another.

It is important to understand qualitatively the geometric meaning of the eight infinitesimal off-shell Hamiltonian gauge transformations and thereby the geometric significance of the related gauge-fixings. i) The transformations generated by the four primary constraints modify the lapse and shift functions which, in turn, determine both how densely the space-like hyper-surfaces Σ_{τ} are distributed in space-time and the appearance of gravito-magnetism on them; ii) the transformations generated by the three super-momentum constraints induce a transition on Σ_{τ} from one given 3-coordinate system to another; iii) the transformation generated by the super-hamiltonian constraint induces a transition from one given a-priori "form" of the 3+1 splitting of M^4 to another (namely, from a given notion of distant simultaneity to another), by operating deformations of the space-like hyper-surfaces in the normal direction.

It should be stressed that the manifest effect of the gauge-fixings related to the above transformations emerges only at the end of the canonical reduction and after the solution of the Einstein-Hamilton equations has been worked out (i.e., on shell). This happens because the role of the gauge-fixings is essentially that of choosing the functional form in which all the gauge variables depend upon the Dirac observables, i.e. - physically - of fixing the form of the inertial potentials of the associated NIF. As anticipated in the Introduction, this important physical aspect is completely lost within the abstract reduced phase space $\tilde{\Omega}_4$, which could play, nevertheless, another role (see Sections V and VI).

It is only after the initial conditions for the *Dirac observables* have been arbitrarily selected on a Cauchy surface that the whole four-dimensional chrono-geometry of the resulting *Einstein "universe"* is *dynamically* determined, including the embedding functions $x^{\mu} = z^{\mu}(\tau, \vec{\sigma})$ (i.e. the *on-shell* shape of Σ_{τ}). In particular, since the transformations generated by the super-hamiltonian modify the rules for the synchronization of distant clocks, *all* the relativistic *conventions*, associated to the 3 + 1 slicing of M^4 in a given *Einstein "universe"*,

turn out to be dynamically-determined, gauge-related options²⁴.

Two important points must be emphasized.

First, in order to carry out the canonical reduction explicitly, before implementing the gauge-fixings we have to perform a basic canonical transformation at the off-shell level, the so-called Shanmugadhasan transformation (Shanmugadhasan, 1973; Lusanna, 1993), moving from the original canonical variables to a new basis including the Dirac observables as a canonical subset²⁵. It should be stressed here that it is not known whether the Shanmugadhasan canonical transformation, and therefore the GR observables, can be defined globally in Christodoulou - Klainermann space-times. In most of the spatially compact space-times this cannot be done for topological reasons. A further problem is that in field theory in general the status of the canonical transformations is still heuristic. Therefore the only tool (viz. the Shanmugadhasan transformation) we have for a systematic search of GR observables in every type of space-time is still lacking a rigorous definition. In conclusion, the mathematical basis of our analysis regarding the objectivity of points is admittedly heuristic, yet our arguments are certainly no more heuristic than the overwhelming majority of the theoretical and/or philosophical claims concerning every model of GR.

The Shanmugadhasan transformation is highly non-local in the metric and curvature variables: although, at the end, for any τ , the Dirac observables are fields indexed by the coordinate point σ^a , they are in fact highly non-local functionals of the metric and the extrinsic curvature over the whole off shell surface Σ_{τ} . We can write, symbolically:

$$q^{r}(\tau, \vec{\sigma}) = \mathcal{F}_{[\Sigma_{\tau}]}{}^{r}[(\tau, \vec{\sigma})| {}^{3}g_{ab}, {}^{3}\pi^{cd}]$$

$$p_{s}(\tau, \vec{\sigma}) = \mathcal{G}_{[\Sigma_{\tau}]_{s}}[(\tau, \vec{\sigma})| {}^{3}g_{ab}, {}^{3}\pi^{cd}], \quad r, s = 1, 2.$$
(4.1)

Second: since, as mentioned, in *spatially compact* space-times the original canonical Hamiltonian in terms of the ADM variables is zero, the Dirac Hamiltonian happens to be written solely in terms of the eight constraints and Lagrangian multipliers. This means, however, that this Hamiltonian generates purely harmless gauge transformations, so that *it cannot engender any real temporal change*. Therefore, in spatially-compact space-times, in a completely fixed Hamiltonian gauge we have a vanishing Hamiltonian, and the canonical Dirac observables are constant of the motion, i.e. τ -independent.

In these models of GR with *spatially-compact* space-times without boundary (nothing is known if there is a boundary) there is the problem of re-introducing the *appearance* of evolution in a frozen picture. Without entering this debated topic [see the viewpoints

 $^{^{24}}$ Unlike the special relativistic case where the various possible conventions are non-dynamical options.

²⁵ In practice, this transformation is adapted to seven of the eight constraints (Lusanna, 2001; DePietri & Lusanna & Martucci & Russo, 2002): they are replaced by seven of the new momenta whose conjugate configuration variables are the gauge variables describing the *lapse* and *shift* functions and the choice of the spatial coordinates on the simultaneity surfaces. The new basis contains the conformal factor (or the determinant) of the 3-metric, which is determined by the super-hamiltonian constraint (though as yet no solution has been found for this equation, also called the Lichnerowicz equation), and its conjugate momentum (the last gauge variable whose variation describes the normal deformations of the simultaneity surfaces).

of Earman (2002,2003), Maudlin (2002), Rovelli (1991,2002) as well as the criticisms of Kuchar (1992,1993) and Unruh (1991)], we only add a remark on the problem of time. In all the globally hyperbolic space-times (the only ones admitting a canonical formulation) there is a mathematical time τ , labeling the simultaneity (and Cauchy) surfaces, which has to be connected to some empirical notion of time (astronomical ephemerides time, laboratory clock,...). In a GR model with the frozen picture there is no physical Hamiltonian governing the evolution in τ ²⁶ and an open problem is how to define a local evolution in terms of a clock built with GR observables (with a time monotonically increasing with τ) and how to parametrize other GR observables in terms of this clock (see the evolving constants of motion and the partial and complete observables of Rovelli (1991, 2002), as well as a lot of different point of views).

Our advantage point, however, is that, in the case of spatially non-compact space-times such as those we are dealing with in this work, the generator of τ -temporal evolution is the weak ADM energy²⁷. Indeed, this quantity does generate real τ -temporal modifications of the canonical variables, which subsequently can be rephrased in terms of some empirical clock monotonically increasing in τ . It's important to stress that the density $\mathcal{E}_{ADM}(\tau, \vec{\sigma})$ of the weak ADM energy $\int d^3\sigma \mathcal{E}_{ADM}(\tau, \vec{\sigma})$ is a gauge-dependent quantity since it contains the potentials of the inertial forces explicitly. This is nothing else than another aspect of the gauge-dependence problem of the energy density in GR.

Thus, the final Einstein-Dirac-Hamilton equations for the Dirac observables are

$$\dot{q}^r = \{q^r, H_{\text{ADM}}\}^*, \quad \dot{p}_s = \{p_s, H_{\text{ADM}}\}^*, \quad r, s = 1, 2,$$
 (4.2)

²⁶ Unless, following Kuchar (1993), one states that the super-Hamiltonian constraint is not a generator of gauge transformations but an effective Hamiltonian instead.

 $^{^{27}}$ The ADM energy is a Noether constant of motion representing the $total\ mass$ of the instantaneous 3-"universe", just one among the ten asymptotic ADM Poincare' charges that, due to the absence of super-translations, are the only asymptotic symmetries existing in Christodoulou-Klainermann spacetimes. Consequently, the Cauchy surfaces Σ_{τ} must tend to space-like hyper-planes, normal to the ADM momentum, at spatial infinity. This means that: (i) such Σ_{τ} 's are the rest frame of the instantaneous 3-"universe"; (ii) asymptotic inertial observers exist and have to be identified with the fixed stars, and (iii) an asymptotic Minkowski metric is naturally defined. This asymptotic background allows us to avoid a split of the metric into a background metric plus a perturbation in the weak field approximation (note that our space-times provide a model of either the solar system or our galaxy but, as yet, not a well-defined model for cosmology). Finally, if gravity is switched off, the Christodoulou-Klainermann space-times collapse to Minkowski space-time and the ADM Poincare' charges become the Poincare' special relativistic generators. These space-times provide, therefore, the natural model of GR for incorporating particle physics which, in every formulation, is a chapter of the theory of representations of the Poincare' group on Minkowski space-time in inertial frames, with the elementary particles identified by the mass and spin invariants. If we change the boundary conditions, allowing the existence of super-translations, the asymptotic ADM Poincare' group is enlarged to the infinite-dimensional asymptotic SPI group (Wald, 1984) and we loose the possibility of defining the spin invariant. Note that in spatially compact space-times with boundary it could be possible to define a boundary Poincare' group (lacking in absence of boundary), but we know of no result about this case. The mathematical background of these results can be found in (Lusanna, 2001; Lusanna & Russo, 2002; DePietri & Lusanna & Martucci & Russo, 2002; Agresti & DePietri & Lusanna & Martucci, 2004) and references therein.

where H_{ADM} is intended as the restriction of the weak ADM energy to Ω_4 and where the $\{\cdot,\cdot\}^*$ are the Dirac brackets.

In conclusion, within the Hamiltonian formulation, we found a class of solutions in which - unlike what has been correctly argued by Earman (Earman, 2002; Belot & Earman, 1999, 2001) for spatially- compact space-times - there is a *real, NIF-dependent, temporal change*. But this of course also means that the *frozen-time* picture, being model dependent, is not a *typical* feature of GR.

On the other hand it is not clear whether the formulation of a cosmological model for GR is necessarily limited to spatially compact space-times without boundary. As already said, our model is suited for the solar system or the galaxy. It cannot be excluded, however, that our asymptotic inertial observers (till now identified with the fixed stars) might be identified with the preferred frame of the cosmic background radiation with our 4-metric including some pre-asymptotic cosmological term.

V. FINDING THE LAST REMNANT OF PHYSICAL OBJECTIVITY: THE INTRINSIC GAUGE AND THE DYNAMICAL INDIVIDUATION OF POINT-EVENTS

We know that only two of the ten components of the metric are physically essential: it seems plausible then to suppose that only this subset can act as an individuating field, and that the remaining components play a different role.

Consider the following four scalars invariant functionals (the eigenvalues of the Weyl tensor), written here in Petrov's compressed notation:

$$w_{1} = \operatorname{Tr}(gWgW),$$

$$w_{2} = \operatorname{Tr}(gW\epsilon W),$$

$$w_{3} = \operatorname{Tr}(gWgWgW),$$

$$w_{4} = \operatorname{Tr}(gWgW\epsilon W),$$

$$(5.1)$$

where g is the 4-metric, W is the Weyl tensor, and ϵ is the Levi–Civita totally antisymmetric tensor.

Bergmann and Komar (Komar, 1958; Bergmann & Komar, 1960; Bergmann, 1961, 1962) proposed a set of invariant intrinsic pseudo-coordinates as four suitable functions of the w_T

$$\hat{I}^{[A]} = \hat{I}^{[A]} [w_T[g(x), \partial g(x)]], \quad A = 0, 1, 2, 3.$$
(5.2)

Indeed, under the hypothesis of no space-time symmetries, the $\hat{I}^{[A]}$ can be used to label the point-events of space-time, at least locally.²⁹ Since they are *scalars*, the $\hat{I}^{[A]}$ are invariant under passive diffeomorphisms (therefore they do not define a coordinate chart in the usual sense, precisely as it happens with radar coordinates).

²⁸ Modulo the equations of motion, the eigenvalues w_T are functionals of the 4-metric and its first derivatives.

²⁹ Problems might arise if we try to extend the labels to the entire space-time: for instance, the coordinates might turn out to be multi-valued.

Clearly, our attempt to use intrinsic coordinates to provide a physical individuation of point-events would prima facie fail in the presence of symmetries, when the $\hat{I}^{[A]}$ become degenerate. This objection was originally raised by Norton (see Norton, 1988, p.60) as a critique to manifold-plus-further-structure (MPFS) substantivalism (see for instance Maudlin, 1988, 1990). Several responses are possible. First, although to this day all the known exact solutions of the Einstein equations admit one or more symmetries, these mathematical models are very idealized and simplified; in a realistic situation (for instance, even with two masses alone) space-time would be filled with the excitations of the gravitational degrees of freedom, and would admit no symmetries at all. Second, the parameters of the symmetry transformations can be used as supplementary individuating fields, since, as noticed by Stachel (1993), they also depend on metric field, through its isometries. Third, and most important, in our analysis of the physical individuation of points we are arguing a question of principle, and therefore we must consider generic solutions of the Einstein equations rather than the null-measure set of solutions with symmetries.

It turns out that the four Weyl scalar invariants can be re-expressed in terms of the ADM variables, namely the lapse N and shift N^a functions, the 3-metric ${}^3g_{ab}$ and its conjugate canonical momentum (the extrinsic curvature ${}^3K_{a,b}$) 30 . Consequently the $\hat{I}^{[A]}$ can be exploited to implement four gauge-fixings constraints involving a hyper-surface Σ_{τ} and its embedding in M^4 . On the other hand, in a completely fixed gauge Γ_8 , the $\hat{I}^{[A]}$ become gauge dependent functions of the Dirac observables of that gauge.

Writing

$$\hat{I}^{[A]}[w_T(g,\partial g)] \equiv \hat{Z}^{[A]}[\hat{w}_T(^3g,^3\pi,N,N^a)], \quad A = 0,1,2,3;$$
(5.3)

and selecting a completely arbitrary, radar, pseudo-coordinate system $\sigma^A \equiv [\tau, \sigma^a]$ adapted to the Σ_{τ} surfaces, we apply the intrinsic gauge-fixing defined by

$$\chi^{A} \equiv \sigma^{A} - \hat{Z}^{[A]} \left[\hat{w}_{T}[^{3}g(\sigma^{B}), ^{3}\pi(\sigma^{D}), N(\sigma^{E}), N^{a}(\sigma^{F})] \right] \approx 0, \quad A, B, D, E, F = 0, 1, 2, 3;$$
 (5.4)

to the super-hamiltonian (A = 0) and the super-momentum (A = 1,2,3) constraints. This is a good gauge-fixing provided that the functions $\hat{Z}^{[A]}$ are chosen to satisfy the fundamental orbit conditions $\{\hat{Z}^{[A]}, \mathcal{H}_B\} \neq 0$, (A, B = 0, 1, 2, 3), which ensure the independence of the χ^A and carry information about the Lorentz signature. Then the complete Γ_8 intrinsic gauge-fixing leads to

$$\sigma^{A} \equiv \tilde{Z}^{[A]}[q^{a}(\sigma^{B}), p_{b}(\sigma^{D})|\Gamma)], \quad A, B, D = 0, 1, 2, 3; \quad a, b = 1, 2;$$
(5.5)

where the notation indexed by $|\Gamma\rangle$ means the functional form assumed in the chosen gauge Γ_8 .

The last equation becomes an *identity* with respect to the σ^A , and amounts, *on-shell*, to a *definition* of the *radar pseudo-coordinates* σ^A as four *scalars* providing a *physical individuation* of any point–event, in the gauge-fixed coordinate system, in terms of the gravitational degrees of freedom q^a and p_b . In this way each of the point–events of space-time is endowed with its own *metrical fingerprint* extracted from the tensor field, i.e., the value of

³⁰ Bergmann and Komar have shown that the four eigenvalues of the spatial part of the Weyl tensor depend only upon the 3-metric and its conjugate momentum.

the four scalar functionals of the *Dirac observables* (exactly four!)³¹. The price that we have paid for this achievement is that we have broken general covariance! This, however, is not a drawback because every choice of 4-coordinates for a point (every gauge-fixing, in the Hamiltonian language), in any procedure whatsoever for solving Einstein's equations, amounts to a breaking of general covariance, by definition. On the other hand the whole extent of general covariance can be recovered by exploiting the gauge freedom.

Note that our construction does *not* depend on the selection of a set of physically preferred intrinsic pseudo-coordinates, because by modifying the functions $I^{[A]}$ we have the possibility of implementing any (adapted) radar-coordinate system. Passive diffeomorphism-invariance reappears in a different suit: we find exactly the same functional freedom of $_PDiff\ M^4$ in the functional freedom of the choice of the $pseudo-coordinates\ Z^{[A]}$ (i.e., of the gauge-fixing). Any adapted radar-coordinatization of the manifold can be seen as embodying the physical individuation of points, because it can be implemented as the Komar-Bergmann $intrinsic\ pseudo-coordinates\ after\ we choose the correct\ Z^{[A]}$ and select the proper gauge.

In conclusion, as soon as the Einstein-Dirac-Hamilton equations are solved in the chosen gauge Γ_8 , starting from given initial values of the Dirac observables on a Cauchy hypersurface Σ_{τ_0} , the evolution in τ throughout M^4 of the Dirac observables themselves, whose dependence on space (and on parameter time) is indexed by the chosen coordinates σ^A , yields the following dynamically-determined effects: i) reproduces the σ^A as the Bergmann-Komar intrinsic pseudo-coordinates; ii) reconstructs space-time as an (on-shell) foliation of M^4 ; iii) defines the associated NIF; iv) determines a simultaneity convention.

Now, what happens if matter is present? Matter changes the Weyl tensor through Einstein's equations and, in the new basis constructed by the Shanmugadhasan transformation, contributes to the separation of gauge variables from Dirac observables through the presence of its own Dirac observables. In this case we have Dirac observables for both the gravitational field and the matter fields, which satisfy coupled Einstein-Dirac-Hamilton equations. Since the gravitational Dirac observables will still provide the individuating fields for point-events according to our procedure, matter will come to influence the evolution of the gravitational Dirac observables and thereby the physical individuation of point-events. Of course, a basic role of matter is the possibility of building apparatuses for the measurement of the gravitational field, i.e. for an empirical localization of point-events. As shown elsewhere (Pauri & Vallisneri, 2002, Lusanna & Pauri, 2004a, 2004b), lacking a dynamical theory of measurement, the epistemic circuit of GR can be approximately closed via an experimental three-steps procedure that, starting from concrete radar measurements and using test-objects, ends up in a complete and empirically coherent intrinsic individuating gauge fixing. In this way, the value of the intrinsic coordinates at a point-event can be extracted (in principle) by an actual experiment designed to measure the w_T .

Finally, let us emphasize that, even in the case with matter, time evolution is still ruled by the weak ADM energy. Therefore, the temporal variation corresponds to a real change and not merely to a harmless gauge transformation as in other models of GR. The latter include, as already stressed in Section IV, the spatially compact space-time without boundary (or simply closed models) which are exploited by Earman (2002). Since in these spatially

³¹ The fact that there are just *four* independent invariants for the vacuum gravitational field should not be regarded as a coincidence. On the contrary, it is crucial for the purpose of point individuation and for the gauge-fixing procedure we are proposing.

compact models the Dirac observables of every completely fixed gauge are τ -independent, the first of the gauge fixings (5.5) is inconsistent: it is impossible to realize the *time*-direction in terms of Dirac observables, and the individuation of point-events breaks down. This is compatible with the Wheeler-DeWitt interpretation according to which we can speak only of a local time evolution (in the direction normal to Σ_{τ}) generated by the super-hamiltonian constraint [see for instance Kuchar (1993)]: in other words the local evolution would coincide with a continuous local change of the convention about distant clock synchronization!

VI. CONCLUDING REMARKS - I: THE SPACE-TIME PHYSICAL TEXTURE

The main results we have so far obtained are: i) a peculiar dis-solution of the Hole Argument; ii) a NIF-dependent physical individuation of point-events in terms of the intrinsic degrees of freedom of the gravitational field (the essential *metrical fingerprint* we were looking for); iii) a NIF-dependent temporal evolution of the physical observables.

While the first claim is asserted for every globally-hyperbolic space-time, the other results are valid with reference to the particular spatially non-compact models of GR we have chosen. In this Section we want to summarize the main features of these three issues and devote a final Section to spend a few words about the philosophical implications of our technical results, having in view the traditional background of the substantivalism/relationism debate as well as the recent debate on structural realism. It will be seen that such implications instantiate a peculiar *holistic and structuralist* view of the general-relativistic space-time that we propose to call "point-structuralism".

Concerning the Hole Argument, our analysis of the correspondence between symmetries of the Lagrangian configurational approach and those of the Hamiltonian formulation has shown the following. Solutions of Einstein's equations that, in the configurational approach, differ within the Hole by elements of the subset ${}_{A}Diff'M^4$, which correspond to mappings among gauge-equivalent Cauchy data, belong to the same 4-qeometry, i.e. the same equivalence class generated by applying all passive diffeomorphisms to any of the original 4metrics: ${}^4Geom = {}^4Riem/{}_PDiff M^4 = {}^4Riem/{}_Q'$. In this case, as seen at the Hamiltonian level, they are simply solutions differing by a harmless Hamiltonian gauge transformation on shell and describing, therefore, the same Einstein "universe". Furthermore, it is possible to engender these allegedly different solutions corresponding to the same "universe", by appropriate choices of the initial gauge fixing (the functions $\hat{Z}^{[A]}$). Since we know that the physical role of the gauge-fixings is essentially that of choosing the functional form of the inertial potentials in the NIF defined by the complete gauge (the *epistemic* part of the game), the "differences" among the solutions generated within the Hole by the allowed active diffeomorphisms amount to the different appearances of the intrinsic gravitational phenomena (the ontic part of the game) in different NIFs. In the end this is what, physically, Leibniz equivalence reduces to.

As already anticipated, our analysis contrasts with Stachel's attitude about the Hole Argument. Leaving aside Stachel's broad perspective about the significance and the possibility of generalizations of the Hole story (see Stachel & Iftime, 2005), let us confine ourselves to few comments about the original Stachel's proposal for the physical individuation of points of M^4 by means of a fully covariant exploitation of the Bergmann-Komar invariants $\hat{I}^{[A]}[w_T[g(x), \partial g(x)]], A = 0, 1, 2, 3$. First of all, remember again that the effect of the Hole Argument reveals itself on solutions of Einstein equations and that the active diffeomorphisms that purportedly maintain the physical identity of the points are, therefore,

dynamical symmetries. Now, how are we guaranteed that the functional dependence of the quantities $\hat{I}^{[A]}[w_T[g(x), \partial g(x)]]$ be concretely characterized as relating to actual solutions of Einstein's equations? Since in the actual case we know that these quantities depend upon 4 Dirac observables and 8 gauge viduate a solution, it follows that this arbitrariness unavoidably transfers itself on the individuation procedure and leaves it undefined. Indeed, speaking of general covariance in an abstract way hides the necessity of getting rid of the above arbitrariness by a gauge-fixing that, in turn, necessarily breaks general covariance. In other words a definite individuation entails a concrete characterization of the *epistemic* part of the game, which is precisely what we have done. The result is, in particular, exactly what Stachel's suggestion was intended to, for our *intrinsic gauge* shows that active diffeomorphisms of the first kind (i.e., those belonging to Q' in their passive interpretation) do map individuations of point-events into physically equivalent individuations. Indeed, since the on-shell Hamiltonian gauge transformation connecting two different gauges is the passive counterpart in Q' of an active diffeomorphisms $D_A \in {}_ADiff'M^4$, it determines the drag-along coordinate transformation \mathcal{T}_{D_A} of Section II connecting the radar 4-coordinates of the two gauges, i.e., the dual view of the active diffeomorphism. While the active diffeomorphism carries-along the identity of points by assumption, its passive view attributes different physically-individuated radar-coordinates to the same (mathematical) point. It is seen, therefore, that for any point-event a given individuation by means of Dirac observables is mapped into a physically-equivalent, NIF-dependent individuation³².

As already noted, it's worth stressing again that the main reason why we succeeded in carrying out a concrete realization of Stachel's original suggestion to its natural end lies in the possibility that the Hamiltonian method offers of working off-shell. In fact, the 4-D active diffeomorphisms, qua dynamical symmetries of Einstein's equations, must act on solutions at every stage of the procedure and fail to display the arbitrary epistemic part of the scalar invariants. On the other hand, the Hamiltonian separation of the gauge variables (characterizing the NIF and ruling the generalized inertial effects), from the Dirac observables (characterizing tidal effects) is an off-shell procedure that brings the wanted metrical fingerprint by working independently of the initial value problem. Once again, this mechanism is a typical consequence of the special role played by gauge variables in GR³³.

³² Note that even the description of two intersecting world-lines - realizing a point-coincidence - is NIF-dependent!

We noted already that, according to a main conjecture advanced elsewhere (Lusanna & Pauri, 2004a, 2004b), a canonical basis should exist having an explicit scalar character. An evaluation of the degrees of freedom in connection with the Newman-Penrose formalism for tetrad gravity (Stewart, 1993) tends to corroborate the conjecture. In the Newman-Penrose formalism we can define ten coordinate-independent quantities, namely the ten Weyl scalars. If we add ten further scalars built using the extrinsic curvature, we have a total of twenty scalars from which one should extract a canonical basis replacing the 4-metric and its conjugate momenta. Consequently, it should be possible to find scalar Dirac observables [the Bergmann observables, see Lusanna & Pauri, 2004b] and scalar gauge variables (were the task of finding a canonical basis of scalars too difficult, a minimal requirement would be to characterize a well-defined family of scalars closing an algebra under Poisson Brackets). Then, the individuating functions of (5.3) would depend on scalars only and the distinction between Dirac and gauge observables would become fully invariant. Yet, the gauge-fixing procedure would always break general covariance and one should not forget, furthermore, that the concept of radar-coordinates contains a built-in frame-dependence (see

Concerning the physical individuation of point-events, what we got in Section V is tantamount to claiming that the physical role of the gravitational field without matter is exactly that of individuating physically the points of M^4 as point-events, by means of the four independent phase-space degrees of freedom. As pointed out above, the mathematical structure of the canonical transformation that separates the Dirac observables from the gauge variables is such that the Dirac observables are highly non-local functionals of the metric and the extrinsic curvature over the whole (off-shell) hyper-surface Σ_{τ} . The same is clearly true for the intrinsic pseudo-coordinates [see Eq.(5.3)]. Since the extrinsic curvature has to do with the embedding of the hyper-surface in M^4 , the Dirac observables do involve geometrical elements external to the Cauchy hyper-surface itself. Furthermore, since the temporal gauge (fixed by the scalar $Z^{[0]}$), refers to a continuous interval of hyper-surfaces, the gauge-fixing identity itself is intrinsically four-dimensional.

At this point we could even say that the existence of physical point-events in our models of general relativity appears to be synonymous with the existence of the Dirac observables for the gravitational field. We advance accordingly the ontological claim that - physically - Einstein's vacuum space-time in our models is literally identifiable with the autonomous degrees of freedom of such a structural field, while the specific (NIF-dependent) functional form of the intrinsic pseudo-coordinates associates such coordinates to the points of M^4 . The intrinsic gravitational degrees of freedom are - as it were - fully absorbed in the individuation of point-events. On the other hand, when matter is present, the individuation methodology maintains its validity and shows how matter comes to influence the physical individuation of point-events.

We would like to surmise that the disclosure of the superfluous structure hidden behind the Leibniz equivalence by means of the physical individuation of point-events, renders even more glaring the ontological diversity of the gravitational field with respect to all other fields, even beyond its prominent causal role. It seems substantially difficult to reconcile the nature of the gravitational field with the standard approach of theories based on a background space-time (to wit, string theory and perturbative quantum gravity in general). Any attempt at linearizing such theories unavoidably leads to looking at gravity from the perspective of a spin-2 theory in which the graviton stands at the same ontological level as other quanta. In the standard approach of background-dependent theories of gravity, photons, gluons and gravitons all live on the stage on an equal footing. From the point of view set forth in this paper, however, non-linear gravitons are at the same time both the stage and the actors within the causal play of photons, gluons, and other material characters such as electrons and quarks.

Finally, let us remark that the class of spatially non-compact models treated in this paper, even if not yet able to describe cosmology ³⁴, has been priviliged by taking into primary consideration the fundamental issue of how to incorporate particle physics in GR, at the classical level to start with. Classical string theories and super-gravity theories include particles, but their quantization requires the introduction of a background space-time to define

Section IV). Finally, the energy density $\mathcal{E}_{ADM}(\tau, \vec{\sigma})$ would remain a NIF-dependent quantity anyway.

³⁴ Let us stress in this connection that in spatially compact cosmologies the use of particle physics (essentially defined in non-compact space-times) for the description of the, e.g., nucleosynthesis implies an *huge extrapolation*. Basically, it is well-known that at the level of quantum field theory in background curved space-times, a useful particle interpretation of states does not, in general, even exist.

the particle Fock space. On the other hand the only well developed form of background-independent quantum gravity (loop quantum gravity), obtained by quantizing either the connection or the loop representation of GR, leads to a quantum formulation inequivalent to Fock space, so that till now it is not known how to incorporate particle physics. We hope that our viewpoint, taking into account the non-inertial aspects of GR, can be developed to the extent to be able to reopen the program of canonical quantization of gravity in a background independent way by quantizing the Dirac observables only³⁵. Note finally that the individuating relation (5.5) is a numerical identity that has a built-in non-commutative structure, deriving from the Dirac-Poisson structure hidden in its right-hand side. The individuation procedure transfers, as it were, the non-commutative Poisson-Dirac structure of the Dirac observables onto the individuated point-events, even though the coordinates on the l.h.s. of the identity are c-number quantities. One could guess that such a feature might deserve some attention in view of quantization, for instance by maintaining that the identity, interpreted as a relation connecting mean values, could still play some role at the quantum level.

A further interesting suggestion comes from the following passage of Bergmann and Komar:

[...] in general relativity the identity of a world point is not preserved under the theory's widest invariance group. This assertion forms the basis for the conjecture that some physical theory of the future may teach us how to dispense with world points as the ultimate constituents of space-time altogether. (Bergmann & Komar, 1972, 27)

Indeed, would it be possible to build a fundamental theory that is grounded in the reduced phase space $\tilde{\Omega}_4$ of the abstract gauge-invariant Dirac observables? This would be an abstract and highly non-local theory of classical gravitation that, transparency aside, would be stripped of all the epistemic machinery (the gauge freedom) which is indispensable for an empirical access to the theory. For any given Einstein's "universe" with its topology, the abstract Dirac fields in $\tilde{\Omega}_4$ are locally functions of the points x of an abstract mathematical manifold \tilde{M}_4 that is the equivalence class of all our concrete realizations of space-time, each one equipped with its gauge-dependent individuation of points, NIF and inertial forces. Such fields must be called intrinsic to the extent that they are no longer NIF-dependent, and synthesize, as it were, the essential properties of all the appearances shown by the gauges. Admittedly, the global existence of $\tilde{\Omega}_4$ over \tilde{M}_4 is subjected to a huge set of mathematical hypotheses which we will not take into account here. Locally, however, the Dirac fields certainly exist and we could introduce a coordinate system defined by their values as intrinsic individuating system for the given "universe".

Let us stress that, once Einstein's equations have been solved, the metric tensor and all of its derived quantities, in particular the light-cone structure, can be re-expressed in terms of *Dirac observables* in a gauge-fixed functional form. Yet, if we look at the reduction procedure the other way around, we could imagine starting with a given choice of initial values for the *Dirac observables* (i.e., the germ of a "universe"), and adding all the required gauge variables as suitable independent variables, so as to obtain at the end a space-time expression for the local field $g_{\mu\nu}(x)$. Since the relation between all tensor expressions and Dirac

³⁵ See Alba & Lusanna, 2005b, for a preliminary attempt to define relativistic and non-relativistic quantum mechanics in non-inertial frames in Galilei and Minkowski space-times, respectively.

observables depends on the gauge, the gauge freedom would represent also the flexibility of the final local description of the deep non-local structure of the theory, a local description that supports the empirical access to the theory. In other words the gauge structure could be seen as playing a crucial role in the re-construction of the concrete spatiotemporal continuum representation from a non-local structure. We see, therefore, that even in the context of classical gravitational theory, the spatiotemporal continuum plays the role of an epistemic precondition of our sensible experience of macroscopic objects, playing a role which is not too dissimilar from that enacted by Minkowski micro-space-time in the local relativistic quantum field theory (see Pauri, 2000). We believe that, from the philosophical point of view, one could recognize much more substance here than what could appear prima facie a simple instantiation of the relationship between canonical structure and locality that pervades contemporary theoretical physics.

Finally, can this basic freedom in the choice of the local realizations be equated with a "taking away from space and time the last remnant of physical objectivity," as Einstein suggested? We believe that, discounting Einstein's "spatial obsession" with realism as locality (and separability), a significant kind of spatio-temporal objectivity survives. It is true that the functional form of the Dirac observables is NIF-dependent; yet, even leaving aside the role of the abstract phase space $\tilde{\Omega}_4$, there is no a-priori physical individuation of the manifold points independently of the metric field, so we cannot say that the individuation procedures corresponding to different gauges individuate different point-events (see footnote 32). A really different physical individuation should only be attributed to different initial conditions for the Dirac observables, (i.e., to a different "universe"). We can, therefore, say that general covariance represents the horizon of a priori possibilities for the physical appearance of space-time, possibilities that must be actualized within any given solution of the dynamical equations in terms of NIFs.

VII. CONCLUDING REMARKS - II: AN INSTANTIATION OF STRUCTURAL REALISM AS "POINT-STRUCTURALISM

We conclude spending a few words about the implications of our results for some issues surrounding the recent debate on scientific structural realism, as well as for the traditional debate on the absolutist/relationist dichotomy.

As well-known, the term *scientific realism* has been interpreted in a number of different ways within the literature on philosophy of science, in connection with the progressive sophistication of our understanding of scientific knowledge. Such ways concern, e.g., realism about *observable or unobservable entities*, and realism about *theories*. A further ramifications of meanings has been introduced more recently by the so called *structural realism* (the only attainable reality are relations between (unobservable) objects), which originated a division between the so-called *epistemic* structural realists (entity realism is unwarranted) and the *ontic* structural realists (the relations exhaust what exists), (see Simon, 2003).

From the logical point of view, we can assume that the concept of *structure* refers to a (stable or not) set of *relations* among a set of some kind of *constituents* that are put in relations (the *relata*). The specification expressed by the notion of *structural realism* introduces *some kind* of ontological distinction between the role of the relations and that of the constituents. At least two main exemplary possibilities present themselves as obvious: (i)

there are relations, in which the constituents are (ontologically) primary and the relation secondary; (ii) there are relations, in which the relation is (ontologically) primary while the constituents are secondary, and this even without any prejudice about the ultimate ontological consistency of the constituents. In the case of *physical* entities, one could cautiously recover in this connection the traditional distinction between *essential* and *non-essential* properties (*accidents*) in order to characterize the *degree of (ontological) primacy* of the relations versus the *relata* and vice versa (and this independently of any metaphysical flavor possibly connected to the above distinctions). For example, one could say that in the extreme case (i) only *accidental* properties of the constituents can depend upon the relational structure, while in the extreme case (ii) at least one *essential* property of the constituents depends upon the relational structure (saying that *all* the essential properties of the relata depend upon the relation would be tantamount to claiming that there exist only relations without constituents, as the *ontic structural realist* has it).

A further complication is connected to the nature of the structure we are considering. For while at the logical level (leaving aside the deep philosophical issue concerning the relationships between mathematical structures and substances) the concept of mathematical structure (e.g. a system of differential equations, or even the bare mathematical manifold of point which provides the first layer of our representations of the real space-time) can be taken to be sufficiently clear for our purposes, the definition of physical structure raises existential philosophical problems immediately. For example, we believe that it is very difficult to define a physical structure without bringing in its constituents, and thereby granting them some kind of existence and defending some sort of (entity realism). Analogously, we believe that it is very difficult to defend structural realism without also endorsing a theory realism of some sort. However, both theses are not universally shared.

Having said this, let us come back to the results we have obtained in the previous Sections. The analysis based on our *intrinsic gauge* has disclosed a remarkable and rich local structure of the general-relativistic space-time for the considered models of GR. In correspondence to every *intrinsic gauge* (5.5) we found a gauge-related *physical individuation of point-events* in terms of the Dirac observables of that gauge, i.e., in terms of the *ontic* part of the gravitational field, as represented in the clothes furnished by the gauge (by the NIF). Moreover, since the gauge-fixing identity (5.5) is four-dimensional we have an instantiation of *metrical holism* which, though local in the temporal dimension and characterized by a *dynamical stratification in 3-hyper surfaces*, is *four-dimensional*.

At this point we can assert that we have a kind of ontology in which the identity of point-events is conferred upon them by a complex relational structure in which they are holistically enmeshed. This relational structure includes all the elements of the complete gauge fixing Γ_8 , summarized by a NIF, and supported by a definite solution of Einstein's equations throughout M_4 , corresponding to given initial values for the Dirac observables in that gauge (a definite Einstein "universe"). The identity of point-events, at this level, should be properly termed as gauge-objective.

It seems, therefore, that we have disclosed a *holistic structure* which is clearly *ontologically prior* to its *constituents*, as to their physical identity, even if we cannot agree with Cao's assertion (see Cao, 2003, p.111) that the constituents, as mere *place-holders*, derive their meaning or *even their existence* from their function and place in the structure. Indeed, at any level of consideration of GR, the practical level above all, one cannot avoid

quantifying over points, and we have just attributed a physical meaning³⁶ to our radarcoordinate indexing of such points which makes point-events as ontologically equivalent to the existence of the gravitational field as an extended entity. Quite in general, we cannot see how a place-holder can have any ontological function in an evolving network of relationships without possessing at least some kind of properties. Even more, let us recover our previous claim that - physically - Einstein's vacuum space-time in our models is literally identifiable with the autonomous degrees of freedom of the gravitational field in vacuum, and moreover that the intrinsic gravitational degrees of freedom are - as it were - fully absorbed in the individuation of point-events. Both conclusions do in fact confer a sort of causal power to the gravitationally-dressed points. Then, we can ask whether all has already been said concerning the identity of points or whether instead some kind of intrinsic individuality survives beneath the variety of descriptions displayed by all the gauge-related NIFs, and common to all these appearances. For indeed this kind of intrinsic identity is just furnished by the abstract Dirac fields residing within the phase-space Ω_4 , which is nothing else that a quotient with respect to all of the concrete realizations and appearances of the NIFs. Accepting this view, we are led to a peculiar space-time structure in which the relation/relata correspondence does not fit with any of the extreme cases listed above, for one could assert that while the abstract essential properties belong to the constituents as seen in Ω_4 (so that abstract point-events in M_4 would be like - as it were - to natural kinds), the totality of the physically concrete accidents are displayed by means of the holistic relational structure. This is the reason why we are proposing to call this peculiar kind of space-time structuralism as point-structuralism.

Summarizing, this view holds that space-time point-events (the relata) do exist as individuals and we continue to quantify over them; however, their properties can be viewed both as extrinsic and relational, being conferred on them in a holistic way by the whole structure of the metric field and the extrinsic curvature on a simultaneity hyper-surface, and, at the same time as intrinsic, being coincident with the autonomous degrees of freedom of the gravitational field represented by the abstract NIF-independent Dirac fields in $\tilde{\Omega}_4$. In this way both the metric field and the point-events maintain their own manner of existence, so that the structural texture of space-time in our models does not force us to abandon an entity realist stance about both the metric field and its points. We must, therefore, refute the thesis according to which metrical relations can exist without their constituents (the point-events).

Concerning the traditional debate on the dichotomy substantivalism/relationism, we believe that our analysis - as a case study limited to the class of space-times dealt with - may offer a tertium-quid solution to the debate by overcoming it. First of all, let us recall that, in remarkable diversity with respect to the traditional historical presentation of Newton's absolutism vis á vis Leibniz's relationism, Newton had a much deeper understanding of the nature of space and time. In two well-known passages of De Gravitatione, Newton expounds what could be defined an original proto-structuralist view of space-time (see also Torretti (1987), and DiSalle (1994)). He writes (our emphasis):

Perhaps now it is maybe expected that I should define extension as substance or accident or else nothing at all. But by no means, for it has its own manner of existence which fits neither

³⁶ Even operationally, in principle (see Pauri & Vallisneri, 2002; Lusanna & Pauri, 2004a, 2004b, already quoted in Section V).

substance nor accidents [...] The parts of space derive their character from their positions, so that if any two could change their positions, they would change their character at the same time and each would be converted numerically into the other qua individuals. The parts of duration and space are only understood to be the same as they really are because of their mutual order and positions (propter solum ordinem et positiones inter se); nor do they have any other principle of individuation besides this order and position which consequently cannot be altered. (Hall & Hall, 1962, p.99, p.103.)

:

On the other hand, in his relationist arguments, Leibniz could exploit the principle of sufficient reason because Newtonian space was *uniform*, as the following passage lucidly explains (our *emphasis*):

Space being *uniform*, there can be neither any *external* nor *internal reason*, by which to distinguish its parts, and to make any choice between them. For, any external reason to discern between them, can only be grounded upon some internal one. Otherwise we should discern what is indiscernible, or choose without discerning. (Alexander (1956), p.39).

Clearly, if the parts of space were real, the Principle of Sufficient Reason would be violated. Therefore, for Leibniz, space is not real. The upshot, however, is that space (space-time) in general relativity far from being *uniform* may possess, as we have seen, a *rich structure*. This is just the reason why - in our sense - it is *real*, and why Leibniz equivalence called upon for general relativity happens to hide the very nature of space-time, instead of disclosing it.

In conclusion, what emerges from our analysis is a kind of new structuralist conception of space-time. Such new structuralism is not only richer than that of Newton, as it could be expected because of the dynamical structure of Einstein space-time, but richer in an even deeper sense. For this new structuralist conception turns out to include elements common to the tradition of both substantivalism (space, and space points, have an autonomous existence independently of other bodies or matter fields) and relationism (the physical meaning of space depends upon the relations between bodies or, in modern language, the specific reality of space depends (also) upon the (matter) fields it contains).

We have seen that the points of general-relativistic space-times, quite unlike the points of the homogeneous Newtonian space, are endowed with a remarkably rich non-point-like and holistic structure furnished by the metric field and its derivatives. Therefore, the independent degrees of freedom of the metric field are able to characterize the "mutual order and positions" of points dynamically, since - as it were - each point-event "is" the "values" of the intrinsic degrees of freedom of the gravitational field. This capacity is even stronger, since such mutual order is altered by the presence of matter. On the other hand, even though the metric field does not embody the traditional notion of substance, it exists and plays a role for the individuation of point-events by means of its *structure*. On the other hand, although one can maintain also the view that the physical properties are conferred to the point-events in a peculiar relational form, our point-structuralism does not support even the standard relationist view. In fact, the holistic relationism we defend does not reduce the whole of spatiotemporal relations to physical relations (i.e. it is not eliminativist), nor it entails that space-time does not exist as such, being reducible to physical relations. It supports a thesis about the nature of identity of point-events we continue to quantify over. These are individuals in a peculiar sense: they exist as autonomous constituents, but one cannot claim that their properties do not depend on the properties of others. Not only relations do exist, but also the carriers of them, even if they do bring intrinsic properties in a very special sense.

We acknowledge that the validity of at least three of our results is restricted to the class of models of GR we worked with. Yet, we were interested in exemplifying a question of principle, so that we can claim that there is a class of models of GR embodying both a real notion of NIF-dependent temporal change, a NIF-dependent physical individuation of points and a new structuralist and holistic view of space-time.

VIII. ACKNOWLEDGMENTS

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