Decoherence during Inflation: the generation of classical inhomogeneities

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We show how the quantum to classical transition of the cosmological fluctuations produced during inflation can be described by means of the influence functional and the master equation. We split the inflaton field into the system-field (long-wavelength modes), and the environment, represented by its own short-wavelength modes. We compute the decoherence times for the system-field modes and compare them with the other time scales of the model. We present the renormalized stochastic Langevin equation for an homogeneous system-field and then we analyze the influence of the environment on the power spectrum for some modes in the system.

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I. INTRODUCTION

The emergence of classical physics from quantum behaviour is important for several physical phenomena in the early Universe. This is beyond the fundamental requirement that only after the Planck time can the metric of the Universe be assumed to be classical. For example, the inflationary era is assumed to have been induced by scalar inflaton fields, with simple potentials [\[1](#page-16-0), [2\]](#page-16-1). Such fields are typically assumed to have classical behaviour, although in principle a full quantum description should be used. In fact, the origin of large scale structure in the Universe can be traced back to quantum fluctuations that, after crossing the Hubble radius, were frozen and became classical, stochastic, inhomogeneities [\[3\]](#page-16-2).

It is generally assumed that several phase transitions have occurred during the expansion of the Universe [\[4\]](#page-16-3). As in the case for the inflaton fields, the (scalar) order parameter fields that describe these transitions are described classically. However, the description of early universe phase transitions from first principles is intrinsically quantum mechanical [\[5](#page-16-4)]. As a specific application [\[6\]](#page-16-5) of the previous point, the very notion of topological defects (e.g. strings and monopoles) that characterize the domain structure after a finite-time transition, and whose presence has consequences for the early universe, is based on this assumption of classical behaviour for the order parameter [\[7](#page-16-6)], as it distributes itself between the several degenerate ground states of the ordered system.

In previous publications, one of us has analyzed the emergence of a classical order parameter during a second order phase transition and the role of decoherence in the process of topological defect formation [\[8](#page-16-7), [9](#page-16-8), [10](#page-16-9), [11](#page-16-10)].

In the present paper our concern is directly related with the first point above, the quantum to classical transition of the inflaton. Any approach must take into account both the quantum nature of the scalar field and the

non-equilibrium aspects of the process [\[12](#page-16-11)]. The problem of the quantum to classical transition in the context of inflationary models was first addressed by Guth and Pi [\[13\]](#page-16-12). In that work, the authors used an inverted harmonic oscillator as a toy model to describe the early time evolution of the inflaton, starting from a Gaussian quantum state centered on the maximum of the potential. They subsequently showed that, according to Schrödinger's equation, the initial wave packet maintains its Gaussian shape (due to the linearity of the model). Since the wave function is Gaussian, the Wigner function is positive for all times. Moreover, it peaks on the classical trajectories in phase space as the wave function spreads. The Wigner function can then be interpreted as a classical probability distribution for coordinates and momenta, showing sharp classical correlations at long times. In other words, the initial Gaussian state becomes highly squeezed and indistinguishable from a classical stochastic process. In this sense, one recovers a classical evolution of the inflaton rolling down the hill.

A similar approach has been used by many authors to describe the appearance of classical inhomogeneities from quantum fluctuations in the inflationary era [\[14](#page-16-13), [15\]](#page-16-14). Indeed, a massless free field ϕ in an expanding universe can be written as $\phi = a^{-1}\psi$ where a is the scale factor and the Fourier modes of the field ψ satisfy the linear equation

$$
\psi_k'' + (k^2 - \frac{a''}{a})\psi_k = 0.
$$
 (1)

For sufficiently long-wavelengths $(k^2 \ll a''/a)$, this equation describes an unstable oscillator. If one considers an initial Gaussian wave function, it will remain Gaussian for all times, and it will spread with time. As with the toy model of Guth and Pi, one can show that classical correlations do appear, and that the Wigner function can again be interpreted as a classical probability distribution in phase space. (It is interesting to note that a similar mechanism can be invoked to explain the origin of a classical, cosmological magnetic field from amplification of quantum fluctuations).

However, classical correlations are only one aspect of

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classical behaviour. It was subsequently recognized that, in order to have a complete classical limit, the role of the environment is crucial, since its interaction with the system distinguishes the field basis as the pointer basis [\[16\]](#page-16-15). [We are reminded that, even for the fundamental problem of the space-time metric becoming classical, simple arguments based on minisuperspace models suggest that the classical treatment is only correct because of the interaction of the metric with other quantum degrees of freedom [\[17\]](#page-16-16).]

While these linear instabilities cited above characterize free fields, the approach fails when interactions are taken into account. Indeed, as shown again in simple quantum mechanical models (e.g. the anharmonic inverted oscillator), an initially Gaussian wave function becomes non-Gaussian when evolved numerically with the Schrödinger equation. The Wigner function now develops negative parts, and its interpretation as a classical probability breaks down [\[10\]](#page-16-9). One can always force the Gaussianity of the wave function by using a Gaussian variational wave function as an approximate solution of the Schrödinger equation, but this approximation deviates significantly from the exact solution as the wave function probes the non-linearities of the potential [\[10](#page-16-9), [18](#page-16-17)].

When interactions are taken into account, classical behaviour is recovered only for "open systems", in which the observable degrees of freedom interact with their environment. When this interaction produces both a diagonalization of the reduced density matrix and a positive Wigner function, the quantum to classical transition is completed [\[19](#page-16-18)].

In Ref. [\[10](#page-16-9)] has been studied an anharmonic inverted oscillator coupled to a high temperature environment. Under some considerations, it was shown that the system becomes classical very quickly, even before the wave function probes the non-linearities of the potential. Being an early time event, the quantum to classical transition can now be studied perturbatively. In general, recoherence effects are not expected [\[20](#page-16-19)]. Taking these facts into account, we have extended the approach to field theory models [\[8](#page-16-7), [9](#page-16-8)]. In field theory, one is usually interested in the long-wavelengths of the order parameter. Even the early universe is replete with fields of all sorts which comprise a rich environment, in the inflationary example, we considered a model in which the system-field interacts with the environment-field, including only its own shortwavelengths. This is enough during inflation. Assuming weak self-coupling constant (a flat inflaton potential) we have shown that decoherence is an event shorter than the time t_{end} , which is a typical time-scale for the duration of inflation. As a result, perturbative calculations are justified [\[9](#page-16-8)]. Subsequent dynamics can be described by a stochastic Langevin equation, the details of which are only known for early times [\[21\]](#page-16-20).

In our approach, the quantum to classical transition is defined by the diagonalization of the reduced density matrix. In phase transitions the separation between long and short-wavelengths is determined by their stability, 2

which depends on the parameters of the potential. During Inflation, this separation is set by the existence of the Hubble radius. Modes cross the apparent horizon during their evolution, and they are usually treated as classical. The main motivation of this paper is to present a formal way to understand this statement within the open quantum system approach. In the last sense, decoherence is the critical ingredient if we are to dynamically demonstrate the quantum-to-classical transition of the open system.

The splitting between short and long-wavelength modes may be done using a time-dependent or timeindependent comoving cut-off as well as a smoother window function [\[21,](#page-16-20) [22](#page-16-21)]. Since any time-dependent splitting produce an effective and arbitrary (split-dependent) interaction between the system and environment degrees of freedom, which can only be discarded by making some additional assumptions, we consider convenient to use a time-independent comoving cut-off, as it has been done in Refs. [\[23](#page-16-22), [24\]](#page-16-23)

The paper is organized as follows. In Section II we introduce our model. This is a theory containing a real system-field, massless and minimally coupled to a fixed de Sitter background. We compute the influence functional by integrating out the environmental sector of the field, composed by the short-wavelength modes. Section III is dedicated to reviewing the evaluation of the master equation and the diffusion coefficients which are relevant in order to study decoherence. In Section IV we analyze the diffusion coefficients and evaluate upper bounds on the decoherence times. As we will see, decoherence takes place before the end of the inflationary period. Section V is concerned with the effective stochastic evolution of the system. We present the renormalized stochastic Langevin equation for an homogeneous system-field and then we analyze the influence of the environment on the power spectrum for some modes in the system. Section VI contains our final remarks. Two short appendices fill in some of the detail. Throughout the paper we use units such that $h = c = 1$.

II. THE INFLUENCE FUNCTIONAL AND THE DENSITY MATRIX

Let us consider a massless quantum scalar field, minimally coupled to a de Sitter spacetime $ds^2 = a(\eta)[d\eta^2$ $d\vec{x}^{2}$ (where η is the conformal time, $d\eta = dt/a(t)$ with t the cosmic time), with a quartic self-interaction. The classical action is given by

$$
S[\phi] = \int d^4x \ a^4(\eta) \ \left[\frac{\phi'^2}{2a^2(\eta)} - \frac{\nabla \phi^2}{2a^2(\eta)} - \lambda \phi^4 \right], \quad (2)
$$

where $a(\eta) = -1/(H\eta)$ and $\phi' = d\phi/d\eta$ $(a(\eta_i) = 1$ [$\eta_i =$ $-H^{-1}$, and H^{-1} is the Hubble radius). Let us make a system-environment field splitting [\[24](#page-16-23)]

$$
\phi = \phi_{<} + \phi_{>} \tag{3}
$$

where the system-field ϕ_{\leq} contains the modes with wave vectors shorter than a critical value $\Lambda \equiv 2\pi/\lambda_c$, while the environment-field ϕ , contains wave vectors longer than Λ. As we set $a(\eta_i) = 1$, a physical length $\lambda_{\text{phys}} = a(\eta)\lambda$ coincides with the corresponding comoving length λ at the initial time. Therefore, the splitting between system and environment gives a system sector constituted by all the modes with physical wavelengths shorter than the critical length λ_c at the initial time η_i .

After splitting, the total action [\(2\)](#page-1-0) can be written as

$$
S[\phi] = S_0^<[\phi_<] + S_0^>[\phi_>] + S_{\rm int}[\phi_<, \phi_>], \qquad (4)
$$

where S_0 denotes the free field action and the interaction terms are given by

$$
S_{\rm int}[\phi_<, \phi_>] = -\lambda \int d^4x \ a^4(\eta) \ \{ \phi_<^4(x) + \phi_>^4(x) + 6\phi_<^2(x)\phi_<^3(x) + 4\phi_<^3(x)\phi_<^3(x) + 4\phi_<^3(x)\phi_<^3(x) \}
$$
(5)

The total density matrix elements (for the system and environment fields) are defined as

$$
\rho[\phi^+_<, \phi^+_- | \phi^-_<, \phi^-_<; \eta] = \langle \phi^+_< \phi^+_- | \hat{\rho}[\eta] | \phi^-_< \phi^-_> \rangle, \qquad (6)
$$

where $|\phi_{\leq}^{\pm}\rangle$ and $|\phi_{\geq}^{\pm}\rangle$ are the eigenstates of the field operators ϕ_{\leq} and ϕ_{\geq} , respectively. For simplicity, we will assume that the interaction is turned on at the initial time η_i and that, at this time, the system and the environment are not correlated (we ignore, for the moment, the physical consequences of such a choice, it has been discussed in [\[9\]](#page-16-8)). Therefore, the total density operator can be written as the product of the density operator for the system and for the environment

$$
\hat{\rho}[\eta_i] = \hat{\rho}_>[\eta_i]\hat{\rho}_<[\eta_i].\tag{7}
$$

We will further assume that the initial state of the environment is the Bunch-Davies vacuum [\[25\]](#page-16-24).

We are interested in the influence of the environment on the evolution of the system. Therefore the reduced density matrix is the object of relevance. It is defined by

$$
\rho_{\rm r}[\phi_{<}^+|\phi_{<}^-;\eta] = \int \mathcal{D}\phi_{>}\rho[\phi_{<}^+,\phi_{>}|\phi_{<}^-,\phi_{>};\eta].\tag{8}
$$

The reduced density matrix evolves in time by means of

$$
\rho_{\rm r}[\phi_{
$$
\times J_{\rm r}[\phi_{
$$
$$

where $J_{\rm r}$ is the reduced evolution operator

$$
J_{\rm r}[\eta|\eta_i] = \int_{\phi_{
$$

The influence functional (or Feynman-Vernon functional) $F[\phi^+_{<}, \phi^-_{<}]$ is defined as

$$
F[\phi^+_{<}, \phi^-_{<}] = \int d\phi^+_{> i} \int d\phi^-_{> i} \rho^{\vphantom{*}}_{> i} \phi^-_{> i} \eta^{\vphantom{*}}_{i} \bigg] \int d\phi^{\vphantom{*}}_{> f}
$$

$$
\times \int_{\phi^+_{> i}}^{\phi^{\vphantom{*}} \phi^+_{> f}} \mathcal{D}\phi^+_{> i} \mathcal{D}\phi^-_{> e^{i\phi^-_{< f}} \{i(S[\phi^+_{>}] + S_{\rm int}[\phi^+_{<}, \phi^+_{>}])\}
$$

$$
\times \exp\left\{-i(S[\phi^-_{>}] + S_{\rm int}[\phi^-_{<}, \phi^-_{>}])\right\}. \tag{11}
$$

This functional takes into account the effect of the environment on the system. The influence functional describes the averaged effect of the environmental degrees of freedom on the system degrees of freedom to which they are coupled. With this functional, one can identify a noise and dissipation kernel related by some kind of fluctuation-dissipation relation. This relation is important when one is interested in possible stationary states where a balance is eventually reached. During inflation we have the field (inflaton) on a very flat potential, away from its minimum, and we are, in general, only interested in the dynamics over some relatively small time. For example, we would neglect dissipation during the slow-roll period; but it is not correct during the eventual reheating phase.

We define the influence action $\delta A[\phi^+_{\leq}, \phi^-_{\leq}]$ and the coarse grained effective action (CGEA) $\widehat{A[\phi^+_<,\phi^-_-]}$ as

$$
F[\phi^+_<, \phi^-_<] = \exp\{i\delta A[\phi^+_<, \phi^-_<]\},\tag{12}
$$

$$
A[\phi^+_<, \phi^-_<] = S[\phi^+_<] - S[\phi^-_<] + \delta A[\phi^+_<, \phi^-_<]. \tag{13}
$$

We will calculate the influence action perturbatively in λ and we will consider only terms up to order λ^2 and one loop in the \hbar expansion. The influence action has the following form [\[24\]](#page-16-23):

$$
\delta A[\phi_{\leq}^{+}, \phi_{\leq}] = \langle S_{\rm int}[\phi_{\geq}^{+}, \phi_{\leq}^{+}] \rangle_{0} - \langle S_{\rm int}[\phi_{\geq}^{-}, \phi_{\leq}^{-}] \rangle_{0} \n- i \langle S_{\rm int}[\phi_{\geq}^{+}, \phi_{\leq}^{+}] S_{\rm int}[\phi_{\geq}^{-}, \phi_{\leq}^{-}] \rangle_{0} \n+ i \langle S_{\rm int}[\phi_{\geq}^{+}, \phi_{\leq}^{+}] \rangle_{0} \langle S_{\rm int}[\phi_{\geq}^{-}, \phi_{\leq}^{-}] \rangle_{0} \n+ \frac{i}{2} \left\{ \langle S_{\rm int}[\phi_{\geq}^{-}, \phi_{\leq}^{-}] \rangle_{0}^{2} - \langle S_{\rm int}[\phi_{\geq}^{-}, \phi_{\leq}^{-}]^{2} \rangle_{0} \right\} \n+ \frac{i}{2} \left\{ \langle S_{\rm int}[\phi_{\geq}^{-}, \phi_{\leq}^{-}] \rangle_{0}^{2} - \langle S_{\rm int}[\phi_{\geq}^{-}, \phi_{\leq}^{-}]^{2} \rangle_{0} \right\}
$$

where $\langle \ \rangle_0$ is the quantum average with respect to the free field action of the environment, defined as

$$
\langle B[\phi^+_{>, \phi^-_{>}]\rangle_0 = \int d\phi^+_{> i} \int d\phi^-_{> i} \int d\phi_{> f} \tag{15}
$$
\n
$$
\times \int_{\phi^+_{> i}}^{\phi > f} D\phi^+_{> f} D\phi^-_{> i} D\phi^-_{> B[\phi^+_{>, \phi^-_{>}]}
$$
\n
$$
\times \exp\left\{i \left(S^{\geq}[\phi^+_{>-}] - S^{\geq}[\phi^-_{>}] \right)\right\} \langle \phi^+_{> i} | \hat{\rho}_\geq [\eta_i] | \phi^-_{> i} \rangle.
$$

Here $\hat{\rho}_>$ is the Bunch-Davies vacuum state assumed for the environment.

The influence functional can be computed, and the result is

$$
Re\delta A = -\lambda \int d^4x_1 \ a^4(\eta) \{2\Delta_4(x_1) - 12\Delta_2(x_1)iG^{\Lambda}_{++}(x_1, x_1) \} + \lambda^2 \int d^4x_1 \int d^4x_2 \ a^4(\eta_1) \ a^4(\eta_2) \ \Theta(\eta_1 - \eta_2) \times \{64\Delta_3(x_1)ReG^{\Lambda}_{++}(x_1, x_2)\Sigma_3(x_2) + 288\Delta_2(x_1)ImG^{\Lambda 2}_{++}(x_1, x_2)\Sigma_2(x_2) \}, \qquad (16)
$$

Im
$$
\delta A = \lambda^2 \int d^4 x_1 \int d^4 x_2 a^4 (\eta_1) a^4 (\eta_2)
$$

\n $\times \{32\Delta_3(x_1) \text{Im} G^{\Lambda}_{++}(x_1, x_2) \Delta_3(x_2)$
\n $- 144\Delta_2(x_1) \text{Re} G^{\Lambda 2}_{++}(x_1, x_2) \Delta_2(x_2) \},$ (17)

where x_j denotes (η_j, \vec{x}_j) , $\Theta(x)$ is the Heaviside step function, and the integrations in time run from η_i to η . $G_{++}^{\Lambda}(x_1, x_2) \equiv i \langle \phi_>}^+(x_1) \phi_>}^+(x_2) \rangle_0$ is the relevant shortwavelength closed time-path correlator (it is proportional to the Feynmann propagator of the environment field, where the integration over momenta is restricted by the presence of the infrared cut-off Λ), and we have defined

$$
\Delta_n = \frac{1}{2} (\phi_{\leq}^{+n} - \phi_{\leq}^{-n}) \quad , \quad \Sigma_n = \frac{1}{2} (\phi_{\leq}^{+n} + \phi_{\leq}^{-n}) , \quad (18)
$$

with $n = 1, 2, 3$.

III. MASTER EQUATION AND DIFFUSION **COEFFICIENTS**

In this Section we obtain the evolution equation for the reduced density matrix (master equation), paying particular attention to the diffusion terms, which are responsible for decoherence. To do so, we closely follow the quantum Brownian motion (QBM) example [\[26,](#page-16-25) [27\]](#page-16-26), translated into quantum field theory [\[8](#page-16-7), [24](#page-16-23)].

The first step in the evaluation of the master equation is the calculation of the density matrix propagator $J_{\rm r}$ from Eq.[\(10\)](#page-2-0). In order to solve the functional integration which defines the reduced propagator, we perform a saddle point approximation, assuming the classical field configuration dominates functional integrals,

$$
J_{\rm r}[\phi_{
$$

where ϕ_{cl}^{\pm} is the solution of the semiclassical equation of motion $\delta Re A/\delta \phi^+_{\leq |\phi^+_{\leq }\Rightarrow \phi^-_{\leq}}=0$ with boundary conditions $\phi_{\leq c}^{\pm}(\eta_i) = \phi_{\leq i}^{\pm}$ and $\phi_{\leq c}^{\pm}(\eta) = \phi_{\leq f}^{\pm}$. Since we are working up to λ^2 order, we can evaluate the influence functional using the solutions of the free field equations. This classical equation is $\phi''_< + 2\mathcal{H}\phi'_< -\nabla^2\phi_< = 0, (\mathcal{H} = a'(\eta)/a(\eta)).$ A Fourier mode $\psi_{\vec{k}}$ of the field $\psi \equiv a(\eta)\phi_{\leq}$, satisfies

$$
\psi_{\vec{k}}'' + \left(k^2 - \frac{2}{\eta^2}\right)\psi_{\vec{k}} = 0, \tag{20}
$$

where we have used the fact that $a''/a = 2/\eta^2$. It is important to note that for long-wavelength modes, $k \ll 2/\eta^2$, Eq. [\(20\)](#page-3-0) describes an unstable (upside-down) harmonic oscillator [\[13](#page-16-12)].

The classical solution for a mode in the system can be written as

$$
\phi_{\vec{k}}^{\pm cl}(\eta') = \phi_{\leq i}^{\pm}(\vec{k})u_1(\eta', \eta) + \phi_{\leq f}^{\pm}(\vec{k})u_2(\eta', \eta), \qquad (21)
$$

where

$$
u_1 = \frac{\sin[k(\eta - \eta')](\frac{1}{k} + k\eta\eta') + \cos[k(\eta - \eta')](\eta' - \eta)}{\sin[k(\eta' - \eta_{i})](\frac{1}{k} + k\eta_{i}\eta') + \cos[k(\eta' - \eta_{i})](\eta_{i} - \eta')},
$$

$$
u_2 = \frac{\sin[k(\eta_{i} - \eta')](\frac{1}{k} + k\eta'\eta_{i}) + \cos[k(\eta' - \eta_{i})](\eta' - \eta_{i})}{\sin[k(\eta_{i} - \eta)](\frac{1}{k} + k\eta\eta_{i}) + \cos[k(\eta - \eta_{i})](\eta - \eta_{i})}.
$$

We will assume that the system-field contains only one Fourier mode with $\vec{k} = \vec{k}_0$. This is a sort of "minisuperspace" approximation for the system-field that will greatly simplify the calculations, therefore we assume

$$
\phi_{\text{ll}}^{\pm}(\vec{x}, \eta') = \phi_{\vec{k}_0}^{\pm \text{cl}}(\eta') \cos(\vec{k}_0 \cdot \vec{x}), \tag{23}
$$

where $\phi_{\vec{k}_0}^{\pm \text{cl}}$ is given by [\(21\)](#page-3-1).

In order to obtain the master equation we must compute the final time derivative of the propagator J_r . After that, all the dependence on the initial field configurations $\phi_{\leq i}^{\pm}$ (coming from the classical solutions $\phi_{\leq i}^{\pm}$) must be eliminated. Following the same procedure outlined in previous publications [\[9](#page-16-8)], we can prove that the free propagator satisfies

$$
\phi_{k_0}^{\pm cl}(\eta')J_0 = \left\{ u_2(\eta', \eta)\phi_{\n
$$
- \frac{2a^2(\eta)u_2'(\eta, \eta)u_1(\eta', \eta)\phi_{\n
$$
\mp \frac{4i u_1(\eta', \eta)V^{-1}}{[a^2(\eta)u_1'(\eta, \eta) - a^2(\eta_i)u_2'(\eta_i, \eta)]}\partial_{\phi_{\n(24)
$$
$$
$$

where a prime now stands for a derivative with respect to η' and the spatial volume V appears because of we are considering only one Fourier mode for the system. These identities allow us to remove the initial field configurations $\phi_{\leq i}^{\pm}$, by expressing them in terms of the final amplitudes $\phi_{\leq f}^{\pm}$ and the derivatives $\partial_{\phi_{\leq f}^{\pm}}$, and obtain the master equation.

The full equation is very complicated and, as for quantum Brownian motion, it depends on the systemenvironment coupling. In what follows we will compute the diffusion coefficients for the different couplings described in the previous section. As we are solely interested in decoherence, it is sufficient to calculate the correction to the usual unitary evolution coming from the imaginary part of the influence action. The result reads

$$
\begin{split}\ni \partial_{\eta}\rho_{r}[\phi_{
$$

where we have defined $\Gamma_1 = \frac{\lambda^2 V}{H^2} (\phi^+_{\leq f})$ $^{3} - \phi_{&f}^{-}$ $^{3})^{2}$ and $\Gamma_2 = \frac{\lambda^2 V}{4} (\phi_{\leq f}^+$ $\frac{2}{\sqrt{5}} - \phi \frac{1}{5} f$ $^{2})^{2}$. The ellipsis denotes other terms coming from the time derivative that not contribute to the diffusive effects. This equation contains time-dependent diffusion coefficients D_j . Up to one loop, only D_1 and D_2 survive. Coefficient D_1 is related to the interaction term $\phi^3_< \phi_>,$ while D_2 to $\phi^2_< \phi^2_>.$ These coefficients can be (formally) written as

$$
D_1(\vec{k_0}, \eta, \Lambda) = \frac{H^2}{2} \int_{\eta_i}^{\eta} d\eta' a^4(\eta) a^4(\eta') F_{\text{cl}}^3(\eta, \eta', k_0) \tag{26}
$$

$$
\times Im G_{++}^{\Lambda}(\eta, \eta', 3\vec{k_0}) \Theta(3k_0 - \Lambda),
$$

and

$$
D_2(\vec{k_0}, \eta, \Lambda) = -36 \int_{\eta_i}^{\eta} d\eta' a^4(\eta) a^4(\eta') F_{\text{cl}}^2(\eta, \eta', k_0) \tag{27}
$$

$$
\times \left[Re G_{++}^{\Lambda 2}(\eta, \eta', 2\vec{k_0}) + 2 \Re G_{++}^{\Lambda 2}(\eta, \eta', 0) \right],
$$

with the function F_{cl} defined by

$$
F_{\rm cl}(\eta, \eta_i, k_0) = \frac{\sin[k_0(\eta - \eta_i)]}{k_0 \eta} + \frac{\eta_i \cos[k_0(\eta - \eta_i)]}{\eta}.
$$
 (28)

The explicit expressions of these coefficients are complicated functions of conformal time, the particular mode k_0 , and the cut-off Λ , and we show them in Appendix [A.](#page-13-0)

It is important to note that here we are only studying the effect of normal diffusion terms, even it is known that anomalous diffusion terms can also be relevant at zero temperature. Analysis done in Ref. [\[27](#page-16-26)] suggests that anomalous diffusion for a supraohmnic environment is only relevant on a small transient and decoherence for unstable long-wavelength modes are driven by normal diffusion coefficients [\[28](#page-16-27)].

IV. DECOHERENCE

Coherences are destroyed by diffusion terms. This process is evident after considering the following appoximate solution to the master equation

$$
\rho_{\rm r} \left[\phi_{\leq}^+, \phi_{\leq}^-, \eta \right] \approx \rho_{\rm r}^{\rm u} [\phi_{\leq}^+, \phi_{\leq}^-, \eta] \times \exp \left[- \sum_j \Gamma_j \int_{\eta_i}^{\eta_f} d\eta \ D_j(k_0, \Lambda, \eta) \right], \quad (29)
$$

where $\rho_{\rm r}^{\rm u}$ is the solution of the unitary part of the master equation (i.e. without environment), and Γ_i includes the coefficients in front each diffusion term in Eq.[\(25\)](#page-3-2). The system will decohere when the non-diagonal elements of the reduced density matrix are much smaller than the diagonal ones.

The decoherence time-scale sets the time after which we have a classical field configuration, and it can be defined as the solution to

$$
1 \approx \sum_{j} \Gamma_{j} \int_{\eta_{i}}^{\eta_{d}} d\eta \ D_{j}(k_{0}, \Lambda, \eta) \qquad (30)
$$

$$
\geq \Gamma_{l} \int_{\eta_{i}}^{\eta_{d}} d\eta \ D_{l}(k_{0}, \Lambda, \eta),
$$

where the inequality is valid for any particular $j = l$. That is, the interactions with the environment have a cumulative effect on the onset of classical behaviour, i.e. the inclusion of a further interaction term reduces the decoherence time η_d . Therefore, in order to find upper bounds to η_d , we define the decoherence time η_{d_j} coming from each diffusion coefficient by

$$
1 \approx \Gamma_j \int_{\eta_i}^{\eta_{d_j}} d\eta \ D_j(k_0, \Lambda, \eta), \tag{31}
$$

with $j = 1,2$.

A. Diffusion terms: Numerical results and analytic approximations

In this subsection we will analyze the behaviour of each diffusion coefficient as a function of the Fourier mode k_0 (considered for the system in Eq.[\(23\)](#page-3-3)) and the cutoff Λ. We will also analyze the temporal evolution of the coefficients and their integration in time for fixed values of k_0 and Λ . We will present simple analytical approximations to the coefficients which can be used in Eq.[\(31\)](#page-4-0) instead of the full expressions to estimate the decoherence time-scale.

We define the dimensionless quantity $\mathcal{N}[k_0, \eta]$ which is the number of e-foldings between the time η_{k_0} when the mode k_0 crosses the Hubble radius (i.e., $|k_0\eta_{k_0}| = 1$) and any time η during inflation,

$$
\mathcal{N}[k_0, \eta] \equiv \ln \left| \frac{\eta_{k_0}}{\eta} \right| = -\ln |k_0 \eta|. \tag{32}
$$

This quantity has the special feature that its sign indicates whether the mode is inside $(\mathcal{N}[k_0, \eta] < 0)$ or outside $(\mathcal{N}[k_0, \eta] > 0)$ the Hubble radius.

Let us first consider the diffusion coefficient D_1 , which comes from the interaction term $\phi^3_< \phi >$. Because of we are considering only one Fourier mode for the system, with wave vector k_0 , and the environment-field contains only modes with $k > \Lambda$, this coefficient is different from zero only if $\Lambda/3 < k_0 < \Lambda$ (i.e., $\phi_<$ is only coupled with the $\vec{k} = 3\vec{k_0}$ mode of the environment).

For this coefficient we can obtain an exact analytical expression from Eq.[\(A3\)](#page-13-1) (see Appendix [A\)](#page-13-0). In Fig. [1](#page-5-0) we have plotted this expression as a function of k_0 for a particular value of the conformal time $(H\eta = -1/2)$. For later times, the graphs are qualitatively similar but D_1 oscillates more rapidly (since we have obtained our results by perturbative calculations, they are not valid at

FIG. 1: Coefficient D_1 (in logarithmic scale) as a function of k_0 for $H\eta = -0.5$.

FIG. 2: Coefficient D_1 (in logarithmic scale) as a function of k_0 for a fixed value of $|k_0\eta| = 1/10$ $(k_0 > H/10)$, where we have also plotted the approximation in Eq.[\(33\)](#page-5-1).

large times). The coefficient decreases with k_0 and takes its maximum value for $k_0 \cong \Lambda/3$, implying that this kind of interaction produce more decoherence for small values of k_0 and, from the above discussion, for small values of Λ. For practical purposes, we consider the following simple approximation to D_1 :

$$
D_1^{\text{approx}}(k_0, \eta, \Lambda) = -\frac{1}{100} \frac{(1+H\eta)}{H^4 \eta^7 k_0^3} \Theta(3k_0 - \Lambda). \quad (33)
$$

As we can see from Fig. [2,](#page-5-2) this approximation is less close to the exact coefficient for big values of k_0 . It is important to note from Eq.[\(31\)](#page-4-0) that if the approximation is a lower bound to the coefficient, then it will be useful to calculate an upper bound to η_{d_1} .

FIG. 3: Temporal integral of D_1 in units of H^{-1} as a function of $\mathcal{N}[k_0, \eta_d]$ for two different values of k_0 . Solid curves are numerical integrations of the exact coefficient, while shortdashed curves are from analytical integration of Eq.[\(33\)](#page-5-1). Note the logarithmic scale on the vertical axis.

According to the definition of η_{d_1} , given k_0 , Λ and Γ_1 , we can estimate this time by integrating D_1 over the conformal time η and plotting this temporal integral as a function of $\mathcal{N}[k_0, \eta_{d_1}]$. Fig. [3](#page-5-3) shows such plots for two particular values of k_0/H , where we have added the curves of the approximation [\(33\)](#page-5-1). As we have illustrated with these examples, the approximation to D_1 is useful to estimate the order of magnitude of the corresponding decoherence time η_{d_1} .

Let us now examine the behaviour of the coefficient D_2 which is associated with the interaction term $\phi^2 < \phi^2 >$. Since the interaction is now quadratic in ϕ_{\geq} , there are no restrictions on the values of k_0 such that $D_2 \neq 0$. Hence this coefficient can affect the coherence of all modes in the system, therefore it is the most important in our model.

The dependence of D_2 with Λ is showed in Fig. [4](#page-6-0) for three different values of k_0 and a fixed time $\eta =$ $-10^{-3}H^{-1}$. As we can see in this figure, for modes with $k_0 \ll \Lambda$ the coefficient is weakly dependent of the value of the critical wave vector Λ .

In Figs. [5](#page-6-1) and [6](#page-6-2) we have plotted the coefficient D_2 as a function of k_0/H for fixed values of η and Λ. Fig. [5](#page-6-1) shows that modes within the Hubble radius $(|k_0\eta| > 1)$ the diffusion coefficient is an oscillatory function and it has a maximum when $k_0 \sim \Lambda$. The same behaviour was noted for conformally coupled fields [\[24\]](#page-16-23). Physically, this fact can be interpreted in terms of particle creation in the environment due to its interaction with the system. For these modes, a very simple but good approximation to D_2 is given by

$$
D_2^{\ell}(\eta) = \frac{27}{2\pi} \frac{1}{(H\eta)^4},\tag{34}
$$

where the upper-script ℓ stands for "local", since it corre-

FIG. 4: Coefficient D_2 (in logarithmic scale) as a function of Λ/H for a fixed conformal time $\eta = -10^{-3}H^{-1}$ and three different values of k_0 ($H/10$, H and $100H$).

FIG. 5: Coefficient D_2 as a function of k_0 for $\Lambda = 20H$ and $H\eta = -1/2.$

sponds to approximate the whole diffusion coefficient by the term in Eq.[\(27\)](#page-4-1) containing the Dirac delta function (see details in Appendix [A\)](#page-13-0). On the other hand, Fig. [6](#page-6-2) shows the dependence of D_2 with k_0 for modes outside the Hubble radius at η . For these modes we note that the diffusive effects are more important for the smallest values of k_0 , which are most sensitive at the expansion of the universe. Note that the classical equation [\(20\)](#page-3-0) for each mode of the free field ψ (defined as $\psi = a(\eta)\phi$) describes an stable (unstable) oscillator if the mode k_0 satisfies $|k_0\eta| >> 1$ ($|k_0\eta| << 1$). Therefore, it was expectable that the diffusion coefficients mirrors this fact.

For modes far outside $(|k_0\eta| << 1)$ the Hubble radius we have found an asymptotic approximation of D_2 , useful

FIG. 6: The same as Fig. [5](#page-6-1) but for modes k_0 outside the Hubble radius at $\eta = -1/2H^{-1}$ and $\Lambda = 2H$.

FIG. 7: Coefficient D_2 (in logarithmic scale) as a function of $\mathcal{N}[k_0, \eta_d]$ for $\Lambda = 300H$ and $k_0 = 100H$. Heavy solid curve is numerically calculated from Eq.[\(27\)](#page-4-1) (see details in Appendix [A\)](#page-13-0). Light solid curve corresponds to the approximation obtained by averaging the local (dotted curve) and the asymptotic (short-dashed curve) approximations.

if $\Lambda \sim k_0$, which can be written as

$$
D_2^a(\eta,\Lambda) = -\frac{1}{\pi^2 H^4 \Lambda^3 \eta^7}.\tag{35}
$$

Fig. [7](#page-6-3) shows a graph of the coefficient D_2 as a function of $\mathcal{N}[k_0, \eta_{d_2}]$ for a particular value of k_0 and Λ , and the curves of the two approximations above. From this figure, we can distinguish two different regimes: one is well described by the local approximation $(|k_0\eta| > 1)$ and the other by the asymptotic one $(|k_0\eta| << 1)$. This behaviour of D_2 indicates that the decoherence process becomes faster once the mode crosses the Hubble radius.

FIG. 8: D_2 and D_2^{approx} as functions of k_0/H for a fixed value of $|k_0\eta| = 1/10$ $(k_0 > H/10)$ and $\Lambda = 100H$. The vertical axis is on a logarithmic scale.

FIG. 9: Temporal integrals of D_2 and D_2^{approx} (on a logarithmic scale and in units of H^{-1}) as functions of $\mathcal{N}[k_0, \eta_d]$, for $\Lambda = 300H$ and two different values of k_0 (100H and 200H).

In addition, we can see that a better approximation to D_2 is obtained by averaging the local and the asymptotic ones, that is

$$
D_2^{approx} = \frac{D_2^a + D_2^{\ell}}{2}, \t\t(36)
$$

which is also showed in the same figure.

The dependence of D_2 with k_0 for a fixed value of $\mathcal{N}[k_0, \eta]$ is shown in Fig. [8,](#page-7-0) in which we see that the approximation is less close to the numerical curve for big values of k_0 , but it bounds D_2 from below.

In Figs. [9](#page-7-1) and [10](#page-7-2) we have plotted the temporal integral of the coefficient D_2 as a function of $\mathcal{N}[k_0, \eta_{d_2}]$ for different values of k_0 and Λ , where we have also plot-

FIG. 10: The same as Fig. [9](#page-7-1) but for smaller values of k_0 $(k_0 = H \text{ and } k_0 = H/10) \text{ and } \Lambda = 2H.$

ted the curves obtained using the approximate expression given in Eq.[\(36\)](#page-7-3). In view of our analysis, it is reasonable to use that approximation to estimate the decoherence time-scale η_{d_2} , at least for values of Λ smaller than $10^3 k_0$.

B. The decoherence time

So far in this Section we have analyzed the behaviour of the diffusion coefficients, and we have shown the approximations considered are useful to estimate the decoherence time associated with each diffusion coefficient (or interaction term). Let us now work at the level of the order of magnitude and apply these results to quantify the decoherence time η_d .

As we have already mentioned, η_{d_1} and η_{d_2} are upper bounds to η_d . The time-scale given by $\eta_d(k_0)$ sets the time after which we are able to distinguish between two different amplitudes of the Fourier mode k_0 within the volume V. Thus, the maximum value of $\eta_d(k_0)$ with k_0 < Λ corresponds to an upper bound to the decoherence time for the system-field (since, in principle, ϕ_{\leq} contains all these modes with amplitudes different from zero).

In order to quantify decoherence times $\eta_{d_{1,2}}$ we have to fix the values of $\Gamma_{1,2}$ (i.e., we have to assume values to λ , $V, \phi_{\leq f}^+$, and $\phi_{\leq f}^-$). For this, as a first approximation and since we are considering a fixed de Sitter background, we will assume that the slow-roll conditions are satisfied at least up to times of the order of the decoherence timescale. We will choose typical values for the parameters of the model (λ, V) and for the elements of the reduced density matrix $(\phi_{\leq f}^+, \phi_{\leq f}^-)$.

The slow-roll conditions are usually written as [\[29,](#page-16-28) [30\]](#page-16-29):

$$
\varepsilon_U = \frac{m_{\rm pl}^2}{16\pi} \left(\frac{U'}{U}\right)^2 \lt 1 , \ \eta_U = \frac{m_{\rm pl}^2}{8\pi} \left(\frac{U''}{U}\right) \lt 1, \ (37)
$$

where $m_{\text{pl}} \equiv G^{-1/2}$ is the Plank mass, $U = \lambda \phi_0^4$, $U' = dU/d\phi_0$, and ϕ_0 is the classical inflaton field. If we also assume that ϕ_0 is homogeneous on physical scales λ_{ph} >> $(\sqrt{\lambda}\phi_0)^{-1}$, these conditions imply that the classsical configuration field ϕ_0 and the Hubble rate satisfy

$$
H^2 \simeq \frac{8\pi U}{3m_{\rm pl}^2} \quad ; \quad \frac{d\phi_0}{dt} \simeq -\frac{U'}{3H}.\tag{38}
$$

Defining the end of the inflationary period setting $\epsilon_U \sim 1$, one can set $\phi_0(N_\eta) \cong \sqrt{(N_\eta + 1)/\pi} m_{\text{pl}}$, where $N_{\eta} = \ln a(\eta_f)/a(\eta)$; typically the e-fold number $N_{\eta_i} \equiv$ $N \sim 60$ [\[29\]](#page-16-28). Thus, we assume the mean value of the system-field at time of decoherence is $\phi_0(N_{\eta_d})$ ($\Sigma_{\phi < f}$) $(\phi^+_{\leq f} + \phi^-_{\leq f})/2 \simeq \phi_0(N_{\eta_d})$. Using this mean value and Eq.[\(38\)](#page-8-0) we can write $H^2 \simeq 8\lambda N_{\eta_d}^2 m_{\text{pl}}^2/(3\pi)$. In order to choose a typical order of magnitude for the fluctuations of the system-field, let us use the fact that the amplitude of the so-called primordial density perturbations δ can be inferred to be of order 10^{-5} , and that $\delta \sim H/\dot{\phi}_0 \delta \phi$, where $\delta\phi$ is the amplitude of the inflaton field fluctuations [\[29](#page-16-28), [30\]](#page-16-29). Thus, with the use of these constraints and Eq.[\(38\)](#page-8-0) we set $\Delta_{\phi_{< f}} \equiv (\phi_{< f}^+ - \phi_{< f}^-) \sim 10^{-5} \phi_0(N_{\eta_d})/N_{\eta_d}$. Since V is the spatial volume inside which there are no coherent superpositions of macroscopically distinguishable states for the system-field, it is reasonable to choose $V = vH^{-3}$, with $v \sim 1$ (which corresponds, if $N \sim 60$, to the comoving volume $V \sim (a_0 H_0)^{-3}$, where H_0 and a_0 are the present values of the Hubble rate and the scalar factor, respectively) .

From previous considerations and according to the con-dition [\(31\)](#page-4-0), the times $\eta_{d_{1,2}}$ can be estimated as the solution to

$$
\int_{\eta_i}^{\eta_{d_1}} H d\eta D_1(\eta) \sim \frac{H}{\Gamma_1} \lesssim 2 \times 10^{15} \frac{(\lambda 10^5)}{v} \left(\frac{N}{60}\right)^5 \tag{39a}
$$

$$
\int_{\eta_i}^{\eta_{d_2}} H d\eta D_2(\eta) \sim \frac{H}{\Gamma_2} \lesssim 9 \times 10^{17} \frac{1}{v} \left(\frac{N}{60}\right)^4, \quad (39b)
$$

where we have used the fact that $N_{\eta_d} \lesssim N$. The last term in [\(39a\)](#page-8-1) follows after taking $(\phi_{\leq f}^{+3} - \phi_{\leq f}^{-3}) \sim \Delta_{\phi_{\leq f}} \Sigma_{\phi_{\leq f}}^2$. For example, from Fig. [10](#page-7-2) we can see that the temporal integral of D_2 takes a value of order 10^{17} for $\mathcal{N}[k_0 =$ H, η_{d_2} between 7 and 8. As D_2 is weakly dependent with Λ for $k_0 < H$, we can say that it is valid for $\Lambda \sim H$. Thus, since D_2 decrease with k_0 for $|k_0\eta| < 1$, we can conclude that the decoherence time for those modes with $k_0 < H$ are smaller.

Substituting the approximation given in Eq.[\(33\)](#page-5-1) into the left-hand side of [\(39a\)](#page-8-1) and assuming that $|H\eta_{d_1}| <<$ 1, we get

$$
t_{d_1} \sim \frac{1}{6H} \ln\left(600\alpha^3 \frac{H}{\Gamma_1}\right) \qquad (40)
$$

$$
\lesssim \frac{7}{H} + \frac{1}{6H} \ln\left(\lambda 10^5\right) + \frac{1}{6H} \ln\left(\frac{\alpha^3}{v}\left(\frac{N}{60}\right)^5\right),
$$

where $t_{d_1} = H^{-1} \ln a(\eta_{d_1})$, and we have defined $k_0 = \alpha H$.

With the use of the approximation in Eq. (36) we obtain a quadratic equation in $x \equiv a^3(\eta_{d_2}) = \exp(3Ht_{d_2})$, which is simple to solve for t_{d_2} . The result is

$$
x \sim \frac{27}{4}\pi\sigma^3 \left(\sqrt{1 + \frac{8}{27\sigma^3} \left[\frac{8H}{9\Gamma_2} + \frac{1}{\pi} + \frac{2}{27\pi^2\sigma^3} \right]} - 1 \right)
$$

<
$$
< \frac{27}{4}\pi\sigma^3 \left(\sqrt{1 + \frac{8}{27\sigma^3} \left[\frac{8H}{9\Gamma_2} + \frac{1}{\pi} + \frac{2}{27\pi^2\sigma^3} \right]} \right), (41)
$$

where $\sigma \equiv \Lambda/H$. For the sake of simplicity let us set $\sigma \sim 1$. Since $H/\Gamma_2 >> 1$, it yields

$$
t_{d_2} \lesssim \frac{1}{6H} \ln \left(12\pi^2 \sigma^3 \frac{H}{\Gamma_2} \right)
$$

$$
\lesssim \frac{7.7}{H} + \frac{1}{6H} \ln \left(\frac{\sigma^3}{v} \left(\frac{N}{60} \right)^4 \right).
$$
 (42)

Assuming $N = H t_{end} \ge 60$ as an estimative scale to the end of inflationary period and values of $\lambda \leq 10^{-5}$, we obtain

$$
\frac{t_{d_1}}{t_{\text{end}}} \lesssim \frac{7}{60} + \frac{1}{120} \ln\left(\frac{\alpha}{v^{1/3}}\right),\tag{43}
$$

which makes sense only if $k_0 < \Lambda < 3k_0$ ($\alpha < \sigma < 3\alpha$), and

$$
\frac{t_{d_2}}{t_{\text{end}}} \lesssim \frac{2}{15} + \frac{1}{120} \ln \left(\frac{\sigma}{v^{1/3}} \right). \tag{44}
$$

From scales t_{d_1} and t_{d_2} we conclude that if one set $\Lambda \lesssim H$, the decoherence time-scale for the system-field is shorter than the minimal duration of inflation for all the wave-vectors within the system sector.

V. EFFECTIVE DYNAMICAL EVOLUTION OF THE SYSTEM-FIELD IN A FIXED DE SITTER BACKGROUND

In this Section we concern ourselves with the time evolution of the system-field. After reviewing the phenomenological way to describe the stochastic dynamical evolution of the system, we present the renormalized "semiclassical-Langevin" equation. This equation can be used to describe the dynamical evolution of the classical configurations of the system-field and hence is useful once all the modes in the system have lost coherence. As a first step to understand the generation of classical inhomogeneities from quantum fluctuations, we then consider a simple situation in which we analyze the influence of the environment on the power spectrum for modes inside the system sector.

A. Effective dynamical evolution, noise and expectation values

The real part of the influence action contains divergent terms and should be renormalized. The imaginary part is finite and is associated with the decoherence process. It is well known that the terms of the imaginary part that come from a given interaction term in the original action can be viewed as arising from a noise source [\[8](#page-16-7), [31\]](#page-16-30). In our case there are two such sources ξ_2 and ξ_3 , which are associated with the interaction terms $\phi^2 < \phi^2$ and $\phi^3 < \phi$ respectively. That is, the imaginary part of the influence action can be rewritten as

$$
Im\delta A = -\ln\left(F[\Delta_2]F[\Delta_3]\right),\tag{45}
$$

where $F[\Delta_n]$ $(n = 2, 3)$ is the characteristic functional of the noise ξ_n , which is related with Gaussian functional probability distribution $P[\xi_n]$ as

$$
F[\Delta_n] = \int D\xi_n \ P[\xi_n] \exp\left\{-i \int d^4x \Delta_n(x)\xi_n(x)\right\},
$$

$$
P[\xi_n] = N_n \exp\left\{-\frac{1}{2} \int d^4x_1 \int d^4x_2 \xi_n(x_1) \times \nu_n^{-1}(x_1, x_2)\xi_n(x_2)\right\},
$$
 (46)

where N_n is a normalization factor and ν_n^{-1} is the functional inverse of the noise kernel ν_n :

$$
\nu_2(x_1, x_2) = -288\lambda^2 a^4(\eta_1) a^4(\eta_2) Re G_{++}^{\Lambda 2}(x_1, x_2), \tag{47a}
$$

$$
\nu_3(x_1, x_2) = 64\lambda^2 a^4(\eta_1) a^4(\eta_2) Im G^{\Lambda}_{++}(x_1, x_2).
$$
 (47b)

The Gaussian noise field $\xi_n(x)$ is completely characterized by

$$
\langle \xi_n(x) \rangle_P = 0,\tag{48a}
$$

$$
\langle \xi_n(x_1)\xi_n(x_2)\rangle_P = \nu_n(x_1, x_2),\tag{48b}
$$

where with $\langle \rangle_P$ we are denoting average over all realizations of $\xi_n(x)$.

The functional variation

$$
\left. \frac{\delta S_{\text{eff}}}{\delta \phi_{\leq}^{+}} \right|_{\phi_{\leq}^{+} = \phi_{\leq}^{-}} = 0, \tag{49}
$$

yields the "semiclassical-Langevin" equation for the system-field, which is only valid once the system has become classical.

With the identifications above, the reduced density matrix can be rewritten as [\[32](#page-16-31)]:

$$
\rho_{r}[\phi_{
$$
\times \rho_{r}[\phi_{
$$
$$

with

$$
\rho_{\rm r}[\phi_{
$$
\times \int_{\phi_{
$$
$$

where the effective action S_{eff} is given by

$$
S_{\text{eff}}[\phi^+_{<}, \phi^-_{<}, \xi_2, \xi_3] = Re\{A[\phi^+_{<}, \phi^-_{<}]\} \qquad (52)
$$

$$
- \int d^4x [\Delta_2(x)\xi_2(x) + \Delta_3(x)\xi_3(x)].
$$

Here A is the CGEA specified in Eq.[\(13\)](#page-2-1). Thus, the full expectation value of any operator $\hat{Q}[\phi_{\leq}]$ can be written as

$$
\langle \hat{Q}[\phi_{<}] \rangle = \int D\xi_2 P[\xi_2] \int D\xi_3 P[\xi_3] \int d\phi_{<}
$$

$$
\times \quad \rho_r[\phi_{<} | \phi_{<}, \xi_2, \xi_3; \eta] \; Q[\phi_{<}]
$$

$$
\equiv \langle \langle \hat{Q}[\phi_{<}] \rangle_q \rangle_P, \tag{53}
$$

where $\langle \rangle_q$ is an usual quantum average for a system-field subjected to external stochastic forces.

B. Renormalized equation of motion for the system

Taking the functional variation as in Eq.[\(49\)](#page-9-0) we obtain

$$
\phi''_{<}(\eta, \vec{x}) - \Delta \phi_{<}(\eta, \vec{x}) + 2 \mathcal{H} \phi'_{<}(\eta, \vec{x}) \n+ 4 \lambda a^2(\eta) \phi_{<}(\eta, \vec{x}) [\phi_{<}^2(\eta, \vec{x}) - 3iG_{++}^{\Lambda}(\eta, \eta, \vec{0})] \n- 96 \lambda^2 a^2(\eta) \int_{\eta_i}^{\eta} d\eta' a^4(\eta') \phi_{<}^2(\eta, \vec{x}) \n\times \int d^3 y \phi_{<}^3(\eta', \vec{y}) ReG_{++}^{\Lambda}(\eta, \eta', \vec{x} - \vec{y}) \n- 288 \lambda^2 a^2(\eta) \int_{\eta_i}^{\eta} d\eta' a^4(\eta') \phi_{<}(\eta, \vec{x}) \n\times \int d^3 y \phi_{<}^2(\eta', \vec{y}) ImG_{++}^{\Lambda 2}(\eta, \eta', \vec{x} - \vec{y}) \n= -\xi_2(\eta, \vec{x}) \frac{\phi_{<}(\eta, \vec{x})}{a^2(\eta)} - \frac{3}{2} \xi_3(\eta, \vec{x}) \frac{\phi_{<}^2(\eta, \vec{x})}{a^2(\eta)}. \tag{54}
$$

This equation contains divergences. In order to renormalize it we use the method of adiabatic substraction with dimensional regularization, which works at the level of the field equation and is particularly useful for solving the equation numerically [\[33\]](#page-16-32). To simplify the task we assume the system-field to be homogeneous enough so that the spatial derivatives and the term in Eq.[\(54\)](#page-9-1) coming from the interaction $\phi > \phi^3$ are negligible. Note that for an homogeneous system-field there is no contribution from this interaction term up to one loop order, due to orthogonality of the Fourier modes (see, for instance, Eq.[\(17\)](#page-3-4)). Details of the renormalization procedure are relegated to Appendix [B.](#page-14-0) The renormalized equation is

$$
\phi''_{\leq}(\eta) + [\Delta M^2(\eta) + \Delta \Sigma(\eta) R] a^2(\eta) \phi_{\leq}(\eta)
$$
(55)
+ 2 H $\phi'_{\leq}(\eta) + 4 [\lambda + \Delta \tilde{\lambda}(\eta)] a^2(\eta) \phi^3_{\leq}(\eta)$
+ $\frac{36\lambda^2}{\pi^2} \phi_{\leq}(\eta) a^2(\eta) \left\{ \int_{\eta_i}^{\eta} d\eta' \phi^2_{\leq}(\eta') \mathcal{J}(\eta, \eta') \right\}$
+ $\int_{\eta_i}^{\eta} d\eta' \frac{\phi_{\leq}(\eta') \phi'_{\leq}(\eta')}{Ha(\eta)} \mathcal{R}(\eta, \eta') \right\} = -\tilde{\xi}_2(\eta) \frac{\phi_{\leq}(\eta)}{a^2(\eta)}.$

where $R = 12H^2$; we have redefined the noise source as

$$
\tilde{\xi}_2(\eta) = \frac{1}{V} \int_V d^3x \, \xi_2(\eta, \vec{x}), \tag{56}
$$

whose correlation function is given by

$$
\langle \tilde{\xi}_2(\eta_1) \tilde{\xi}_2(\eta_2) \rangle_P = a^2(\eta_1) a^2(\eta_2) \frac{36\lambda^2}{\pi^2 V} \left\{ \frac{\pi}{2} \delta[(\eta_1 - \eta_2)] + \frac{\cos[2\Lambda(\eta_1 - \eta_2)]}{\Lambda} \left[\frac{2}{\eta_1 \eta_2} + \frac{(\eta_1 - \eta_2)^2}{3 \eta_1^2 \eta_2^2} + \frac{1}{3 \Lambda^2 \eta_1^2 \eta_2^2} \right] - (\pi - 2 \mathcal{S}i[2\Lambda|\eta_1 - \eta_2|]) \left[\frac{|\eta_1 - \eta_2|^3}{3 \eta_1^2 \eta_2^2} + \frac{|\eta_1 - \eta_2|}{\eta_1 \eta_2} \right] - \sin[2\Lambda(\eta_1 - \eta_2)] \left[\frac{1}{2(\eta_1 - \eta_2)} - \frac{2(\eta_1 - \eta_2)}{3 \Lambda^2 \eta_1^2 \eta_2^2} \right] \right\};
$$

and we have also defined the following functions:

$$
\Delta \tilde{\lambda}(\eta) = \Delta \lambda + \frac{9\lambda^2}{2\pi^2} \left[\frac{\gamma}{2} - \ln \left| \frac{a(\eta)\mu}{2\Lambda} \right| \right],
$$
 (57a)

$$
\Delta \Sigma(\eta) = \Delta \xi - \frac{\lambda}{4\pi^2} \left[\frac{\gamma}{2} - \ln \left| \frac{a(\eta)\mu}{2\Lambda} \right| \right],\tag{57b}
$$

$$
\Delta M^{2}(\eta) = \Delta m^{2} - \frac{18\lambda^{2}}{\pi^{2}} \frac{a^{2}(\eta_{i})}{a^{2}(\eta)} \phi_{<}^{2}(\eta_{i})
$$
(57c)

$$
\times Ci[2\Lambda(\eta-\eta_i)] - \frac{3\lambda\Lambda^2}{2\pi^2 a^2(\eta)},
$$

$$
\mathcal{R}(\eta,\eta') = \frac{\eta}{\eta'^2} Ci[2\Lambda(\eta-\eta')],
$$
 (57d)

$$
\mathcal{J}(\eta, \eta') = \mathcal{J}_1(\eta, \eta') + \mathcal{J}_2(\eta, \eta') + \mathcal{J}_3(\eta, \eta'), \tag{57e}
$$

$$
\mathcal{J}_1(\eta, \eta') = C i [2\Lambda(\eta - \eta')] \left[\frac{2(\eta^3 - {\eta'}^3)}{3{\eta'}^4} + \frac{\eta^2}{\eta'}^3 \right], \quad (57f)
$$

$$
\mathcal{J}_2(\eta, \eta') = \frac{2(\eta - \eta')}{3\eta'^4 \Lambda^2} \left[\cos[2\Lambda(\eta - \eta')] \right]
$$
(57g)

$$
\sin[2\Lambda(\eta - \eta')] \right]
$$

$$
-\frac{\sin[2\Lambda(\eta-\eta')] }{2\Lambda(\eta-\eta')} ,
$$

$$
\mathcal{J}_3(\eta,\eta') = -\frac{\sin[2\Lambda(\eta-\eta')] }{\Lambda} \left[\frac{(\eta-\eta')^2}{3\eta'^4} + \frac{2\eta}{\eta'^3} \right], (57h)
$$

where $Ci[x]$ is the cosine integral function [\[34](#page-16-33)]. Notice that these functions are logarithmically divergent in the limit $\Lambda \to 0$, which is the well-known infrared divergence [\[33\]](#page-16-32).

The functions above contain useful information. Particularly, they allows us to examine the conditions under which the loop expansion breaks down. In order to estimate the time after which the one-loop terms become of the same order of magnitude as the classical ones we may compare their time-dependent parts. For example, from Eq.[\(57\)](#page-10-0) we see that the time-dependent part of $\Delta\lambda(\eta)$ $(\Delta \Sigma(\eta))$ is the order λ (one) for $Ht \sim 1/\lambda$ and $\Delta M^2(\eta)$ is important only at the initial time $(\eta \sim \eta_i)$. In the same way, we can use the term $\lambda a^2 \phi^3_{\leq}$ to compare it with the remaining ones. In Fig. [11](#page-10-1) we have plotted the \mathcal{J}_i functions, where we can see that \mathcal{J}_1 dominates all others. It

FIG. 11: $\mathcal J$ and $\mathcal J_{1,2,3}$ as functions of $\mathcal N[\Lambda,\eta'] \equiv -\ln|\Lambda\eta'|$ for $\Lambda \eta = -10^{-2} \left(\mathcal{N}[\Lambda, \eta] \right) = \ln 100 \simeq 4.605$.

allow us to make the following approximation:

$$
\int_{\eta_i}^{\eta} d\eta' \phi^2_{\lt}(\eta') \mathcal{J}(\eta, \eta') \simeq \phi^2_{\lt}(\eta) \int_{\eta_i}^{\eta} d\eta' \mathcal{J}_1(\eta, \eta')
$$

$$
\sim -\phi^2_{\lt}(\eta) \frac{1}{3} (\ln |\Lambda \eta|)^2, \quad (58)
$$

where the last term is a simple long-time asymptotic expression. With the use of this approximation we obtain that the two terms in question are of the same order when $|\ln |\Lambda \eta|| \sim 1/\sqrt{\lambda}$ and, for $\Lambda \sim H$, $Ht \sim 1/\sqrt{\lambda}$.

As it is shown in Fig. [12,](#page-11-0) the R function peaks at $\eta \sim \eta'$ and hence we can approximate

$$
\int_{\eta_i}^{\eta} d\eta' \frac{\phi_{\leq}(\eta')\phi'_{\leq}(\eta')}{Ha(\eta)} \mathcal{R} \simeq \frac{\phi_{\leq}(\eta)\phi'_{\leq}(\eta)}{Ha(\eta)} \int_{\eta_i}^{\eta} d\eta' \mathcal{R}_{\geq} 9
$$

$$
\sim \frac{\phi_{\leq}(\eta)\phi'_{\leq}(\eta)}{Ha(\eta)} \times (\gamma + \ln|2\Lambda \eta|), \tag{60}
$$

where the last expression corresponds to a long-time approximation. As for the $\mathcal J$ term, using the long-time approximation we get $\ln |\Lambda \eta| \sim \phi \langle H/(\lambda \phi \rangle)$. If in addition we assume that the slow-roll conditions are satisfied, this time can be estimated as $|\ln |\Lambda \eta|| \sim 1/(\lambda \varepsilon_U)$ and, for $\Lambda \sim H$, $Ht \sim 1/(\lambda \varepsilon_U)$, where ε_U is the slow-roll parameter given in Eq.[\(37\)](#page-7-4).

From previous discussion, we can see that the additional terms are only important at times longer than the typical time-scale associated with the decoherence process.

C. Generation of inhomogeneities: role of the noise

Having reached this point, we ask ourself about the primordial inhomogeneities. Certainly, the model we are

FIG. 12: R as a function of $\mathcal{N}[\Lambda, \eta']$ for $\Lambda \eta = -10^{-2}$ $(\mathcal{N}[\Lambda, \eta] = \ln 100 \simeq 4.605).$

considering is too simplified to obtain any property of the power spectrum of such inhomogeneities. Nevertheless, we can discuss some relevant aspects about the way of connecting the initial quantum fluctuations with those which eventually become classical and statistical. With this purpose, let us compute the power spectrum for some of the Fourier modes in the system working in an approximate way and making certain assumptions, but taking into account the decoherence process.

According to the results of the previous Section, the decoherence process occurs at different rates for each mode in the system. Provided that decoherence is more effective for modes outside the Hubble radius, we consider the following situation. We suppose that the "relevant" modes are in the Bunch-Davies state (i.e., in equilibrium with the environment) at the initial time η_i . Consequently, the matrix elements of the initial density operator for each of these modes are Gaussian functions in a mode-amplitude basis. As a first approximation we neglect the non-linearities which might affect the Gaussian form of those matrix elements.

We are now interested only in computing the power spectrum of those "relevant" modes up to \hbar and λ^2 order. To carry this out, we split the system-field as $\phi₀(\eta) + \delta\phi₀$, where we identify $\phi₀(\eta)$ as a classical background field which satisfies slow-roll conditions. The power spectrum of the quantum field fluctuations $\delta\phi$ may be expressed as $P_{\phi}(k) = 2\pi^2 k^{-3} \Delta_{\phi}^2(k)$, with $\Delta^2_{\phi}(k)$ defined by

$$
\langle \delta \phi_{<}(\vec{x}) \delta \phi_{<}(\vec{x} + \vec{r}) \rangle = \int d^3k \, \frac{\Delta_{\phi}^2(k)}{4\pi k^3} \exp(-i\vec{k} \cdot \vec{r}), \tag{61}
$$

where $\langle \rangle$ is the full average in Eq.[\(53\)](#page-9-2).

Consistently with the assumption that the "relevant" matrix elements remain Gaussian, we expand the semiclassical equation [\(54\)](#page-9-1) up to linear order in the mode

amplitude of interest $\delta\phi<(\vec{k})$. Through this procedure we obtain

$$
\phi_0''(\eta) + 2\mathcal{H}\phi_0'(\eta) + 4\lambda a^2 \phi_0^3(\eta) = 0,
$$
\n(62a)

$$
\delta\phi''_{\leq}(\vec{k},\eta) + [k^2 + 12\lambda a^2 \phi_0^2(\eta)] \delta\phi_{\leq}(\vec{k},\eta) + 2\mathcal{H}\delta\phi'_{\leq}(\vec{k},\eta) = -\frac{\xi_2(\vec{k},\eta)}{a^2} \phi_0(\eta),
$$
(62b)

where we have discarded the terms which do not contribute to the power spectrum up to \hbar order. The term with the ξ_3 noise source gives a zero contribution due to our approximations and the orthogonality of the Fourier modes. It is important to note the presence of the ξ_2 noise source, which ensure the decoherence process occurs as the matrix elements are evolved in time.

In order to obtain the power spectrum let us split $\delta\phi$ _<(\vec{k}, η) = $\delta\phi$ ^{ξ}(\vec{k}, η) + $\delta\phi$ ^{g}(\vec{k}, η), with $\delta\phi$ ^{ξ}(\vec{k}, η) = $\langle \delta \phi \langle \vec{k}, \eta \rangle \rangle_q$, where $\langle \rangle_q$ is the quantum average defined in Eq.[\(53\)](#page-9-2). Because of the assumption of linearity, this quantum average satisfies the semiclassical equation [\(62b\)](#page-11-1), whose solution can be written as the sum of the homogeneous solution $\delta \phi^h_{\leq}(k, \eta)$ and a particular solution $\delta \phi^p_{\lt}(\vec{k}, \eta)$. The former is given by

$$
\delta \phi^h_{\leq}(k,\eta) = a^{-1}(\eta) [\alpha_k \sqrt{|\eta|} J_\nu + \beta_k \sqrt{|\eta|} J_{-\nu}], \qquad (63)
$$

 $\sqrt{\frac{9}{4}-\epsilon}$, with $\epsilon = 6\lambda\phi_0^2/H^2$. Setting $\nu \simeq \frac{3}{2}$, a particular where α_k and β_k are constants of integration, and $\nu =$ solution is

$$
\delta \phi_{\leq}^p(\vec{k}, \eta) = -\int_{\eta_i}^{\eta} d\eta' g(k, \eta, \eta') \xi_2(\vec{k}, \eta') \phi_0(\eta'), \quad (64)
$$

where

$$
g(k,\eta,\eta') = \frac{1}{a(\eta)a(\eta')} \left[\frac{\sin[k(\eta-\eta')] }{k} \left(1 + \frac{1}{k^2 \eta \eta'} \right) - \frac{\cos[k(\eta-\eta')] }{k^2 \eta \eta'} (\eta-\eta') \right].
$$
 (65)

With the use of the initial conditions $\langle \delta \phi \langle \vec{k}, \eta_i \rangle \rangle_q =$ $\langle \delta \dot{\phi}_{\leq}(\vec{k}, \eta_i) \rangle_q = 0$, we obtain that $\langle \delta \phi_{\leq}^{\xi}(\vec{k}, \eta) \rangle_P = 0$, and thus $\delta \phi \xi(\vec{k}, \eta) = \delta \phi \xi(\vec{k}, \eta)$. Within these approximations, the result is analogous to that for the linear quantum Brownian motion (QBM) [\[27,](#page-16-26) [35\]](#page-16-34). Therefore, the quantity $\Delta_{\phi}^2(k)$ can be written so that it receives two contributions:

$$
\Delta_{\phi}^2(k) = \Delta_{\phi^q}^2(k) + \Delta_{\phi^\xi}^2(k). \tag{66}
$$

The first one comes from the unitary evolution of the initial density matrix, i.e., it is the usual quantum result for the case of the free field [\[29](#page-16-28)]: $\Delta_{\phi^q}^2(k) = (H/2\pi)^2 (1 +$ $(k^2\eta^2)$. The second one appears due to the ξ_2 noise source and can be computed through

$$
\langle \delta \phi \xi(\vec{k_1}, \eta) \delta \phi \xi(\vec{k_2}, \eta) \rangle_P = \int_{\eta_i}^{\eta} d\eta_1 \int_{\eta_i}^{\eta} d\eta_2 \phi_0(\eta_1) \phi_0(\eta_2)
$$

$$
\times \langle \xi_2(\vec{k_1}, \eta_1) \xi_2^*(\vec{k_2}, \eta_2) \rangle_P
$$

$$
\times g(k_1, \eta, \eta_1) g(k_2, \eta, \eta_2), \quad (67)
$$

FIG. 13: $\Delta_{\phi^{\xi}}^2/(\lambda \phi_0)^2$ as a function of $\mathcal{N}[k_0, \eta]$ for $\Lambda = 5H$ and two different values of k_0 . Note that the vertical axis is on a logarithmic scale.

where

$$
\langle \xi_2(\vec{k_1}, \eta_1) \xi_2^*(\vec{k_2}, \eta_2) \rangle_P = -288\lambda^2 a^4(\eta_1) a^4(\eta_1) \times (2\pi)^3 \delta^3(\vec{k_1} - \vec{k_2}) Re G_{++}^{\Lambda 2}(\eta_1, \eta_2, \vec{k_1}).
$$
\n(68)

Thus $\Delta_{\phi^{\xi}}^2(k)$ can be expressed as

$$
\Delta_{\phi^{\xi}}^{2}(k) = -\lambda^{2} \frac{144}{\pi^{2}} k^{3} \int_{\eta_{i}}^{\eta} d\eta_{1} \int_{\eta_{i}}^{\eta} d\eta_{2} a^{4}(\eta_{1}) a^{4}(\eta_{2})
$$

$$
\times \phi_{0}(\eta_{1}) \phi_{0}(\eta_{2}) g(k, \eta, \eta_{1}) g(k, \eta, \eta_{2})
$$

$$
\times \text{Re} G_{++}^{\Lambda 2}(\eta_{1}, \eta_{2}, \vec{k}). \tag{69}
$$

The Fourier transform $Re G_{++}^{\Lambda2}(\eta_1, \eta_2, \vec{k})$ (where $k < \Lambda$) can be obtained from Eq.[\(A9\)](#page-14-1) (see Appendix [A\)](#page-13-0) replacing $2k_0$ by k.

Since the additional contribution $\Delta_{\phi^{\xi}}^2(k)$ to the power spectrum is of order λ^2 , it is expected to be negligible. As we are assuming that ϕ_0 is a slowly varying field, we can compare the relative order of magnitude of both contribution setting $\phi_0(\eta_1) \sim \phi_0(\eta_2) \sim \phi_0$ and $H^2 \sim \lambda \phi_0^4/m_{\rm pl}^2$, so that ϕ_0 can be taken out from the inte-gration in Eq.[\(69\)](#page-12-0). Thereby, when $|k\eta| < 1$, the contributions $\Delta_{\phi^{\xi}}^2(k)$ is negligible compared to the usual $\Delta_{\phi^q}^2(k)$ if $(\Delta_{\phi^{\xi}}(k)/\lambda \phi_0)^2 << \lambda^{-1}(\phi_0/m_{\rm pl})^2$. Since the last quotient is usually much bigger than one, this condition is typically satisfied if $(\Delta_{\phi^{\xi}}(k)/\lambda \phi_0)^2 < 1$. As in the example shown in Fig. [13,](#page-12-1) this is the case and hence the additional contribution can be neglected.

On the other hand, the usual contribution $\Delta_{\phi^q}^2(k)$ is independent of k for a fixed value of $k\eta$, corresponding to a nearly scale-invariant spectrum, whereas $\Delta_{\phi^{\xi}}^2(k)$ depends on k and Λ (see Figs. [14](#page-12-2) and [15\)](#page-12-3). To see this more

FIG. 14: $\Delta_{\phi\xi}^2/(\lambda\phi_0)^2$ as a function of k_0/H for $\Lambda = 5H$ and $|k_0\eta| = 0.001.$

FIG. 15: $\Delta_{\phi\xi}^2/(\lambda\phi_0)^2$ as a function of Λ/H for $k_0 = 2H$ and $|k_0\eta| = 0.001.$

clearly, it is useful to rewrite $\Delta_{\phi^{\xi}}^2(k)$ as

$$
\Delta_{\phi^{\xi}}^{2}(k) = -\frac{144}{\pi^2} \int_{k\eta_i}^{k\eta} \frac{dx_1}{x_1^4} \int_{k\eta_i}^{k\eta} \frac{dx_2}{x_2^4} \phi_0(\eta_1) \phi_0(\eta_2)
$$

$$
\times f(k\eta, x_1) f(k\eta, x_2) F(x_1, x_2, \frac{\Lambda}{k}), \qquad (70)
$$

where $f \equiv k^3 Hg$ and $F \equiv k^3 H^{-1} Re G_{++}^{\Lambda 2}$, which depend on k not only through the $k\eta$ combination, but also through the combinations $k\eta_i$ and Λ/k . It is well-known that a finite duration of the inflation stage produces a departure of the power spectrum from the scale-invariant one [\[2\]](#page-16-1). The breaking of the scale invariance by the presence of the infrared cut-off Λ is similar to the one found in Ref. [\[36](#page-16-35)].

It is important to note that we have assumed a Gaus-

sian initial condition for the reduced density matrix elements and that the Gaussian form of these elements is not affected by the non-linearities of the interactions. However, as was pointed out in Ref. [\[10](#page-16-9)], it is expectable that when the non-linearities become important, the Gaussian approximation breaks down and therefore the associated Wigner functional becomes non-positive. Nevertheless, as well as for a given mode the Gaussian approximation remains valid up to times longer than the decoherence time-scale, one can use the classical description for it even in the non-linear regime.

VI. FINAL REMARKS

Let us summarize the results contained in this paper. After the integration of the high frequency modes in Section II, we obtained the CGEA for the modes whose wave vector is shorter than a critical value Λ. From the imaginary part of the CGEA we obtained, in Section III, the diffusion coefficients of the master equation. System and environment are two sectors of a single scalar field, and the results depend on the "size" of these sectors, which is fixed by the cut-off Λ .

To analyze the decoherence process, in Section IV we evaluated the diffusion coefficients and its integrations over the conformal time. It allows us to conclude that if we consider a cut-off $\Lambda \sim H$, those modes with wave vector $k_0 \ll \Lambda$ are the more affected by diffusion through one of the coefficients. For these modes, we saw that the effect is weakly dependent of the critical wave vector Λ . If one consider a cut-off $\Lambda > H$, and modes $H < k_0 < \Lambda$, diffusive effects are larger for those modes in the system whose wave vector is close to the critical Λ [\[24](#page-16-23)].

We presented the complete expression of the diffusive terms (in Appendix A), and also some simple analytical approximations to the coefficients, which are useful to make an evaluation of the decoherence time-scale. We performed an extensive analysis of the evaluation of the time-scale for the decoherence process for a typical case of interest. In such a case, we obtained that for a given mode $k_0 < \Lambda \leq H$, decoherence is effective by the time in which inflation is ending.

In Section V we analyzed the effective evolution of the system. Assuming an homogeneous system-field we first presented the explicit form of the renormalized stochastic Langevin equation and we analyzed the relative importance of the terms appearing in that equation due to the system-environment interaction. From such analysis we conclude that those terms are of the same order of magnitude than the classical ones for times longer than the typical decoherence time-scale.

We them considered inhomogeneity generation in a simple particular situation, in which we analyzed the influence of the environment on the power spectrum for some modes in the system. In that situation, splitting the spectrum as the sum of the contribution coming from the unitary evolution of the Bunch-Davies initial condi-

tion plus the one which appears because of the systemenvironment interaction, we found that the latter is negligible compared to the former. In spite of this result, we have remarked that the system-environment interaction is essential to have a complete quantum to classical transition which allows a late-time classical treatment of the system degrees of freedom.

As for future work, we consider that it is worth applying the same procedure used in this paper to inflationary models involving two or more interacting fields, such as some hybrid model [\[37\]](#page-16-36).

VII. ACKNOWLEDGMENTS

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APPENDIX A: DIFFUSION COEFFICIENTS

In this appendix we describe some technical details about the computation of the diffusion coefficients D_1 and D_2 , starting from Eq.[\(26\)](#page-4-2) and [\(27\)](#page-4-1) respectively.

In order to evaluate the coefficient D_1 , we have to perform the Fourier transform of the imaginary part of the propagator for the environment G_{++}^{Λ} (see Eq.[\(26\)](#page-4-2)). This propagator can be expressed in terms of the mode functions ϕ_k given by

$$
\phi_{\vec{k}} = \frac{1}{(2\pi)^{3/2}} \frac{\exp\{-ik\eta\}}{\sqrt{2k}a(\eta)} \left(1 - \frac{i}{k\eta}\right) \exp\{i\vec{k}\cdot\vec{x}\}
$$

$$
\equiv \tilde{\phi}_k(\eta) \exp\{i\vec{k}\cdot\vec{x}\}, \tag{A1}
$$

which corresponds to the Bunch-Davies vacuum assumed for the environment-field. The imaginary part of the propagator reads

$$
ImG^{\Lambda}_{++}(x_1, x_2) = \int_{k>\Lambda} d^3k \ Re{\{\phi_{\vec{k}}(x_1)\phi_{\vec{k}}^*(x_2)\}}. \quad (A2)
$$

Substituting this expression into Eq.[\(26\)](#page-4-2) we obtain

$$
D_1(\vec{k_0}, \eta, \Lambda) = \frac{\tilde{k_0}^4}{2} \int_x^{\tilde{k_0}} dy \left[\frac{\cos[3\Delta_{xy}]}{6x^3y^3} \left(1 + \frac{1}{9xy} \right) + \frac{\sin[3\Delta_{xy}]\Delta_{xy}}{18x^4y^4} \right] F_{cl}^3(x, y) \Theta(3 - \tilde{\Lambda}), \text{(A3)}
$$

with

$$
F_{cl}(x,y) = \frac{\sin[\Delta_{xy}]}{x} + \frac{y \cos[\Delta_{xy}]}{x}, \quad (A4)
$$

where to reduce the notation we have defined the following set of dimensionless variables:

$$
\tilde{k_0} = \frac{k_0}{H}
$$
, $\tilde{\Lambda} = \frac{\Lambda}{k_0}$, $y = |k_0 \eta'|$, $x = |k_0 \eta|$, $\Delta_{xy} = x - y$.

The integrations above can be performed exactly and the result can be numerically integrated over the conformal time η.

To obtain D_2 from Eq.[\(27\)](#page-4-1) we need to compute the Fourier transform of the real part of $G_{++}^{\Lambda 2}$ in $\vec{k} = 2\vec{k_0}$ and $\vec{k} = \vec{0}$. This real part is given by

$$
Re G_{++}^{\Lambda 2}(\eta, \eta', 2\vec{k_0}) = -(2\pi)^3 \int_{k>\Lambda} d^3k \int_{k'>\Lambda} d^3k'
$$

$$
\times \delta^3 \left(\vec{k} + \vec{k}' - 2\vec{k_0}\right) Re{\{\tilde{\phi}_k(\eta)\tilde{\phi}_k^*(\eta')\tilde{\phi}_{k'}(\eta)\tilde{\phi}_{k'}^*(\eta')\}}, (A5)
$$

where $\tilde{\phi}_k$ is the time-dependent mode function defined in Eq.[\(A1\)](#page-13-2) and $\tilde{\phi}_k^*$ its complex conjugate. This Fourier transform can be expressed in terms of integrals of the form,

$$
I_{n,m}^C = \frac{k_0^{m+n-3}}{\pi} \int_{k>\Lambda} d^3k \int_{k'>\Lambda} d^3k' \delta^3 \left(\vec{k} + \vec{k}' - 2\vec{k_0} \right)
$$

$$
\times \frac{\cos[(k+k')(\eta - \eta')] }{k^nk'^m}
$$

$$
= \int_{\tilde{\Lambda}+2}^{+\infty} \frac{du}{u^{n-1}} \int_{u-2}^{u+2} \frac{dz}{z^{m-1}} \cos[(u+z)\Delta_{xy}]
$$

$$
+ \int_{\tilde{\Lambda}}^{\tilde{\Lambda}+2} \frac{du}{u^{n-1}} \int_{\tilde{\Lambda}}^{u+2} \frac{dz}{z^{m-1}} \cos[(u+z)\Delta_{xy}], \quad (A6)
$$

where n and m are integer numbers (only $m = 3$ result to be necessary). The second equality follows after the change of variables $u = k_0^{-1}k$ and $z = k_0^{-1}|\vec{k} - 2\vec{k_0}| =$ $k_0^{-1}\sqrt{k^2-4kk_0\cos(\theta)+4k_0^2}$, where θ is the angle between \vec{k} and $\vec{k_0}$. We also define the integral $I_{n,m}^S$ as the one obtained from $I_{n,m}^C$ by replacing the cosine function by sine.

An integration of $I_{n,m}^{C,S}$ by parts yields

$$
I_{n,m}^C + \Delta_{xy} \frac{I_{n,m-1}^S}{2-m} = \int_{\tilde{\Lambda}}^{+\infty} \frac{du}{u^{n-1}} \frac{\cos[2(u+1)\Delta_{xy}]}{(2-m)(u+2)^{m-2}} - \int_{\tilde{\Lambda}+2}^{+\infty} \frac{du}{u^{n-1}} \frac{\cos[2(u-1)\Delta_{xy}]}{(2-m)(\tilde{\Lambda}+u)^{m-2}} - \int_{\tilde{\Lambda}}^{\tilde{\Lambda}+2} \frac{du}{u^{n-1}} \frac{\cos[2(u+1)\Delta_{xy}]}{(2-m)\tilde{\Lambda}^{m-2}},
$$
(A7)

$$
I_{n,m}^{S} - \Delta_{xy} \frac{I_{n,m-1}^{C}}{2-m} = -\int_{\tilde{\Lambda}}^{+\infty} \frac{du}{u^{n-1}} \frac{\sin[2(u+1)\Delta_{xy}]}{(2-m)(u+2)^{m-2}} + \int_{\tilde{\Lambda}+2}^{+\infty} \frac{du}{u^{n-1}} \frac{\sin[2(u-1)\Delta_{xy}]}{(2-m)(\tilde{\Lambda}+u)^{m-2}} + \int_{\tilde{\Lambda}}^{\tilde{\Lambda}+2} \frac{du}{u^{n-1}} \frac{\sin[2(u+1)\Delta_{xy}]}{(2-m)\tilde{\Lambda}^{m-2}}.
$$
(A8)

With the use of these properties and definitions, the Fourier transform [\(A5\)](#page-14-2) becomes

$$
ReG_{++}^{\Lambda 2}(\eta, \eta', 2\vec{k_0}) = \frac{-H^4 x^2 y^2}{8(2\pi)^2 k_0^3} \left\{ I_A + \frac{2I_B}{xy} + \frac{I_C}{x^2 y^2} \right\} (A9)
$$

where we have defined:

$$
I_A = I_{11}^C \tag{A10a}
$$

$$
I_B = I_{31}^C - \Delta_{xy} I_{21}^S \tag{A10b}
$$

$$
I_C = I_{33}^C - 2\Delta_{xy} I_{32}^S - \Delta_{xy}^2 I_{22}^C
$$
 (A10c)

These last integrals are easily computed, with the result

$$
I_A = 2\pi\delta(\Delta_{xy}) + \frac{\cos[2(\tilde{\Lambda} + 1)\Delta_{xy}] - \cos[2\tilde{\Lambda}\Delta_{xy}]}{\Delta_{xy}^2}, \text{(A11)}
$$

\n
$$
I_B = \cos[2\Delta_{xy}] \left\{ Ci[2(\tilde{\Lambda} + 2)|\Delta_{xy}|| - Ci[2\tilde{\Lambda}|\Delta_{xy}|| \right\} - \sin[2|\Delta_{xy}|| \left\{ \pi - Si[2(\tilde{\Lambda} + 2)|\Delta_{xy}|| - Si[2\tilde{\Lambda}|\Delta_{xy}|| \right\} + \frac{\sin[2(\tilde{\Lambda} + 1)\Delta_{xy}]}{\tilde{\Lambda}\Delta_{xy}} - \frac{\sin[2\tilde{\Lambda}\Delta_{xy}]}{\tilde{\Lambda}\Delta_{xy}}, \text{(A12)}
$$

$$
I_C = \left\{ \pi - Si[2(\tilde{\Lambda} + 2)|\Delta_{xy}|] - Si[2\tilde{\Lambda}|\Delta_{xy}|] \right\}
$$

\n
$$
\times \left(\cos[2\Delta_{xy}]|\Delta_{xy}| - \frac{\sin[2|\Delta_{xy}|]}{2} \right)
$$

\n
$$
+ \left\{ Ci[2(\tilde{\Lambda} + 2)|\Delta_{xy}|] - Ci[2\tilde{\Lambda}|\Delta_{xy}|] \right\}
$$

\n
$$
\times \left(\sin[2\Delta_{xy}]\Delta_{xy} + \frac{\cos[2\Delta_{xy}]}{2} \right)
$$

\n
$$
+ \frac{\cos[2\tilde{\Lambda}\Delta_{xy}]}{\tilde{\Lambda}^2} - \frac{\cos[2(\tilde{\Lambda} + 1)\Delta_{xy}]}{\tilde{\Lambda}}, \qquad (A13)
$$

where $\mathcal{S}i[x]$ and $\mathcal{C}i[x]$ are the sine and the cosine integral, respectively [\[34](#page-16-33)].

The Fourier transform in $\vec{k} = \vec{0}$ is much simpler to compute, and the result is

$$
ReG_{++}^{\Lambda 2}(\eta, \eta', \vec{0}) = \frac{-H^4 x^2 y^2}{2(2\pi)^2 k_0^3} \left\{ \frac{\pi}{2} \delta \left(\Delta_{xy} \right) \right.- \sin[2\tilde{\Lambda} \Delta_{xy}] \left(\frac{1}{2\Delta_{xy}} + \frac{2}{3} \frac{\Delta_{xy}}{\tilde{\Lambda}^2 x^2 y^2} \right) + \frac{\cos[2\tilde{\Lambda} \Delta_{xy}]}{\tilde{\Lambda}} \left(\frac{2}{xy} + \frac{\Delta_{xy}^2}{3x^2 y^2} + \frac{1}{3\tilde{\Lambda}^2 x^2 y^2} \right) - \left(\pi - 2\mathcal{S}i[2\tilde{\Lambda} |\Delta_{xy}|] \right) \frac{|\Delta_{xy}|}{xy} \left(\frac{\Delta_{xy}^2}{3xy} + 1 \right) \right\}
$$
(A14)

Substitution of Eqs. $(A9)$ and $(A14)$ into Eq. (27) yields an expression for D_2 that allows to evaluate it and its integration over the conformal time η numerically.

APPENDIX B: RENORMALIZATION

In this Appendix we obtain the renormalized semiclassical equation for the system-field given in Eq.[\(55\)](#page-9-3). To do so, we follow the same procedure as for the renormalization of the evolution equation for the mean value $\langle \phi \rangle$ [\[33\]](#page-16-32).

Starting with the bare action for the field ϕ , the semiclassical equation for the homogeneous system-field ϕ reads

$$
\phi''_{<}(\eta) + [m_0^2 + \xi_0 R] a^2(\eta) \phi_{<}(\eta) + 2\mathcal{H} \phi'_{<}(\eta) \n+4\lambda_0 a^2(\eta) \phi_{<}^3(\eta) - 12\lambda a^2(\eta) \phi_{<}(\eta) iG^{\Lambda}_{++}(\eta, \eta, \vec{0}) \n-288\lambda^2 a^2(\eta) \phi_{<}(\eta) \int_{\eta_i}^{\eta} d\eta' a^4(\eta') \int d^3y \phi_{<}^2(\eta') \n\times Im G^{\Lambda 2}_{++}(\eta, \eta', \vec{x} - \vec{y}) = -\xi_2(\eta, \vec{x}) \frac{\phi_{<}(\eta)}{a^2(\eta)},
$$
\n(B1)

where m_0 , ξ_0 and λ_0 are the bare constants, and $R =$ 12 H^2 . Here we have replaced λ_0 by the renormalized constant λ in the one-loop terms.

Let us add and substrate the following terms to the left-hand side of Eq.[\(B1\)](#page-15-0):

$$
12\lambda a^2(\eta)\phi_<(\eta)\langle \hat{\phi}_>^2 \rangle_{ad2} \tag{B2}
$$

where $\langle \hat{\phi}_>^2 \rangle_{ad2}$ is the adiabatic expansion of the expectation value $\langle \hat{\phi}^2_{\geq} \rangle$ up to the second adiabatic order [\[33\]](#page-16-32).

In what follows we will show how the infinities in Eq.[\(B1\)](#page-15-0) are cancelled by the terms subtracted. We will compute the divergent terms added via dimensional regularization, which will thus be able to be absorbed in the bare parameters as usual.

The expansion of $\langle \hat{\phi}_>^2 \rangle$ up to the second adiabatic order yields:

$$
\langle \phi^2 \rangle_{ad2} \equiv \langle \phi^2 \rangle_{ad2}^F + \langle \phi^2 \rangle_{ad2}^I,
$$
 (B3)

with

$$
\langle \phi_{>}^2 \rangle_{ad2}^F = \frac{a}{2} \int_{k>\Lambda} \frac{d^3k}{(2\pi a)^3} \left\{ \frac{1}{\omega_k} - \frac{\left(\xi_0 - \frac{1}{6}\right) a^2 R}{2\omega_k^3} \right\} \quad \text{(B4a)}
$$

$$
+ \frac{m_0^2}{4\omega_k^5} [a'^2 + aa''] - \frac{5m_0^4}{8\omega_k^7} (aa')^2 \right\},
$$

$$
\langle \phi_{>}^2 \rangle_{ad2}^I = -\frac{a}{2} \int_{k>\Lambda} \frac{d^3k}{(2\pi a)^3} \frac{6\lambda_0 a^2}{\omega_k^3} \phi_{<}^2. \quad \text{(B4b)}
$$

where $\omega_k^2 = k^2 + a^2 m_0^2$, and the integrations over the wave vector \vec{k} are restricted by $k > \Lambda$.

In order to use dimensional regularization, we perform the integrations above over all wave vectors with $k \geq 0$ and then we subtract the ones restricted by $k < \Lambda$ with the result

$$
\langle \phi^2 \rangle_{ad2}^F = -\frac{1}{4\pi^2} \frac{R}{12} \left[\frac{1}{n-4} + \frac{\gamma}{2} - \ln \left| \frac{a(\eta)\mu}{2\Lambda} \right| \right] - \frac{\Lambda^2}{8\pi^2 a^2(\eta)},
$$
 (B5a)

$$
\langle \phi_{>}^2 \rangle_{ad2}^I = \frac{3\lambda}{2\pi^2} \phi_{<}^2(\eta) \left[\frac{1}{n-4} + \frac{\gamma}{2} - \ln \left| \frac{a(\eta)\mu}{2\Lambda} \right| \right], \text{ (B5b)}
$$

where γ is the Euler's constant and we have replaced the bare parameters with their renormalized counterparts

 $(m = 0, \xi = 0 \text{ and } \lambda)$. It is important to notice that only after subtracting the bare parameters can be replaced by the renormalized ones, since one of the integrals over $k < \Lambda$ has an infrared logarithmic divergence.

Writing the bare parameters in terms of the renormalized ones plus conterterms,

$$
m_0^2 = 0 + \delta m^2 \ , \ \xi_0 = 0 + \delta \xi \ , \ \lambda_0 = \lambda + \delta \lambda,
$$

we can see that the divergences appearing for $n \to 4$ are cancelled with the use of the following counterterms:

$$
\delta m^2 = \Delta m^2,\tag{B6a}
$$

$$
\delta \xi = \frac{\lambda}{4\pi^2} \left[\frac{1}{n-4} \right] + \Delta \xi, \tag{B6b}
$$

$$
\delta \lambda = -\frac{9\lambda^2}{2\pi^2} \left[\frac{1}{n-4} \right] + \Delta \lambda, \tag{B6c}
$$

where Δm^2 , $\Delta \xi$ and $\Delta \lambda$ remain finite as $n \to 4$.

Replacing the bare parameters by the renormalized ones in the integrals over $k > \Lambda$ of Eq.[\(B4\)](#page-15-1) we obtain

$$
\langle \phi^2 \rangle_{ad2}^F = -iG_{++}^{\Lambda}(\eta, \eta, \vec{0}), \tag{B7a}
$$

$$
\langle \phi^2 \rangle_{ad2}^I = -\frac{3\lambda}{2\pi^2} \int_{k>\Lambda} \frac{dk}{k} \phi^2 0(\eta).
$$
 (B7b)

From equation above it is simple to note that the order λ contribution in Eq.[\(B1\)](#page-15-0) is completely cancelled by the one with $\langle \phi^2 \rangle_{ad2}^F$ (see Eq.[\(B2\)](#page-15-2)).

In order to separate the divergent part from the order λ^2 contribution of Eq.[\(B1\)](#page-15-0), we write the propagator as

$$
\int d^3y Im G^{\Lambda 2}_{++}(\eta, \eta', \vec{x}, \vec{y}) = -(2\pi)^3 \int_{k>\Lambda} d^3k Im[\tilde{\phi}_{\vec{k}}^2 \tilde{\phi}_{\vec{k}}^{*2}]
$$

=
$$
- \int_{k>\Lambda} \frac{dk}{8\pi^2} \left\{ \frac{\cos[2k(\eta - \eta')]}{a^2(\eta)a^2(\eta')} \left[\frac{2(\eta - \eta')}{k\eta\eta'} + \frac{2(\eta - \eta')}{k^3\eta^2\eta'^2} \right] - \frac{\sin[2k(\eta - \eta')]}{a^2(\eta)a^2(\eta')} \left[1 + \frac{1}{k^4\eta^2\eta'^2} + \frac{2}{k^2\eta\eta'} - \frac{(\eta - \eta')^2}{k^2\eta^2\eta'^2} \right] \right\}
$$

\equiv $I_D + I_{ND}$,

where $\tilde{\phi}_{\vec{k}}$ and $\tilde{\phi}_{\vec{k}}^*$ are the mode function and its complex conjugate respectively, given in Eq.[\(A1\)](#page-13-2). The only divergent term is

$$
I_D \equiv \int_{k>\Lambda} \frac{dk}{8\pi^2} \frac{\sin[2k(\eta - \eta')] }{a^2(\eta)a^2(\eta')}.
$$
 (B8)

With the use of the definition of I_D and I_{ND} , we can write the order λ^2 contribution as

$$
- 288 \lambda^2 a^2(\eta) \int_{\eta_i}^{\eta} d\eta' a^4(\eta') \phi_<(\eta) \int d^3 y \phi_<^2(\eta') Im G^{\Lambda 2}_{++}
$$

\n
$$
\equiv 12 \lambda a^2(\eta) \phi_<(\eta) \left([\langle \phi_>^2 \rangle]_D^I + [\langle \phi_>^2 \rangle]_{ND}^I \right),
$$
 (B9)

with

$$
\begin{split}\n& [\langle \phi_{>}^2 \rangle]_D^I = -24\lambda \int_{\eta_i}^{\eta} d\eta' \ a^4(\eta') \phi_{<}^2(\eta') I_D \\
&= -\frac{3\lambda}{\pi^2} \int_{\eta_i}^{\eta} d\eta' \frac{a^2(\eta') \phi_{<}^2(\eta')}{a^2(\eta)} \int_{k>\Lambda} dk \ \sin[2k(\eta - \eta')] \\
&= -\frac{3\lambda}{2\pi^2} \phi_{<}^2(\eta) \int_{k>\Lambda} \frac{dk}{k} + \frac{3\lambda}{2\pi^2} \frac{a^2(\eta_i) \phi_{<}^2(\eta_i)}{a^2(\eta)} \\
&\times \int_{k>\Lambda} \frac{dk}{k} \cos[2k(\eta - \eta_i)] \\
&+ \frac{3\lambda}{2\pi^2} \int_{\eta_i}^{\eta} d\eta' \frac{(a^2(\eta') \phi_{<}^2(\eta'))'}{a^2(\eta)} \int_{k>\Lambda} \frac{dk}{k} \cos[2k(\eta - \eta')],\n\end{split}
$$

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where the last equality follows after performing an integration by parts. In this expression we can see that the first term after the last equality is $\langle \phi^2 \rangle_{ad2}^I$ and hence all infinities are cancelled. Finally, performing the I_{ND} integral and reordering the terms we obtain the explicit form of the renormalized equation given in Eq.[\(55\)](#page-9-3).

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