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## Dimensionally Regulated Graviton 1-Point Function in de Sitter

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### ABSTRACT

We use dimensional regularization to compute the 1PI 1-point function of quantum gravity at one loop order in a locally de Sitter background. As with other computations, the result is a finite constant at this order. It corresponds to a small positive renormalization of the cosmological constant.

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# 1 Introduction

Since its inception, dimensional regularization [1, 2] has been an extraordinarily useful technique because it preserves continuous symmetries that do not depend upon the special properties of a certain dimension. Although its use is ubiquitous in flat space, the technique is not so simply applied in curved backgrounds because one must know the propagators in an arbitrary dimension. This is no problem for determining divergences [3, 4, 5, 6, 7] because these are universal, but it can be a real problem for extracting the finite parts of the 1-Particle-Irreducible (1PI) functions which incorporate all information about a quantum field theory.

The difficulty appears even in as simple a background as de Sitter for as simple a diagram as the 1PI graviton 1-point function. Up to some factors this is the same quantity which is often termed the expectation value of the graviton stress-tensor. It represents the back-reaction of virtual gravitons upon the (de Sitter) background. Ford was the first to compute it at one loop order in the context of a general search for secular infrared corrections to the effective cosmological constant [8]. Owing to the inherently infrared character of the effect he sought, Ford worked in  $D = 3 + 1$  dimensions and evaluated only a certain part of the graviton stress tensor canonically in a physical gauge. He found a small, time independent, negative shift of the cosmological constant.

Finelli, Marozzi, Vacca and Venturi have recently computed the full graviton stress tensor using adiabatic regularization [9]. Although their result is also independent of time, they find a small positive shift of the cosmological constant. Because they computed slightly different things it is not clear there is any disagreement between this result and Ford's. In any case, both effects can be absorbed into counterterms, and *must* be so absorbed if the universe is to inflate at the background Hubble constant.

The purpose of this paper is to exploit a new result for the  $D$ -dimensional graviton propagator in a locally de Sitter background [10] to compute the one loop graviton 1-point function (Fig. 1) using dimensional regularization. The point is not to check previous results but rather to test the new formalism in a setting where we know what it should give: namely, a finite, time independent renormalization of the cosmological constant. Although this is, of necessity, a technical paper, there are two important physical motivations for developing the new formalism. We shall digress briefly to explain these before becoming immersed in technicalities.

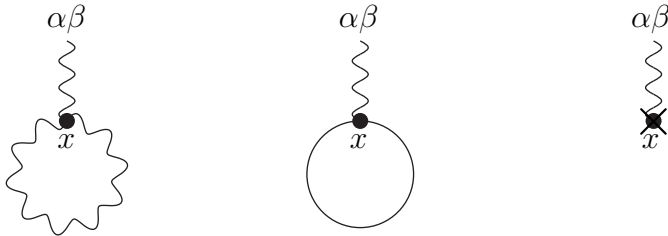


Fig. 1: One loop contributions to the graviton 1-point function.

Our first motivation is to extend Ford’s search past one loop order. The genesis of back-reaction during inflation (for which de Sitter is a paradigm) is that the expansion of spacetime continually rips long wavelength gravitons out of the vacuum. There is no secular back-reaction at one loop because the enormous growth of the total energy of these gravitons is canceled by the inflationary expansion of the 3-volume. Hence the one loop energy density should amount to only a positive constant. At the next order there must be gravitational interactions *between* the newly produced gravitons. Whereas the one loop effect is a constant — which must be subsumed into a cosmological counterterm for the universe to begin inflation at the background Hubble constant — the two loop effect should grow because each newly emerged graviton experiences the gravitational fields of all the gravitons produced within its past light-cone [11]. A decade-old computation of the 1PI graviton 1-point function at two loop order does indicate such an effect [12] but this calculation had to be done in  $D = 3 + 1$  dimensions using a cutoff on the co-moving 3-momentum. It is important to make the computation with an invariant regularization in order to ensure that spurious secular behavior is not being injected by what is effectively a time dependent ultraviolet cut-off. If the expected secular back-reaction occurs it could lead to a realistic model of inflation in which the (old) problem of the cosmological constant is resolved [13].

Our second purpose is to facilitate a general study of graviton-mediated, quantum effects during inflation. Massless, minimally coupled (MMC) scalars and gravitons are unique in achieving masslessness without classical conformal invariance. This allows both particles to be produced copiously during inflation, which is the ultimate source of primordial cosmological scalar [14] and tensor [15] perturbations. It has recently been realized that interactions involving even a single, undifferentiated MMC scalar can result in vastly strengthened quantum effects during inflation. Reliable, dimensionally reg-

ulated results have been obtained in three models:

1. For a MMC scalar with a quartic self-interaction both the expectation value of the stress tensor [16, 17] and the self-mass-squared [18] have been evaluated at one and two loop orders. This model shows a violation of the weak energy condition in which inflationary particle production drives the scalar up its potential and induces a curious sort of time-dependent mass.
2. When a complex MMC scalar is coupled to electromagnetism it has been possible to compute the one loop vacuum polarization [19, 20] and use the result to solve the quantum corrected Maxwell equations [21]. Although photon creation is suppressed during inflation, this model shows a vast enhancement of the 0-point energy of super-horizon photons which may serve to seed cosmological magnetic fields [22, 23, 24].
3. When a real MMC scalar is Yukawa coupled to a massless Dirac fermion it has been possible to compute the one loop fermion self-energy and use it to solve the quantum corrected Dirac equation [25]. The resulting model shows explosive creation of fermions. A recent one loop computation of the scalar self-mass-squared indicates that the scalar cannot develop a large enough mass quickly enough to prevent the super-horizon fermion modes from becoming fully populated [26].

Analogous graviton effects should be suppressed by the higher dimension of the respective couplings. On the other hand, they should be universal. In particular, graviton-mediated effects do not depend upon the existence of a minimally coupled scalar with an unnaturally light mass.

Having motivated the exercise, we close this introduction with an outline. In section 2 we work out the Feynman rules. The actual computation is described in section 3 and our conclusions are presented in section 4.

## 2 Feynman Rules

The purpose of this section is to give the Feynman rules necessary for evaluating the diagrams of Fig. 1. We begin by expressing the invariant action in terms of a conformally rescaled graviton field. At this point it is simple to read off the various 3-graviton vertex operators needed for the first diagram. In order to get the propagators we fix the gauge with a convenient

variant of the de Donder gauge fixing term of flat space. That determines the ghost and graviton propagators. We close with the graviton-ghost-anti-ghost vertex operators.

In  $D$  spacetime dimensions the Einstein-Hilbert Lagrangian is,

$$\mathcal{L} = \frac{1}{16\pi G} \left( R - (D-2)\Lambda \right) \sqrt{-g}. \quad (1)$$

The unique, maximally symmetric solution for positive  $\Lambda$  is known as de Sitter space. In order to regard this as a paradigm for inflation we work on a portion of the full de Sitter manifold known as the open conformal coordinate patch. The invariant element for this is,

$$ds^2 = a^2 \left( -d\eta^2 + d\vec{x} \cdot d\vec{x} \right) \quad \text{where} \quad a(\eta) = -\frac{1}{H\eta}, \quad (2)$$

and the  $D$ -dimensional Hubble constant is  $H \equiv \sqrt{\Lambda/(D-1)}$ . Note that the conformal time  $\eta$  runs from  $-\infty$  to zero.

We define the graviton field  $h_{\mu\nu}(x)$  as the perturbation of the conformally rescaled metric,

$$g_{\mu\nu}(x) \equiv a^2 \left( \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) \right) \equiv a^2 \tilde{g}_{\mu\nu}, \quad (3)$$

where  $\kappa^2 \equiv 16\pi G$  is the loop-counting parameter of quantum gravity. By convention, graviton indices are raised and lowered with the Lorentz metric:  $h^\mu{}_\nu \equiv \eta^{\mu\rho} h_{\rho\nu}$ ,  $h^{\mu\nu} \equiv \eta^{\mu\rho} \eta^{\nu\sigma} h_{\rho\sigma}$  and  $h \equiv \eta^{\mu\nu} h_{\mu\nu}$ . However,  $\tilde{g}^{\mu\nu}$  denotes the full matrix inverse of  $\tilde{g}_{\mu\nu}$ ,

$$\tilde{g}^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^\mu{}_\rho h^{\rho\nu} - \dots \quad (4)$$

With these conventions we can extract a surface term from the invariant Lagrangian and write it in the form [27],

$$\begin{aligned} \mathcal{L} - \text{Surface} &= \left(\frac{D}{2}-1\right) H a^{D-1} \sqrt{-\tilde{g}} \tilde{g}^{\rho\sigma} \tilde{g}^{\mu\nu} h_{\rho\sigma,\mu} h_{\nu 0} \\ &+ a^{D-2} \sqrt{-\tilde{g}} \tilde{g}^{\alpha\beta} \tilde{g}^{\rho\sigma} \tilde{g}^{\mu\nu} \left\{ \frac{1}{2} h_{\alpha\rho,\mu} h_{\beta\sigma,\nu} - \frac{1}{2} h_{\alpha\beta,\rho} h_{\sigma\mu,\nu} + \frac{1}{4} h_{\alpha\beta,\rho} h_{\mu\nu,\sigma} - \frac{1}{4} h_{\alpha\rho,\mu} h_{\beta\sigma,\nu} \right\}. \end{aligned} \quad (5)$$

We can read the graviton 3-point interaction off from expression (5),

$$\begin{aligned} \mathcal{L}^{(3)} &= \left(\frac{D}{2}-1\right) \kappa H a^{D-1} \left\{ \frac{1}{2} h h_{,\mu} h^{\mu 0} - h_{\alpha\beta} h^{\alpha\beta,\mu} h_{\mu 0} - h^{\mu\nu} h_{,\mu} h_{\nu 0} \right\} \\ &+ \kappa a^{D-2} \left\{ \frac{1}{4} h h^{\alpha\beta,\mu} h_{\mu\alpha,\beta} - h^{\alpha\beta} h_{\alpha\mu,\nu} h^{\mu\nu}{}_{,\beta} - \frac{1}{2} h^{\alpha\beta} h_{\alpha\mu}{}^{,\nu} h_{\beta\nu}{}^{,\mu} - \frac{1}{4} h h_{,\mu} h^{\mu\nu}{}_{,\nu} \right. \\ &+ \frac{1}{2} h^{\alpha\beta} h_{\alpha\beta,\mu} h^{\mu\nu}{}_{,\nu} + \frac{1}{2} h^{\alpha\beta} h_{,\alpha} h_{\mu\beta}{}^{,\mu} + \frac{1}{2} h^{\alpha\beta} h_{\alpha\mu,\beta} h^{\mu\nu} + \frac{1}{8} h h^{\mu\nu} h_{,\mu} - \frac{1}{2} h^{\alpha\beta} h_{\alpha\beta,\mu} h^{\mu\nu}{}_{,\nu} \\ &\left. - \frac{1}{4} h^{\alpha\beta} h_{,\alpha} h_{,\beta} - \frac{1}{8} h h^{\alpha\beta,\mu} h_{\alpha\beta,\mu} + \frac{1}{2} h^{\alpha\beta} h_{\alpha\mu,\nu} h_{\beta}{}^{\mu,\nu} + \frac{1}{4} h^{\alpha\beta} h_{\mu\nu,\alpha} h^{\mu\nu}{}_{,\beta} \right\}. \end{aligned} \quad (6)$$

In deriving the associated vertex operators we must account for the indistinguishability of gravitons. This would ordinarily be accomplished by fully symmetrizing each interaction, which turns out to give over 70 distinct terms. For the pure graviton loop in Fig. 1 it is wasteful to first sum over these possibilities and then divide by the symmetry factor of 2 to compensate for overcounting. The more efficient strategy is to symmetrize the vertex only on line #1 and dispense with the symmetry factor.

To obtain the partially symmetrized vertices one first takes any of the terms from (6) and permutes graviton #1 over the three possibilities. As an example, consider the term  $\frac{1}{4}\kappa a^{D-2} h h^{\alpha\beta,\mu} h_{\mu\alpha,\beta}$ . Denoting graviton #1 by a breve, we obtain the following three terms,

$$\frac{1}{4}\kappa a^{D-2} \breve{h} h^{\alpha\beta,\mu} h_{\mu\alpha,\beta} + \frac{1}{4}\kappa a^{D-2} \breve{h}_{\mu\alpha,\beta} h h^{\alpha\beta,\mu} + \frac{1}{4}\kappa a^{D-2} \breve{h}^{\alpha\beta,\mu} h_{\mu\alpha,\beta} h . \quad (7)$$

One then assigns the remaining two gravitons in each term as #2 and #3 in any way. For example, from (7) we could infer the following three vertex operators,

$$\begin{aligned} \frac{1}{4}\kappa a^{D-2} \eta^{\alpha_1\beta_1} \partial_3^{(\alpha_2} \eta^{\beta_2)} \partial_2^{\alpha_3} \partial_2^{\beta_3)} \quad , \quad & \frac{1}{4}\kappa a^{D-2} \eta^{\alpha_2\beta_2} \partial_1^{(\alpha_3} \eta^{\beta_3)} \partial_3^{\alpha_1} \partial_3^{\beta_1)} \\ \text{and} \quad & \frac{1}{4}\kappa a^{D-2} \eta^{\alpha_3\beta_3} \partial_2^{(\alpha_1} \eta^{\beta_1)} \partial_1^{\alpha_2} \partial_1^{\beta_2)} . \end{aligned} \quad (8)$$

These are Vertex Operators #10, #11 and #12, respectively, in Table 1.

Our gauge fixing term is an analogue of the de Donder term used in flat space [27],

$$\mathcal{L}_{GF} = -\frac{1}{2} a^{D-2} \eta^{\mu\nu} F_\mu F_\nu , \quad F_\mu \equiv \eta^{\rho\sigma} \left( h_{\mu\rho,\sigma} - \frac{1}{2} h_{\rho\sigma,\mu} + (D-2) H a h_{\mu\rho} \delta_\sigma^0 \right). \quad (9)$$

Because space and time components are treated differently it is useful to have an expression for the purely spatial parts of the Minkowski metric and the Kronecker delta,

$$\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta_\mu^0 \delta_\nu^0 \quad \text{and} \quad \bar{\delta}_\nu^\mu \equiv \delta_\nu^\mu - \delta_\nu^0 \delta_\nu^0 . \quad (10)$$

The quadratic part of  $\mathcal{L} + \mathcal{L}_{GF}$  can be partially integrated to take the form  $\frac{1}{2} h^{\mu\nu} D_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma}$ , where the kinetic operator is,

$$\begin{aligned} D_{\mu\nu}{}^{\rho\sigma} \equiv & \left\{ \frac{1}{2} \bar{\delta}_\mu^{(\rho} \bar{\delta}_\nu^{\sigma)} - \frac{1}{4} \eta_{\mu\nu} \eta^{\rho\sigma} - \frac{1}{2(D-3)} \delta_\mu^0 \delta_\nu^0 \delta_0^\rho \delta_0^\sigma \right\} D_A \\ & + \delta_{(\mu}^0 \bar{\delta}_{\nu)}^{(\rho} \delta_0^{\sigma)} D_B + \frac{1}{2} \left( \frac{D-2}{D-3} \right) \delta_\mu^0 \delta_\nu^0 \delta_0^\rho \delta_0^\sigma D_C , \end{aligned} \quad (11)$$

#	Vertex Operator	#	Vertex Operator
1	$\frac{(D-2)}{4}\kappa H a^{D-1}\eta^{\alpha_1\beta_1}\eta^{\alpha_2\beta_2}\partial_2^{(\alpha_3}\delta_0^{\beta_3)}$	22	$\frac{1}{2}\kappa a^{D-2}\eta^{\alpha_2(\alpha_3}\eta^{\beta_3)\beta_2}\partial_3^{(\alpha_1}\partial_1^{\beta_1)}$
2	$\frac{(D-2)}{4}\kappa H a^{D-1}\eta^{\alpha_2\beta_2}\eta^{\alpha_3\beta_3}\partial_3^{(\alpha_1}\delta_0^{\beta_1)}$	23	$\frac{1}{2}\kappa a^{D-2}\eta^{\alpha_3(\alpha_1}\eta^{\beta_1)\beta_3}\partial_1^{(\alpha_2}\partial_2^{\beta_2)}$
3	$\frac{(D-2)}{4}\kappa H a^{D-1}\eta^{\alpha_3\beta_3}\eta^{\alpha_1\beta_1}\partial_1^{(\alpha_2}\delta_0^{\beta_2)}$	24	$\frac{1}{2}\kappa a^{D-2}\partial_2^{(\alpha_1}\eta^{\beta_1)(\alpha_3}\partial_3^{\beta_3)}\eta^{\alpha_2\beta_2}$
4	$-\frac{(D-2)}{2}\kappa H a^{D-1}\eta^{\alpha_1(\alpha_2}\eta^{\beta_2)\beta_1}\partial_2^{(\alpha_3}\delta_0^{\beta_3)}$	25	$\frac{1}{2}\kappa a^{D-2}\partial_3^{(\alpha_2}\eta^{\beta_2)(\alpha_1}\partial_1^{\beta_1)}\eta^{\alpha_3\beta_3}$
5	$-\frac{(D-2)}{2}\kappa H a^{D-1}\eta^{\alpha_2(\alpha_3}\eta^{\beta_3)\beta_2}\partial_3^{(\alpha_1}\delta_0^{\beta_1)}$	26	$\frac{1}{2}\kappa a^{D-2}\partial_1^{(\alpha_3}\eta^{\beta_3)(\alpha_2}\partial_2^{\beta_2)}\eta^{\alpha_1\beta_1}$
6	$-\frac{(D-2)}{2}\kappa H a^{D-1}\eta^{\alpha_3(\alpha_1}\eta^{\beta_1)\beta_3}\partial_1^{(\alpha_2}\delta_0^{\beta_2)}$	27	$\frac{1}{2}\kappa a^{D-2}\partial_2^{(\alpha_1}\eta^{\beta_1)(\alpha_2}\partial_3^{\beta_3)}\eta^{\alpha_3\beta_3}$
7	$-\frac{(D-2)}{2}\kappa H a^{D-1}\delta_0^{(\alpha_3}\eta^{\beta_3)(\alpha_1}\partial_2^{\beta_1)}\eta^{\alpha_2\beta_2}$	28	$\frac{1}{2}\kappa a^{D-2}\partial_3^{(\alpha_2}\eta^{\beta_2)(\alpha_3}\partial_1^{\beta_3)}\eta^{\alpha_1\beta_1}$
8	$-\frac{(D-2)}{2}\kappa H a^{D-1}\delta_0^{(\alpha_1}\eta^{\beta_1)(\alpha_2}\partial_3^{\beta_2)}\eta^{\alpha_3\beta_3}$	29	$\frac{1}{2}\kappa a^{D-2}\partial_1^{(\alpha_3}\eta^{\beta_3)(\alpha_1}\partial_2^{\beta_1)}\eta^{\alpha_2\beta_2}$
9	$-\frac{(D-2)}{2}\kappa H a^{D-1}\delta_0^{(\alpha_2}\eta^{\beta_2)(\alpha_3}\partial_1^{\beta_3)}\eta^{\alpha_1\beta_1}$	30	$\frac{1}{8}\kappa a^{D-2}\eta^{\alpha_1\beta_1}\eta^{\alpha_2\beta_2}\eta^{\alpha_3\beta_3}\partial_2 \cdot \partial_3$
10	$\frac{1}{4}\kappa a^{D-2}\eta^{\alpha_1\beta_1}\partial_3^{(\alpha_2}\eta^{\beta_2)(\alpha_3}\partial_2^{\beta_3)}$	31	$\frac{1}{4}\kappa a^{D-2}\eta^{\alpha_1\beta_1}\eta^{\alpha_2\beta_2}\eta^{\alpha_3\beta_3}\partial_3 \cdot \partial_1$
11	$\frac{1}{4}\kappa a^{D-2}\eta^{\alpha_2\beta_2}\partial_1^{(\alpha_3}\eta^{\beta_3)(\alpha_1}\partial_3^{\beta_1)}$	32	$-\frac{1}{2}\kappa a^{D-2}\eta^{\alpha_1(\alpha_2}\eta^{\beta_2)\beta_1}\eta^{\alpha_3\beta_3}\partial_2 \cdot \partial_3$
12	$\frac{1}{4}\kappa a^{D-2}\eta^{\alpha_3\beta_3}\partial_2^{(\alpha_1}\eta^{\beta_1)(\alpha_2}\partial_1^{\beta_2)}$	33	$-\frac{1}{2}\kappa a^{D-2}\eta^{\alpha_2(\alpha_3}\eta^{\beta_3)\beta_2}\eta^{\alpha_1\beta_1}\partial_3 \cdot \partial_1$
13	$-\kappa a^{D-2}\partial_3^{(\alpha_1}\eta^{\beta_1)(\alpha_2}\eta^{\beta_2)(\alpha_3}\partial_2^{\beta_3)}$	34	$-\frac{1}{2}\kappa a^{D-2}\eta^{\alpha_3(\alpha_1}\eta^{\beta_1)\beta_3}\eta^{\alpha_2\beta_2}\partial_1 \cdot \partial_2$
14	$-\kappa a^{D-2}\partial_1^{(\alpha_2}\eta^{\beta_2)(\alpha_3}\eta^{\beta_3)(\alpha_1}\partial_3^{\beta_1)}$	35	$-\frac{1}{4}\kappa a^{D-2}\partial_2^{(\alpha_1}\partial_3^{\beta_1)}\eta^{\alpha_2\beta_2}\eta^{\alpha_3\beta_3}$
15	$-\kappa a^{D-2}\partial_2^{(\alpha_3}\eta^{\beta_3)(\alpha_1}\eta^{\beta_1)(\alpha_2}\partial_1^{\beta_2)}$	36	$-\frac{1}{2}\kappa a^{D-2}\partial_3^{(\alpha_2}\partial_1^{\beta_2)}\eta^{\alpha_3\beta_3}\eta^{\alpha_1\beta_1}$
16	$-\frac{1}{2}\kappa a^{D-2}\partial_3^{(\alpha_2}\eta^{\beta_2)(\alpha_1}\eta^{\beta_1)(\alpha_3}\partial_2^{\beta_3)}$	37	$-\frac{1}{8}\kappa a^{D-2}\eta^{\alpha_1\beta_1}\eta^{\alpha_2(\alpha_3}\eta^{\beta_3)\beta_2}\partial_2 \cdot \partial_3$
17	$-\kappa a^{D-2}\partial_1^{(\alpha_3}\eta^{\beta_3)(\alpha_2}\eta^{\beta_2)(\alpha_1}\partial_3^{\beta_1)}$	38	$-\frac{1}{4}\kappa a^{D-2}\eta^{\alpha_2\beta_2}\eta^{\alpha_3(\alpha_1}\eta^{\beta_1)\beta_3}\partial_3 \cdot \partial_1$
18	$-\frac{1}{4}\kappa a^{D-2}\eta^{\alpha_1\beta_1}\eta^{\alpha_2\beta_2}\partial_2^{(\alpha_3}\partial_3^{\beta_3)}$	39	$\frac{1}{2}\kappa a^{D-2}\eta^{\alpha_1(\alpha_2}\eta^{\beta_2)(\alpha_3}\eta^{\beta_3)(\beta_1}\partial_2 \cdot \partial_3$
19	$-\frac{1}{4}\kappa a^{D-2}\eta^{\alpha_2\beta_2}\eta^{\alpha_3\beta_3}\partial_3^{(\alpha_1}\partial_1^{\beta_1)}$	40	$\kappa a^{D-2}\eta^{\alpha_1(\alpha_2}\eta^{\beta_2)(\alpha_3}\eta^{\beta_3)(\beta_1}\partial_3 \cdot \partial_1$
20	$-\frac{1}{4}\kappa a^{D-2}\eta^{\alpha_3\beta_3}\eta^{\alpha_1\beta_1}\partial_1^{(\alpha_2}\partial_2^{\beta_2)}$	41	$\frac{1}{4}\kappa a^{D-2}\partial_2^{(\alpha_1}\partial_3^{\beta_1)}\eta^{\alpha_2(\alpha_3}\eta^{\beta_3)\beta_2}$
21	$\frac{1}{2}\kappa a^{D-2}\eta^{\alpha_1(\alpha_2}\eta^{\beta_2)\beta_1}\partial_2^{(\alpha_3}\partial_3^{\beta_3)}$	42	$\frac{1}{2}\kappa a^{D-2}\partial_3^{(\alpha_2}\partial_1^{\beta_2)}\eta^{\alpha_3(\alpha_1}\eta^{\beta_1)\beta_3}$

Table 1: Vertex operators contracted into  $h_{\alpha_1\beta_1}h_{\alpha_2\beta_2}h_{\alpha_3\beta_3}$  with  $h_{\alpha_1\beta_1}$  external.

and the three scalar differential operators are,

$$D_A \equiv \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) , \quad (12)$$

$$D_B \equiv \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) - \frac{1}{D} \left( \frac{D-2}{D-1} \right) R \sqrt{-g} , \quad (13)$$

$$D_C \equiv \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) - \frac{2}{D} \left( \frac{D-3}{D-1} \right) R \sqrt{-g} . \quad (14)$$

The associated ghost Lagrangian is,

$$\mathcal{L}_{\text{gh}} \equiv -a^{D-2} \bar{\omega}^\mu \delta F_\mu , \quad (15)$$

$$\begin{aligned} &= \bar{\omega}^\mu \left( \bar{\delta}_\mu^\nu D_A + \delta_\mu^0 \delta_0^\nu D_B \right) \omega_\nu - 2\kappa a^{D-2} \bar{\omega}^{\mu,\nu} \left( h^\rho_{(\mu} \partial_{\nu)} + \frac{1}{2} h_{\mu\nu}{}^{,\rho} - H a h_{\mu\nu} \delta_0^\rho \right) \omega_\rho \\ &\quad + \kappa \left( a^{D-2} \bar{\omega}^\mu \right)_{,\mu} \left( h^{\rho\sigma} \partial_\sigma + \frac{1}{2} h_\sigma{}^{\sigma,\rho} - H a h \delta_0^\rho \right) \omega_\rho . \end{aligned} \quad (16)$$

The ghost and graviton propagators in this gauge take the form of a sum of constant index factors times scalar propagators,

$$i \left[ {}_\mu \Delta_\nu \right] (x; x') = \bar{\eta}_{\mu\nu} i \Delta_A (x; x') - \delta_\mu^0 \delta_\nu^0 i \Delta_B (x; x') , \quad (17)$$

$$i \left[ {}_{\mu\nu} \Delta_{\rho\sigma} \right] (x; x') = \sum_{I=A,B,C} \left[ {}_{\mu\nu} T_{\rho\sigma}^I \right] i \Delta_I (x; x') . \quad (18)$$

The three scalar propagators invert the various scalar kinetic operators,

$$D_I \times i \Delta_I (x; x') = i \delta^D (x - x') \quad \text{for} \quad I = A, B, C , \quad (19)$$

and we will presently give explicit expressions for them. The index factors in the graviton propagator are,

$$\left[ {}_{\mu\nu} T_{\rho\sigma}^A \right] = 2 \bar{\eta}_{\mu(\rho} \bar{\eta}_{\sigma)\nu} - \frac{2}{D-3} \bar{\eta}_{\mu\nu} \bar{\eta}_{\rho\sigma} , \quad (20)$$

$$\left[ {}_{\mu\nu} T_{\rho\sigma}^B \right] = -4 \delta_{(\mu}^0 \bar{\eta}_{\nu)(\rho} \delta_{\sigma)}^0 , \quad (21)$$

$$\left[ {}_{\mu\nu} T_{\rho\sigma}^C \right] = \frac{2}{(D-2)(D-3)} \left[ (D-3) \delta_\mu^0 \delta_\nu^0 + \bar{\eta}_{\mu\nu} \right] \left[ (D-3) \delta_\rho^0 \delta_\sigma^0 + \bar{\eta}_{\rho\sigma} \right] . \quad (22)$$

With these definitions and equation (19) for the scalar propagators it is straightforward to verify that the graviton propagator (18) indeed inverts the gauge-fixed kinetic operator,

$$D_{\mu\nu}{}^{\rho\sigma} \times i \left[ {}_{\rho\sigma} \Delta^{\alpha\beta} \right] (x; x') = \delta_\mu^{(\alpha} \delta_\nu^{\beta)} i \delta^D (x - x') . \quad (23)$$



The scalar propagators can be expressed in terms of the following function of the invariant length  $\ell(x; x')$  between  $x^\mu$  and  $x'^\mu$ ,

$$y(x; x') \equiv 4 \sin^2\left(\frac{1}{2}H\ell(x; x')\right) = aa'H^2\left(\|\vec{x}-\vec{x}'\|^2 - (|\eta-\eta'| - i\delta)^2\right). \quad (24)$$

The most singular term for each case is the propagator for a massless, conformally coupled scalar[28],

$$i\Delta_{\text{cf}}(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Gamma\left(\frac{D}{2}-1\right)\left(\frac{4}{y}\right)^{\frac{D}{2}-1}. \quad (25)$$

The  $A$ -type propagator obeys the same equation as that of a massless, minimally coupled scalar. It has long been known that no de Sitter invariant solution exists [29]. If one elects to break de Sitter invariance while preserving homogeneity and isotropy (this is known as the ‘‘E(3)’’ vacuum [30]), the minimal solution is [16, 17],

$$\begin{aligned} i\Delta_A(x; x') &= i\Delta_{\text{cf}}(x; x') \\ &+ \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\left\{\frac{D}{D-4}\frac{\Gamma^2(\frac{D}{2})}{\Gamma(D-1)}\left(\frac{4}{y}\right)^{\frac{D}{2}-2}\pi\cot\left(\frac{\pi}{2}D\right)+\ln(aa')\right\} \\ &+ \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\sum_{n=1}^{\infty}\left\{\frac{1}{n}\frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})}\left(\frac{y}{4}\right)^n - \frac{1}{n-\frac{D}{2}+2}\frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)}\left(\frac{y}{4}\right)^{n-\frac{D}{2}+2}\right\}. \end{aligned} \quad (26)$$

The  $B$ -type and  $C$ -type propagators possess de Sitter invariant (and also unique) solutions [10],

$$\begin{aligned} i\Delta_B(x; x') &= i\Delta_{\text{cf}}(x; x') - \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\sum_{n=0}^{\infty}\left\{\frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})}\left(\frac{y}{4}\right)^n \right. \\ &\quad \left. - \frac{\Gamma(n+\frac{D}{2})}{\Gamma(n+2)}\left(\frac{y}{4}\right)^{n-\frac{D}{2}+2}\right\}, \end{aligned} \quad (27)$$

$$\begin{aligned} i\Delta_C(x; x') &= i\Delta_{\text{cf}}(x; x') + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\sum_{n=0}^{\infty}\left\{(n+1)\frac{\Gamma(n+D-3)}{\Gamma(n+\frac{D}{2})}\left(\frac{y}{4}\right)^n \right. \\ &\quad \left. - \left(n-\frac{D}{2}+3\right)\frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+2)}\left(\frac{y}{4}\right)^{n-\frac{D}{2}+2}\right\}. \end{aligned} \quad (28)$$

These expressions might seem daunting but they are actually simple to use because the infinite sums vanish in  $D = 4$ , and each term in these sums

#	Vertex Operator	#	Vertex Operator
1	$-\kappa a^{D-2} \eta^{\alpha_2(\alpha_1 \eta^{\beta_1}) \alpha_3} \partial_2 \cdot \partial_3$	6	$\frac{1}{2} \kappa a^{D-2} \eta^{\alpha_1 \beta_1} \partial_2^{\alpha_2} \partial_1^{\alpha_3}$
2	$-\kappa a^{D-2} \eta^{\alpha_3(\alpha_1 \partial_2^{\beta_1})} \partial_3^{\alpha_2}$	7	$-\kappa H a^{D-1} \eta^{\alpha_1 \beta_1} \partial_2^{\alpha_2} \delta_0^{\alpha_3}$
3	$-\kappa a^{D-2} \eta^{\alpha_2(\alpha_1 \partial_2^{\beta_1})} \partial_1^{\alpha_3}$	8	$-(D-2) \kappa H a^{D-1} \eta^{\alpha_3(\alpha_1 \partial_3^{\beta_1})} \delta_0^{\alpha_2}$
4	$2 \kappa H a^{D-1} \eta^{\alpha_2(\alpha_1 \partial_2^{\beta_1})} \delta_0^{\alpha_3}$	9	$-\frac{(D-2)}{2} \kappa H a^{D-1} \eta^{\alpha_1 \beta_1} \partial_1^{\alpha_3} \delta_0^{\alpha_2}$
5	$\kappa a^{D-2} \eta^{\alpha_3(\alpha_1 \partial_3^{\beta_1})} \partial_2^{\alpha_2}$	10	$(D-2) \kappa H^2 a^D \eta^{\alpha_1 \beta_1} \delta_0^{\alpha_2} \delta_0^{\alpha_3}$

Table 2: Vertex operators contracted into  $h_{\alpha_1 \beta_1} \bar{\omega}_{\alpha_2} \omega_{\alpha_3}$ .

goes like a positive power of  $y(x; x')$ . This means the infinite sums can only contribute when multiplied by a divergent term, and even then only a small number of terms can contribute. Note also that the  $B$ -type and  $C$ -type propagators agree with the conformal propagator in  $D = 4$ .

The graviton-ghost-anti-ghost vertex operators can be read off from the order  $\kappa$  terms of  $\mathcal{L}_{\text{gh}}$  in expression (16). Because the three fields are distinct there is no need for symmetrization. Table 2 gives the ten vertex operators which result.

The final diagram in Fig. 1 represents a renormalization of the cosmological constant. We compute it by expanding the relevant counterterm to first order in the graviton field,

$$-\frac{(D-2)\delta\Lambda}{16\pi G} \sqrt{-g} = -\frac{(D-2)\delta\Lambda a^D}{\kappa^2} \left(1 + \frac{1}{2}\kappa h + \dots\right). \quad (29)$$

Hence the final diagram of Fig. 1 makes the following contribution,

$$-\frac{i(\frac{D}{2}-1)\delta\Lambda a^D}{\kappa} \eta^{\alpha\beta}. \quad (30)$$

By writing the sum of the first two diagrams in this form we can express our final result as a graviton stress tensor for comparison with the computations of Ford [8] and Finelli, Marozzi, Venturi and Vacca [9].

### 3 The Computation

The purpose of this section is to describe the calculation. We begin by explaining generally how one assembles the components of the previous section

to evaluate the first two diagrams of Fig. 1. We next give the four contractions of each of the three index factors in the graviton propagator. We also give the results of taking the coincidence limits of zero, one and two derivatives of each of the three scalar propagators. The graviton vertex operators turn out to possess a simple structure when organized into ten groups. A representative of each group is reduced. Finally, the sum is taken and shown to give a small, positive shift in the cosmological constant.

The graviton loop (first diagram of Fig. 1) consists of a sum of the coincidence limits of ( $i$  times) the vertex operators from Table 1 acting on the graviton propagator,

$$\left(\text{Graviton Loop}\right)^{\alpha\beta} = \sum_{i=1}^{42} \lim_{x' \rightarrow x} iV_i^{\alpha\beta\mu\nu\rho\sigma} \times i\left[\mu\nu\Delta_{\rho\sigma}\right](x; x'). \quad (31)$$

For example, the contribution from Vertex Operator #1 in Table 1 is,

$$\left(\text{Graviton Loop}\right)_1^{\alpha\beta} = \lim_{x' \rightarrow x} i\left(\frac{D-2}{4}\right)\kappa Ha^{D-1}\eta^{\alpha\beta}\eta^{\mu\nu}\partial'^{\rho}\delta_0^{\sigma} \times i\left[\mu\nu\Delta_{\rho\sigma}\right](x; x'). \quad (32)$$

Similarly, the ghost loop (second diagram of Fig. 1) consists of minus the sum of the coincidence limits of ( $i$  times) the vertex operators from Table 2 acting on the ghost propagator,

$$\left(\text{Ghost Loop}\right)^{\alpha\beta} = -\sum_{i=1}^{10} \lim_{x' \rightarrow x} iV_i^{\alpha\beta\mu\nu} \times i\left[\mu\Delta_{\nu}\right](x; x'). \quad (33)$$

For example, the contribution from Vertex Operator #1 on Table 2 is,

$$\left(\text{Ghost Loop}\right)_1^{\alpha\beta} = \lim_{x' \rightarrow x} i\kappa a^{D-2}\eta^{\alpha\mu}\eta^{\beta\nu}\partial\cdot\partial' \times i\left[\mu\Delta_{\nu}\right](x; x'). \quad (34)$$

The only subtle point is that derivatives with respect to the external line must be partially integrated back on the entire diagram. For example, the contribution from Vertex Operator #42 of Table 1 is,

$$\left(\text{Graviton Loop}\right)_{42}^{\alpha\beta} = -\partial^{\mu} \left\{ \lim_{x' \rightarrow x} \frac{i}{2}\kappa a^{D-2}\eta^{\alpha\rho}\eta^{\beta\sigma}\partial^{\nu} \times i\left[\mu\nu\Delta_{\rho\sigma}\right](x; x') \right\}. \quad (35)$$

From an examination of the vertex operators in Table 1 it is apparent that we must take four generic contractions of the three index factors  $[\mu\nu T_{\rho\sigma}^I]$  which make up the graviton propagator,

$$\eta^{\alpha\rho}\eta^{\beta\sigma}\eta^{\mu\nu} \quad , \quad \eta^{\rho\sigma}\eta^{\mu\nu} \quad , \quad \eta^{\mu\rho}\eta^{\nu\sigma} \quad , \quad \eta^{\alpha\mu}\eta^{\nu\rho}\eta^{\sigma\beta}. \quad (36)$$

For the  $A$ -type index factor these contractions give,

$$\begin{aligned}
\eta^{\alpha\rho}\eta^{\beta\sigma}\eta^{\mu\nu} [\mu\nu T_{\sigma\rho}^A] &= -\frac{4}{D-3}\bar{\eta}^{\alpha\beta}, \\
\eta^{\rho\sigma}\eta^{\mu\nu} [\mu\nu T_{\sigma\rho}^A] &= -4\left(\frac{D-1}{D-3}\right), \\
\eta^{\mu\rho}\eta^{\nu\sigma} [\mu\nu T_{\sigma\rho}^A] &= (D^2-3D-2)\left(\frac{D-1}{D-3}\right), \\
\eta^{\alpha\mu}\eta^{\nu\rho}\eta^{\sigma\beta} [\mu\nu T_{\sigma\rho}^A] &= \left(\frac{D^2-3D-2}{D-3}\right)\bar{\eta}^{\alpha\beta}. \tag{37}
\end{aligned}$$

The four contractions of the  $B$ -type index factor are,

$$\begin{aligned}
\eta^{\alpha\rho}\eta^{\beta\sigma}\eta^{\mu\nu} [\mu\nu T_{\sigma\rho}^B] &= 0, \quad \eta^{\rho\sigma}\eta^{\mu\nu} [\mu\nu T_{\sigma\rho}^B] = 0, \\
\eta^{\mu\rho}\eta^{\nu\sigma} [\mu\nu T_{\sigma\rho}^B] &= 2(D-1), \quad \eta^{\alpha\mu}\eta^{\nu\rho}\eta^{\sigma\beta} [\mu\nu T_{\sigma\rho}^B] = -(D-1)\delta_0^\alpha\delta_0^\beta + \bar{\eta}^{\alpha\beta}. \tag{38}
\end{aligned}$$

And the four contractions of the  $C$ -type index factor give,

$$\begin{aligned}
\eta^{\alpha\rho}\eta^{\beta\sigma}\eta^{\mu\nu} [\mu\nu T_{\sigma\rho}^C] &= \frac{4}{(D-2)(D-3)} [(D-3)\delta_0^\alpha\delta_0^\beta + \bar{\eta}^{\alpha\beta}], \\
\eta^{\rho\sigma}\eta^{\mu\nu} [\mu\nu T_{\sigma\rho}^C] &= \frac{8}{(D-2)(D-3)}, \\
\eta^{\mu\rho}\eta^{\nu\sigma} [\mu\nu T_{\sigma\rho}^C] &= 2\frac{(D^2-5D+8)}{(D-2)(D-3)}, \\
\eta^{\alpha\mu}\eta^{\nu\rho}\eta^{\sigma\beta} [\mu\nu T_{\sigma\rho}^C] &= \frac{2}{(D-2)(D-3)} [-(D-3)^2\delta_0^\alpha\delta_0^\beta + \bar{\eta}^{\alpha\beta}]. \tag{39}
\end{aligned}$$

We also require the coincidence limits of zero, one or two derivatives acting on each of the scalar propagators. For the  $A$ -type propagator these are,

$$\lim_{x'\rightarrow x} i\Delta_A(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ -\pi \cot\left(\frac{\pi}{2}D\right) + 2\ln(a) \right\}, \tag{40}$$

$$\lim_{x'\rightarrow x} \partial_\mu i\Delta_A(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times Ha\delta_\mu^0, \tag{41}$$

$$\lim_{x'\rightarrow x} \partial_\mu \partial'_\nu i\Delta_A(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\left(\frac{D-1}{D}\right)H^2 g_{\mu\nu}. \tag{42}$$

The analogous coincidence limits for the  $B$ -type propagator are actually finite in  $D = 4$  dimensions,

$$\lim_{x' \rightarrow x} i\Delta_B(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\frac{1}{D-2}, \quad (43)$$

$$\lim_{x' \rightarrow x} \partial_\mu i\Delta_B(x; x') = 0, \quad (44)$$

$$\lim_{x' \rightarrow x} \partial_\mu \partial'_\nu i\Delta_B(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times \frac{1}{D} H^2 g_{\mu\nu}. \quad (45)$$

The same is true for the coincidence limits of the  $C$ -type propagator,

$$\lim_{x' \rightarrow x} i\Delta_C(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times \frac{1}{(D-2)(D-3)}, \quad (46)$$

$$\lim_{x' \rightarrow x} \partial_\mu i\Delta_C(x; x') = 0, \quad (47)$$

$$\lim_{x' \rightarrow x} \partial_\mu \partial'_\nu i\Delta_C(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\frac{2}{(D-2)D} H^2 g_{\mu\nu}. \quad (48)$$

Table 3 gives the contribution to the first diagram of Fig. 1 from each of the 42 graviton vertex operators. Although the 25 nonzero contributions might seem bewilderingly varied they in fact derive from just ten distinct groups, each of which sums to a simple result. We begin with Vertex Operators #2 and #5, which derive from a single derivative of the  $A$ -type propagator. The reduction of Vertex Operator #2 is,

$$\left(\text{Graviton Loop}\right)_2^{\alpha\beta} = i\left(\frac{D-2}{4}\right) \kappa H a^{D-1} \eta^{\mu\nu} \eta^{\rho\sigma} \delta_0^\alpha \partial'^\beta \times i[\mu\nu \Delta_{\rho\sigma}], \quad (49)$$

$$= i\left(\frac{D-2}{4}\right) \kappa H a^{D-1} \times -4\left(\frac{D-1}{D-3}\right) \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -H a \delta_0^\alpha \delta_0^\beta, \quad (50)$$

$$= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times \frac{(D-2)(D-1)}{(D-3)} \delta_0^\alpha \delta_0^\beta. \quad (51)$$

The contribution from Vertex Operator #5 is just  $\frac{1}{2}(D^2-3D-2)$  times this, and the pole at  $D = 3$  cancels in their sum,

$$\left(\text{Graviton Loop}\right)_{2+5}^{\alpha\beta} = \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times \frac{1}{2}(D-2)(D-1)D \delta_0^\alpha \delta_0^\beta. \quad (52)$$

#	Vertex Contribution	#	Vertex Contribution
1	0	22	$-(D^2-3D-2)\frac{(D-1)^2}{2(D-3)}\delta_0^\alpha\delta_0^\beta$
2	$\frac{(D-2)(D-1)}{(D-3)}\delta_0^\alpha\delta_0^\beta$	23	0
3	$\frac{(D-1)}{(D-3)(D-2)}\eta^{\alpha\beta}$	24	$2\frac{(D^2-2D+2)}{(D-2)^2D}\bar{\eta}^{\alpha\beta} - \frac{4}{(D-2)^2D}\delta_0^\alpha\delta_0^\beta$
4	0	25	0
5	$(D^2-3D-2)\frac{(D-2)(D-1)}{2(D-3)}\delta_0^\alpha\delta_0^\beta$	26	0
6	$-\frac{(D-1)}{(D-3)(D-2)}\bar{\eta}^{\alpha\beta} - \frac{(D-1)}{(D-2)}\delta_0^\alpha\delta_0^\beta$	27	$2\frac{(D^2-2D+2)}{(D-2)^2D}\bar{\eta}^{\alpha\beta} - \frac{4}{(D-2)^2D}\delta_0^\alpha\delta_0^\beta$
7	0	28	0
8	0	29	0
9	$[-\frac{1}{2}(D-1)^2 + \frac{(D-1)}{(D-2)}]\eta^{\alpha\beta}$	30	$\frac{D}{2}\frac{(D^2-3D+4)}{(D-2)^2}\eta^{\alpha\beta}$
10	$A(D)\eta^{\alpha\beta}$	31	$-\frac{(D-1)^2}{(D-3)}\eta^{\alpha\beta}$
11	0	32	$-2\frac{(D^2-2D+2)}{(D-2)^2}\bar{\eta}^{\alpha\beta} + \frac{4}{(D-2)^2}\delta_0^\alpha\delta_0^\beta$
12	0	33	$-(D^2-3D-2)\frac{(D-1)^2}{2(D-3)}\eta^{\alpha\beta}$
13	$B(D)\bar{\eta}^{\alpha\beta} + C(D)\delta_0^\alpha\delta_0^\beta$	34	$2\frac{(D-1)}{(D-3)}\bar{\eta}^{\alpha\beta}$
14	0	35	$-\frac{(D^2-3D+4)}{(D-2)^2}\eta^{\alpha\beta}$
15	0	36	0
16	$\frac{1}{2}B(D)\bar{\eta}^{\alpha\beta} + \frac{1}{2}C(D)\delta_0^\alpha\delta_0^\beta$	37	$-\frac{D}{2}A(D)\eta^{\alpha\beta}$
17	0	38	$\frac{(D-1)}{(D-3)}\bar{\eta}^{\alpha\beta}$
18	$-\frac{(D^2-3D+4)}{(D-2)^2}\eta^{\alpha\beta}$	39	$-\frac{D}{2}B(D)\bar{\eta}^{\alpha\beta} - \frac{D}{2}C(D)\delta_0^\alpha\delta_0^\beta$
19	$-\frac{(D-1)^2}{(D-3)}\delta_0^\alpha\delta_0^\beta$	40	$(D^2-3D-2)\frac{(D-1)}{(D-3)}\bar{\eta}^{\alpha\beta}$
20	0	41	$A(D)\eta^{\alpha\beta}$
21	$2\frac{(D^2-2D+2)}{(D-2)^2D}\bar{\eta}^{\alpha\beta} - \frac{4}{(D-2)^2D}\delta_0^\alpha\delta_0^\beta$	42	0

Table 3: Contributions from the graviton loop with an overall factor of  $\frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}$  removed. The three constants are  $A(D)\equiv -\frac{1}{4}(D-1)^2+1+\frac{1}{(D-2)^2}$ ,  $B(D)\equiv (D-1)-\frac{1}{D}-\frac{1}{(D-2)}-\frac{2}{(D-2)^2}$  and  $C(D)\equiv 1+\frac{2}{D}-\frac{3}{(D-2)}+\frac{2}{(D-2)^2}$ .

The contributions from Vertex Operators #19 and #22 involve a partially integrated derivative acting back on a derivative of the  $A$ -type propagator. The contribution from Vertex Operator #19 is,

$$\left(\text{Graviton Loop}\right)_{19}^{\alpha\beta} = -\partial^\alpha \left\{ -\frac{i}{4}\kappa a^{D-2}\eta^{\mu\nu}\eta^{\rho\sigma}\partial'^\beta \times i[\mu\nu\Delta_{\rho\sigma}] \right\}, \quad (53)$$

$$= \partial_0 \left\{ -\frac{i}{4}\kappa H a^{D-2} \times -4\left(\frac{D-1}{D-3}\right) \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -H a \delta_0^\alpha \delta_0^\beta \right\}, \quad (54)$$

$$= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\frac{(D-1)^2}{(D-3)} \delta_0^\alpha \delta_0^\beta. \quad (55)$$

The contribution from Vertex Operator #22 is just  $\frac{1}{2}(D^2-3D-2)$  times this, and the pole at  $D=3$  again cancels in their sum,

$$\left(\text{Graviton Loop}\right)_{19+22}^{\alpha\beta} = \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\frac{1}{2}(D-1)^2 D \delta_0^\alpha \delta_0^\beta. \quad (56)$$

Hence the first four vertex operators we have considered contribute,

$$\left(\text{Graviton Loop}\right)_{\substack{2+5 \\ +19+22}}^{\alpha\beta} = \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\frac{1}{2}(D-1)D \delta_0^\alpha \delta_0^\beta. \quad (57)$$

The contributions from Vertex Operators #31 and #33 are proportional to  $\eta^{\alpha\beta}$  with a partially integrated derivative contracted into a derivative of the  $A$ -type propagator. The contribution from Vertex Operator #31 is,

$$\left(\text{Graviton Loop}\right)_{31}^{\alpha\beta} = -\partial_\gamma \left\{ \frac{i}{4}\kappa a^{D-2}\eta^{\alpha\beta}\eta^{\mu\nu}\eta^{\rho\sigma}\partial'^\gamma \times i[\mu\nu\Delta_{\rho\sigma}] \right\}, \quad (58)$$

$$= -\partial_0 \left\{ \frac{i}{4}\kappa H a^{D-2} \times -4\left(\frac{D-1}{D-3}\right) \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -H a \eta^{\alpha\beta} \right\}, \quad (59)$$

$$= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\frac{(D-1)^2}{(D-3)} \eta^{\alpha\beta}. \quad (60)$$

In what must by now seem a familiar pattern, the contribution from Vertex Operator #33 is just  $\frac{1}{2}(D^2-3D-2)$  times this, and the pole at  $D=3$  cancels in their sum,

$$\left(\text{Graviton Loop}\right)_{31+33}^{\alpha\beta} = \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\frac{1}{2}(D-1)^2 D \eta^{\alpha\beta}. \quad (61)$$

The contributions from Vertex Operators #34, #38 and #40 also have a partially integrated derivative contracted into a derivative of the  $A$ -type propagator, but their free indices reside inside the propagator. The contribution from Vertex Operator #34 is,

$$\left(\text{Graviton Loop}\right)_{34}^{\alpha\beta} = -\partial_\gamma \left\{ -\frac{i}{2} \kappa a^{D-2} \eta^{\alpha\rho} \eta^{\mu\nu} \eta^{\beta\sigma} \partial^{\gamma} \times i [\mu\nu \Delta_{\rho\sigma}] \right\}, \quad (62)$$

$$= -\partial_0 \left\{ -\frac{i}{2} \kappa H a^{D-2} \times \left( \frac{-4}{D-3} \right) \bar{\eta}^{\alpha\beta} \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -H a \right\}, \quad (63)$$

$$= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times 2 \left( \frac{D-1}{D-3} \right) \bar{\eta}^{\alpha\beta}. \quad (64)$$

The contribution from Vertex Operator #38 is half this, and the contribution from #40 is  $\frac{1}{2}(D^2 - 3D - 2)$  times it. Hence the three contributions sum to,

$$\left(\text{Graviton Loop}\right)_{34+38+40}^{\alpha\beta} = \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ (D-1)D + \left( \frac{D-1}{D-3} \right) \right\} \bar{\eta}^{\alpha\beta}. \quad (65)$$

Vertex Operator is one of those with only a single derivative. It contributes,

$$\left(\text{Graviton Loop}\right)_6^{\alpha\beta} = -\delta_0^\mu \partial^\nu \left\{ -i \left( \frac{D-2}{2} \right) \kappa a^{D-1} \eta^{\alpha\rho} \eta^{\beta\sigma} \times i [\mu\nu \Delta_{\rho\sigma}] \right\}, \quad (66)$$

$$= \partial_0 \left\{ -i \left( \frac{D-2}{2} \right) \kappa a^{D-1} \times [{}_{00}T_C^{\alpha\beta}] \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times \frac{1}{(D-3)(D-2)} \right\}, \quad (67)$$

$$= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ -\frac{(D-1)}{(D-3)(D-2)} \bar{\eta}^{\alpha\beta} - \left( \frac{D-1}{D-2} \right) \delta_0^\alpha \delta_0^\beta \right\}. \quad (68)$$

The sum of this with the three previous terms is free of the pole at  $D = 3$ ,

$$\left(\text{Graviton Loop}\right)_{\substack{6+34 \\ +38+40}}^{\alpha\beta} = \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ (D-1)D \bar{\eta}^{\alpha\beta} + \left( \frac{D-1}{D-2} \right) \eta^{\alpha\beta} \right\}. \quad (69)$$

The contributions from Vertex Operators #10, #37 and #41 are all proportional to  $\eta^{\alpha\beta}$  times the coincidence limit of a double derivative. The reduction for #10 is,

$$\left(\text{Graviton Loop}\right)_{10}^{\alpha\beta} = \frac{i}{4} \kappa a^{D-2} \eta^{\alpha\beta} \eta^{\mu\rho} \partial^\sigma \partial^{\nu} \times i [\mu\nu \Delta_{\rho\sigma}], \quad (70)$$



$$\begin{aligned}
&= \frac{i}{4} \kappa a^{D-2} \eta^{\alpha\beta} \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times H^2 a^2 \times \eta^{\mu\rho} \eta^{\nu\sigma} \\
&\quad \times \left\{ -\left(\frac{D-1}{D}\right) [\mu\nu T_{\rho\sigma}^A] + \frac{1}{D} [\mu\nu T_{\rho\sigma}^B] - \frac{2}{(D-2)D} [\mu\nu T_{\rho\sigma}^C] \right\}, \quad (71)
\end{aligned}$$

$$\begin{aligned}
&= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ -\frac{1}{4}(D-1)^2 + 1 + \frac{1}{(D-2)^2} \right\} \eta^{\alpha\beta}. \quad (72)
\end{aligned}$$

We define the bracketed constant in the final expression as  $A(D)$ . Vertex Operator #41 gives the same, and #37 gives  $-\frac{D}{2}$  times that of #10. So the three of them sum to be  $-\frac{1}{2}(D-4)$  times (72).

The contributions from Vertex Operators #18, #30 and #35 are also proportional to  $\eta^{\alpha\beta}$  times the coincidence limit of a double derivative, but with the other tensor contraction of the propagator. The reduction for #18 is,

$$(\text{Graviton Loop})_{18}^{\alpha\beta} = -\frac{i}{4} \kappa a^{D-2} \eta^{\alpha\beta} \eta^{\mu\nu} \partial^\rho \partial^\sigma \times i [\mu\nu \Delta_{\rho\sigma}], \quad (73)$$

$$\begin{aligned}
&= -\frac{i}{4} \kappa a^{D-2} \eta^{\alpha\beta} \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times H^2 a^2 \times \eta^{\mu\nu} \eta^{\rho\sigma} \\
&\quad \times \left\{ -\left(\frac{D-1}{D}\right) [\mu\nu T_{\rho\sigma}^A] + \frac{1}{D} [\mu\nu T_{\rho\sigma}^B] - \frac{2}{(D-2)D} [\mu\nu T_{\rho\sigma}^C] \right\}, \quad (74)
\end{aligned}$$

$$\begin{aligned}
&= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ -1 - \frac{1}{(D-2)} - \frac{2}{(D-2)^2} \right\} \eta^{\alpha\beta}. \quad (75)
\end{aligned}$$

Vertex Operator #35 gives the same, and #30 gives  $-\frac{D}{2}$ , so the three of them again sum to be  $-\frac{1}{2}(D-4)$  times the first. Of course we can add these to the preceding three to find,

$$\begin{aligned}
(\text{Graviton Loop})_{\substack{10+37+41 \\ +18+30+35}}^{\alpha\beta} &= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \\
&\quad \times \frac{1}{2}(D-4) \left\{ \frac{1}{4}(D-1)^2 + \frac{1}{(D-2)} + \frac{1}{(D-2)^2} \right\} \eta^{\alpha\beta}. \quad (76)
\end{aligned}$$

The contributions from Vertex Operators #13, #16 and #39 involve the coincidence limit of a double derivative, but times one of the contractions in which free indices reside on the propagator. The reduction for #13 is,

$$(\text{Graviton Loop})_{13}^{\alpha\beta} = -i\kappa a^{D-2} \eta^{\alpha\mu} \eta^{\nu\rho} \partial^\sigma \partial^\beta \times i [\mu\nu \Delta_{\rho\sigma}], \quad (77)$$

$$\begin{aligned}
&= -i\kappa a^{D-2} \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times H^2 a^2 \times \eta^{\alpha\mu} \eta^{\nu\rho} \eta^{\beta\sigma} \\
&\quad \times \left\{ -\left(\frac{D-1}{D}\right) [\mu\nu T_{\rho\sigma}^A] + \frac{1}{D} [\mu\nu T_{\rho\sigma}^B] - \frac{2}{(D-2)D} [\mu\nu T_{\rho\sigma}^C] \right\}, \quad (78)
\end{aligned}$$

$$= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ B(D) \bar{\eta}^{\alpha\beta} + C(D) \delta_0^\alpha \delta_0^\beta \right\}, \quad (79)$$

where the  $D$ -dependent constants in (79) are,

$$B(D) = (D-1) - \frac{2}{D} - \frac{1}{(D-2)} - \frac{2}{(D-2)^2}, \quad (80)$$

$$C(D) = 1 + \frac{2}{D} - \frac{3}{(D-2)} + \frac{2}{(D-2)^2}. \quad (81)$$

Vertex Operator #16 gives half of (79), and #39 gives  $-\frac{D}{2}$  times (79), so the three contributions sum to  $-\frac{1}{2}(D-3)$  times (79).

The contributions from Vertex Operators #21, #24, #27 and #32 have the same structure but with the other contraction of the propagator. The reduction for #21 is,

$$(\text{Graviton Loop})_{21}^{\alpha\beta} = \frac{i}{2} \kappa a^{D-2} \eta^{\alpha\mu} \eta^{\beta\nu} \partial^\rho \partial^\sigma \times i [\mu\nu \Delta_{\rho\sigma}], \quad (82)$$

$$\begin{aligned}
&= \frac{i}{2} \kappa a^{D-2} \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times H^2 a^2 \times \eta^{\alpha\mu} \eta^{\beta\nu} \eta^{\rho\sigma} \\
&\quad \times \left\{ -\left(\frac{D-1}{D}\right) [\mu\nu T_{\rho\sigma}^A] + \frac{1}{D} [\mu\nu T_{\rho\sigma}^B] - \frac{2}{(D-2)D} [\mu\nu T_{\rho\sigma}^C] \right\}, \quad (83)
\end{aligned}$$

$$= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ 2 \frac{(D^2-2D+2)}{(D-2)^2 D} \bar{\eta}^{\alpha\beta} - \frac{4}{(D-2)^2 D} \delta_0^\alpha \delta_0^\beta \right\}, \quad (84)$$

Vertex Operators #24 and #27 each give the same, and #32 gives  $-D$  times (84), so the four contributions sum to  $-(D-3)$  times (84). At this stage we note that the pole at  $D=0$  cancels when the contributions from the preceding seven vertex operators are summed,

$$\begin{aligned}
&(\text{Graviton Loop})_{\substack{13+16+39 \\ +21+24+27+32}}^{\alpha\beta} = \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \\
&\quad \times \left\{ -\frac{1}{2}(D-3) D \bar{\eta}^{\alpha\beta} + \frac{1}{2}(D-3) \left[ 1 - \frac{1}{(D-2)} - \frac{2}{(D-2)^2} \right] \eta^{\alpha\beta} \right\}. \quad (85)
\end{aligned}$$

Adding this to (76) results in cancellation of the double pole at  $D=2$ ,

$$\begin{aligned} & \left( \text{Graviton Loop} \right)_{\substack{\alpha\beta \\ 10+37+41 \\ +18+30+35}} + \left( \text{Graviton Loop} \right)_{\substack{\alpha\beta \\ 13+16+39 \\ +21+24+27+32}} = \frac{i\kappa H^D a^D \Gamma(D-1)}{(4\pi)^{\frac{D}{2}} \Gamma(\frac{D}{2})} \\ & \times \left\{ -\frac{1}{2}(D-3)D\bar{\eta}^{\eta\beta} + \left[ -\frac{1}{(D-2)} + \frac{D}{2} - \frac{3}{2} + \frac{1}{8}(D-4)(D-1)^2 \right] \eta^{\eta\beta} \right\}! \quad (86) \end{aligned}$$

Finally, Vertex Operators #3 and #9 are proportional to  $\eta^{\alpha\beta}$  times a single derivative integrated back on the coincidence limit of the undifferentiated propagator. For Vertex Operator #3 the reduction is,

$$\left( \text{Graviton Loop} \right)_3^{\alpha\beta} = -\delta_0^\mu \partial^\nu \left\{ i \left( \frac{D-2}{4} \right) \kappa H a^{D-1} \eta^{\alpha\beta} \eta^{\rho\sigma} \times i [\mu\nu \Delta_{\rho\sigma}] \right\}, \quad (87)$$

$$= \partial_0 \left\{ i \left( \frac{D-2}{4} \right) \kappa H a^{D-1} \times \frac{4\eta^{\alpha\beta}}{(D-2)} \times \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times \frac{1}{(D-3)(D-2)} \right\}, \quad (88)$$

$$= \frac{i\kappa H^D a^D \Gamma(D-1)}{(4\pi)^{\frac{D}{2}} \Gamma(\frac{D}{2})} \times \frac{(D-1)}{(D-3)(D-2)} \eta^{\alpha\beta}. \quad (89)$$

The contribution from Vertex Operator #9 involves the other contraction of the propagator. The pole at  $D=2$  cancels when the two are summed,

$$\left( \text{Graviton Loop} \right)_{3+9}^{\alpha\beta} = \frac{i\kappa H^D a^D \Gamma(D-1)}{(4\pi)^{\frac{D}{2}} \Gamma(\frac{D}{2})} \left\{ \left( \frac{D-1}{D-3} \right) - \frac{1}{2}(D-1)^2 \right\} \eta^{\alpha\beta}. \quad (90)$$

The entire graviton loop sums to,

$$\begin{aligned} \left( \text{Graviton Loop} \right)^{\alpha\beta} &= \frac{i\kappa H^D a^D \Gamma(D-1)}{(4\pi)^{\frac{D}{2}} \Gamma(\frac{D}{2})} \left\{ \frac{1}{2} D(D+1) \bar{\eta}^{\alpha\beta} - \frac{1}{2} (D-1) D \delta_0^\alpha \delta_0^\beta \right. \\ & \left. + \left[ \left( \frac{D-1}{D-3} \right) - \frac{1}{2} (D-1)^2 (D+1) + \frac{1}{2} (D-1) + \frac{1}{8} (D-4)(D-1)^2 \right] \eta^{\alpha\beta} \right\}. \quad (91) \end{aligned}$$

Note that all exotic denominators have canceled, save for a lone factor of  $1/(D-3)$  from Vertex Operator #3. Because this multiplies  $\eta^{\alpha\beta}$  it can be absorbed into a harmless renormalization of the cosmological constant.

The contributions from the ghost loop are comparatively simple to evaluate. They are listed in Table 4. Their sum is,

$$\left( \text{Ghost Loop} \right)^{\alpha\beta} = \frac{i\kappa H^D a^D \Gamma(D-1)}{(4\pi)^{\frac{D}{2}} \Gamma(\frac{D}{2})} \left\{ \frac{1}{2} (D-1) \eta^{\alpha\beta} - D \bar{\eta}^{\alpha\beta} \right\}. \quad (92)$$

#	Vertex Contribution	#	Vertex Contribution
1	$-(D-1)\bar{\eta}^{\alpha\beta} - \delta_0^\alpha \delta_0^\beta$	6	0
2	$-\frac{(D-1)}{D}\bar{\eta}^{\alpha\beta} - \frac{1}{D}\delta_0^\alpha \delta_0^\beta$	7	0
3	0	8	0
4	0	9	$\frac{1}{2}(D-1)\eta^{\alpha\beta}$
5	$\frac{(D-1)}{D}\bar{\eta}^{\alpha\beta} + \frac{1}{D}\delta_0^\alpha \delta_0^\beta$	10	$-\eta^{\alpha\beta}$

Table 4: Contributions from the ghost loop with an overall factor of  $\frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}$  removed.

Adding (91) and (92) gives the total for the two primitive graphs of Fig. 1,

$$\begin{aligned}
(\text{Primitive})^{\alpha\beta} &= \frac{i\kappa H^D a^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ \left( \frac{D-1}{D-3} \right) \right. \\
&\quad \left. - \frac{1}{2}(D-2)(D-1)(D+1) + \frac{1}{8}(D-4)(D-1)^2 \right\} \eta^{\alpha\beta}. \quad (93)
\end{aligned}$$

Because all noncovariant terms have canceled the entire one loop result can be absorbed into a renormalization of the cosmological constant,

$$\delta\Lambda = \frac{\kappa^2 H^D}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-2)}{\Gamma(\frac{D}{2})} \left\{ 2 \left( \frac{D-1}{D-3} \right) - (D-2)(D-1)(D+1) + \frac{1}{4}(D-4)(D-1)^2 \right\}. \quad (94)$$

In fact it *must* be so absorbed if our renormalization condition is that the universe begins inflation with Hubble constant  $H$ . Hence our result for the one loop 1PI 1-point function is zero!

## 4 Discussion

We have used dimensional regularization to compute the 1PI graviton 1-point function at one loop order about a locally de Sitter background. Like other computations, our result can be expressed as a finite shift in the background cosmological constant. We can write this as the negative of the cosmological counterterm  $\delta\Lambda$  that would be needed to cancel the effect and enforce the

elementary consistency condition that the universe begins inflation at the background Hubble constant. Recall that the Newtonian expectation is [11],

$$-\delta\Lambda_{\text{Newt}} = +\frac{1}{16} \frac{\kappa^2 H^4}{\pi^2} . \quad (95)$$

Our result is a factor of 24 larger,

$$-\delta\Lambda_{\substack{\text{TW} \\ D=4}} = +\frac{3}{2} \frac{\kappa^2 H^4}{\pi^2} . \quad (96)$$

By comparison, the recent result by Finelli, Marozzi, Venturi and Vacca is [9],

$$-\delta\Lambda_{\text{FMVV}} = +\frac{361}{1920} \frac{\kappa^2 H^4}{\pi^2} . \quad (97)$$

When one corrects for the normalization of the graviton, Ford's result is [8],

$$-\delta\Lambda_{\text{Ford}} = -\frac{\kappa^2 H^4}{\pi^2} . \quad (98)$$

The failure of any of these results to agree seems arise to from having sometimes computed different things and sometimes used different techniques. The Newtonian model derives from an estimate for just the infrared contributions under the assumption that each graviton polarization contributes to the vacuum energy like a massless, minimally coupled scalar. Ford's result is a direct computation of what gravitons contribute to the vacuum, but only from infrared gravitons. By contrast, the result of Finelli, Marozzi, Venturi and Vacca includes ultraviolet gravitons, which can of course induce additional constant shifts in the vacuum energy. Our result also includes the full theory, but in a different gauge and with a different regularization. It has long been known that even the finite parts of counterterms can disagree in different gauges and with different regularization techniques. On the *physical* result everyone agrees: the one loop effect can be absorbed into a counterterm.

So there are good grounds for believing we have succeeded in dimensionally regulating quantum gravity about de Sitter background. An obvious first application for this formalism is to re-compute the one loop graviton self-energy that was previously obtained using a cutoff on the co-moving 3-momentum [31]. One would then like to study the quantum-corrected, linearized Einstein equations,

$$D_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma}(x) - \int d^4 x' [\mu\nu\Sigma^{\rho\sigma}](x; x') h_{\rho\sigma}(x') = -\frac{\kappa}{2a^2} T_{\mu\nu}(x) . \quad (99)$$

The solution with zero stress tensor describes how one loop corrections modify free gravitons.

Solving the quantum-corrected, linearized Einstein equations with the stress tensor of a point mass would determine how one loop corrections affect the long range force law. In this regard it is interesting to note that there is no simple dimensional argument that one loop corrections must be negligible at large distances the way they must be in flat space [32, 33]. In de Sitter background the universal one loop factor of  $\kappa^2$  can be balanced by a factor of  $H^2$ , rather than just the  $1/r^2$  of flat space. Loop corrections can also acquire factors of the number of e-foldings since the onset of inflation. So it seems entirely possible for the long range force in de Sitter background to acquire a secular proportional correction of the form  $\kappa^2 H^2 \ln(a)$ , which could become nonperturbatively strong over a very long period of inflation.

Other obvious, and fairly simple, applications for the new formalism are computing the quantum gravitational contributions to the one loop scalar self-mass-squared and to the one loop fermion self-energy. Both models show enhanced quantum effects from scalar couplings [18, 25] so it is reasonable to expect enhanced effects from gravitons. These studies are on-going and results should be available soon.

A more complicated but timely application would be applying dimensional regularization to modified gravity models involving inverse powers the Ricci scalar which have been invoked to explain the recent phase of cosmological acceleration [34]. A heroic computation of the one loop effective potential (as a function of constant curvature) has recently been carried out using generalized zeta function regularization [35]. Because nonlinear functions of only the Ricci scalar would just change the scalar part of the graviton propagator it should not be prohibitively difficult to generalize our methods to these models.

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