Extra dimensions, preferred frames and ether-drift experiments

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Abstract

Models with extra space-time dimensions produce, tipically, a 4D effective theory whose vacuum is not exactly Lorentz invariant but can be considered a physical medium whose refractive index is determined by the gravitational field. This leads to a version of relativity with a preferred frame and to look for experimental tests with the new generation of ether-drift experiments using rotating cryogenic optical resonators. Considering various types of cosmic motion, we formulate precise predictions for the modulations of the signal induced by the Earth's rotation and its orbital revolution around the Sun. We also compare with recent experimental results that might represent the first modern experimental evidence for a preferred frame.

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1. Introduction

Models with extra space-time dimensions [1] represent an interesting approach toward a consistent quantum theory of gravity and its conceptual unification with the other interactions. A characteristic feature of such models is to predict tipically a speed of gravity $c_g \neq c$ thus leading, in the 4D effective theory, to a version of relativity where there is a preferred frame Σ , the one associated with the isotropic value of c_g . At the same time, through the coupling to gravitons, the induced Lorentz-violations [2] will extend to the other sectors of the theory. Intuitively, the effect of gravitons transforms the vacuum into a physical medium with a non-trivial refractive index $\mathcal{N}_{\text{vacuum}} \neq 1$. Thus, if light propagates isotropically in Σ , on the Earth there would be a small anisotropy

$$\frac{\delta c}{c} \sim (\mathcal{N}_{\text{vacuum}} - 1) \frac{v_{\text{earth}}^2}{c^2} \tag{1}$$

 v_{earth} being the Earth's velocity with respect to Σ .

The aim of this paper is to explore the observable consequences of this scenario by comparing with the ether-drift experiments and, in particular, with the new generation where (vacuum) cryogenic optical resonators are maintained under active rotation. If there were a preferred frame Σ , one should be able to detect periodic modulations of the signal as those associated with the typical angular frequency defined by the Earth's rotation.

To this end we shall compare with the results of the Düsseldorf experiment [3] that indeed indicate a definite non-zero modulation associated with the Earth's rotation and might represent the first modern experimental evidence for a preferred frame. We shall also describe how, taking data in different periods of the year, one can obtain precise informations to restrict the class of possible Earth's cosmic motions.

2. General formalism

In general the observable implications of a speed of gravity $c_g \neq c$ have a considerable model dependence due to the many possible ways of embedding in curved space-time different graviton and photon light-cone conditions [4]. Restricting to flat space, the parameter $\epsilon = c_g - 1$, introduced to parameterize the difference of the speed of gravity c_g from the basic parameter $c \equiv 1$ entering Lorentz transformations, is a naturally small parameter. Also, one can safely restrict to the case $\epsilon > 0$, in view of the strong constraints placed by the absence of gravitational Cherenkov radiation in cosmic rays [5]. As a convenient framework for our analysis, we shall follow the authors of Ref.[6] and introduce a set of effective Minkowski tensors $\hat{\eta}(i)_{\mu\nu}$

$$\hat{\eta}(i)_{\mu\nu} = \eta_{\mu\nu} - \kappa_i v_\mu v_\nu \tag{2}$$

Here $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$, v_{μ} is the 4-velocity of S' with respect to a preferred frame Σ while κ_i represent generalized Fresnel's drag coefficients for particles of type *i* originating from their interactions with the gravitons. In this way, the energy-momentum relation in a given frame S' can be expressed as

$$p^{\mu}p^{\nu}\hat{\eta}(i)_{\mu\nu} + m^{2}(i) = 0$$
(3)

For photons this becomes

$$p^{\mu}p^{\nu}\hat{\eta}(\gamma)_{\mu\nu} = 0 \tag{4}$$

with $\hat{\eta}(\gamma)_{\mu\nu} = \eta_{\mu\nu} - \kappa_{\gamma} v_{\mu} v_{\nu}$ and with a photon energy that, in the S' frame, depends on the direction between the photon momentum and the S' velocity **v** with respect to Σ .

To obtain the photon energy spectrum, we shall follow Jauch and Watson [7] who worked out the quantization of the electromagnetic field in a moving medium. They noticed that the procedure introduces unavoidably a preferred frame, the one where the photon energy does not depend on the direction of propagation, and which is "usually taken as the system for which the medium is at rest". However, such an identification reflects the point of view of Special Relativity with no preferred frame. Therefore, we shall adapt their results to our case where the photon energy does not depend on the angle in some frame Σ . In this way, in a moving frame S', we get the radiation field Hamiltonian

$$H_0 = \sum_{r=1,2} \int d^3 \mathbf{p} \left[\hat{n}_r(\mathbf{p}) + \frac{1}{2} \right] E(|\mathbf{p}|, \theta)$$
(5)

where $\hat{n}_r(\mathbf{p})$ is the photon number operator and

$$E(|\mathbf{p}|,\theta) = \frac{\kappa_{\gamma} v_0 \zeta + \sqrt{|\mathbf{p}|^2 (1 + \kappa_{\gamma} v_0^2) - \kappa_{\gamma} \zeta^2}}{1 + \kappa_{\gamma} v_0^2} \tag{6}$$

with

$$\zeta = \mathbf{p} \cdot \mathbf{v} = |\mathbf{p}| |\mathbf{v}| \cos \theta \tag{7}$$

 $\theta \equiv \theta_{\text{lab}}$ being the angle defined, in the S' frame, between the photon momentum and the S' velocity **v** with respect to Σ . Notice that only one of the two roots of Eq.(4) appears and the energy is not positive definite in connection with the critical velocity $1/\sqrt{1+\kappa_{\gamma}}$ defined by the occurrence of the Cherenkov radiation.

Using the above relation, the one-way speed of light in the S' frame depends on θ (we replace $v = |\mathbf{v}|$ and $v_0^2 = 1 + v^2$)

$$\frac{E(|\mathbf{p}|,\theta)}{|\mathbf{p}|} = c_{\gamma}(\theta) = \frac{\kappa_{\gamma} v \sqrt{1 + v^2} \cos \theta + \sqrt{1 + \kappa_{\gamma} + \kappa_{\gamma} v^2 \sin^2 \theta}}{1 + \kappa_{\gamma} (1 + v^2)}$$
(8)

This is different from the v = 0 result, in the Σ frame, where the energy does not depend on the angle

$$\frac{E^{(\Sigma)}(|\mathbf{p}|)}{|\mathbf{p}|} = c_{\gamma} = \frac{1}{\mathcal{N}_{\text{vacuum}}}$$
(9)

and the speed of light is simply rescaled by the inverse of the vacuum refractive index

$$\mathcal{N}_{\text{vacuum}} = \sqrt{1 + \kappa_{\gamma}} \tag{10}$$

Working to $\mathcal{O}(\kappa_{\gamma})$ and $\mathcal{O}(v^2)$, one finds in the S' frame

$$c_{\gamma}(\theta) = \frac{1 + \kappa_{\gamma} v \cos \theta - \frac{\kappa_{\gamma}}{2} v^2 (1 + \cos^2 \theta)}{\sqrt{1 + \kappa_{\gamma}}}$$
(11)

This expression differs from Eq.(6) of Ref.[8], for the replacement $\cos \theta \rightarrow -\cos \theta$ and for the relativistic aberration of the angles. In Ref.[8], in fact, the one-way speed of light in the S' frame was parameterized in terms of the angle $\theta \equiv \theta_{\Sigma}$, between the velocity of S' and the direction of propagation of light, as defined in the Σ frame. In this way, starting from Eq.(11), replacing $\cos \theta \rightarrow -\cos \theta$ and using the aberration relation

$$\cos(\theta_{\rm lab}) = \frac{-v + \cos \theta_{\Sigma}}{1 - v \cos \theta_{\Sigma}}$$
(12)

one re-obtains Eq.(6) of Ref.[8] in terms of $\theta = \theta_{\Sigma}$.

Finally, using Eq.(11), the two-way speed of light (in terms of $\theta = \theta_{lab}$) is

$$\bar{c}_{\gamma}(\theta) = \frac{2c_{\gamma}(\theta)c_{\gamma}(\pi+\theta)}{c_{\gamma}(\theta) + c_{\gamma}(\pi+\theta)} \sim 1 - \left[\kappa_{\gamma} - \frac{\kappa_{\gamma}}{2}\sin^{2}\theta\right]v^{2}$$
(13)

Therefore, re-introducing, for sake of clarity, the speed of light entering Lorentz transformations, $c = 2.997...10^{10}$ cm/s, one can define the RMS [9, 10] parameter $(1/2 - \beta + \delta)$. This is used to parameterize the anisotropy of the speed of light *in the vacuum*, through the relation

$$\frac{\bar{c}_{\gamma}(\pi/2+\theta) - \bar{c}_{\gamma}(\theta)}{\langle \bar{c}_{\gamma} \rangle} \sim (1/2 - \beta + \delta) \frac{v^2}{c^2} \cos(2\theta)$$
(14)

so that one can relate κ_{γ} to $(1/2 - \beta + \delta)$ through

$$(1/2 - \beta + \delta) = \frac{\kappa_{\gamma}}{2} \tag{15}$$

Now, in Ref.[6], estimates of κ_{γ} were obtained by computing the coupling of photons to gravitons including the first few graviton loops. In this way, one obtains typical values $\kappa_{\gamma} = \mathcal{O}(10^{-10})$ or smaller.

However, in principle, besides the graviton loops, another class of effects arise when considering the propagation of photons in a *background* gravitational field, such as on the Earth's surface. As it is well known, resumming such tree-level background graviton graphs leads to the realm of classical General Relativity where such interaction effects can be re-absorbed into a re-definition of the space-time metric that depends on the external gravitational potential. However, comparing the local distortions of space-time with the density variations of a medium, these effects can also be incorporated into an effective refractive index. We have only to take into account that $c_g \neq 1$ and that there might be a preferred frame Σ where light propagates isotropically.

Now, for a static gravitational field the first modification is trivial. In fact, the timeaveraged scalar graviton propagator

$$\lim_{T \to \infty} \langle D(\mathbf{r}, t) \rangle_T = \lim_{T \to \infty} \int_{-T}^{+T} dt \int \frac{d^3 \mathbf{p}}{(2\pi)^3} e^{i\mathbf{p} \cdot \mathbf{r}} \int \frac{dp_0}{2\pi i} \frac{e^{-ip_0 t}}{c_g^2 \mathbf{p}^2 - p_0^2 - i\epsilon} = \frac{1}{4\pi^2 c_g^2 r}$$
(16)

is just rescaled by an overall factor $1/c_g^2$. This is an unobservable change where one simply replaces the Newton constant $G_N^{(0)}$ (for $c_g = 1$) with $G_N^{(0)}/c_g^2$ and the gravitational potential $\varphi^{(0)}$ with $\varphi^{(0)}/c_g^2 \equiv \varphi$.

The second modification, on the other hand, requires to re-consider the traditional point of view on the energy of a photon in a gravitational field. For instance, let us consider the Earth's gravitational field and an observer S' placed on the Earth's surface (but otherwise in free fall with respect to any other gravitational field). According to standard General Relativity, light is seen to propagate isotropically by S'. In fact, introducing the Newtonian potential

$$\varphi = -\frac{G_N M_{\text{earth}}}{c^2 R_{\text{earth}}} \sim -0.7 \cdot 10^{-9} \tag{17}$$

and considering the weak-field isotropic form of the metric [11]

$$ds^{2} = (1+2\varphi)dt^{2} - (1-2\varphi)(dx^{2} + dy^{2} + dz^{2})$$
(18)

the energy of a photon for S' is generally assumed to be

$$E(|\mathbf{p}|) = c_{\gamma}|\mathbf{p}| \tag{19}$$

(as in Eq.(9)) in terms of the effective vacuum refractive index \mathcal{N}_{vacuum} in the gravitational field

$$c_{\gamma} = \frac{1}{\mathcal{N}_{\text{vacuum}}} \sim 1 + 2\varphi \tag{20}$$

This type of reasoning has to be modified in the presence of a preferred frame Σ . In fact, it is now perfectly legitimate [12] to ask whether photons are seen to propagate isotropically by the S' observer placed on the Earth's surface or by the Σ observer. In the latter case, the S' energy would *not* be given by Eq.(19) but would rather be given by Eq.(6) with a value

$$\kappa_{\gamma} = \mathcal{N}_{\text{vacuum}}^2 - 1 \sim 28 \cdot 10^{-10} \tag{21}$$

corresponding to a RMS parameter

$$(1/2 - \beta + \delta) \sim \mathcal{N}_{\text{vacuum}} - 1 \sim 14 \cdot 10^{-10}$$
 (22)

In this sense, as with the graviton loops considered in Ref.[6], a background gravitational field transforms the (local) vacuum into a physical medium where the speed of light differs from the parameter c entering Lorentz transformations. If there were a preferred frame, one should detect an anisotropy of the two-way speed of light in modern ether-drift experiments.

3. Cosmic motions and ether-drift experiments

In modern ether-drift experiments, one measures the relative frequency shift $\delta\nu$ of two vacuum cryogenic optical resonators under the Earth's rotation [13] or upon active rotations of the apparatus [3]. If there is a preferred frame Σ , using Eqs.(13) and (14), the frequency shift of two orthogonal optical resonators to $\mathcal{O}(\frac{v^2}{c^2})$ can be expressed as

$$\frac{\delta\nu(\theta)}{\nu} = \frac{\bar{c}_{\gamma}(\pi/2 + \theta) - \bar{c}_{\gamma}(\theta)}{\langle\bar{c}_{\gamma}\rangle} = \frac{A}{\nu}\cos(2\theta)$$
(23)

where $\theta = 0$ indicates the direction of the ether-drift and the amplitude of the signal is given by

$$\frac{A}{\nu} = (1/2 - \beta + \delta)\frac{v^2}{c^2}$$
(24)

v denoting the projection of the Earth's velocity with respect to Σ in the plane of the interferometer.

Notice that, in principle, one might also consider the possibility of measuring the frequency shift with light propagating in a medium of refractive index $N_{\text{medium}} \sim 1$. In this case, by continuity, very small deviations of the refractive index from the vacuum value cannot

qualitatively change the main result that light propagates isotropically in Σ and not in the moving frame S' where the interferometer is at rest. On the other hand, substantial changes of the refractive index, as for instance for $\mathcal{N}_{\text{medium}} \sim 3$ which is the relevant one for the resonating cavities of Ref.[14], might induce a transition to a completely different regime where it is the medium itself to set up the frame where light propagates isotropically. For this reason, in principle, different types of ether-drift experiments might provide qualitatively different informations on the very existence of Σ .

To compare with the vacuum experiment of Ref.[3], it is convenient to re-write Eq.(23) in the form of Ref.[3] where the frequency shift at a given time t is expressed as

$$\frac{\delta\nu[\theta(t)]}{\nu} = \hat{B}(t)\sin 2\theta(t) + \hat{C}(t)\cos 2\theta(t)$$
(25)

 $\theta(t)$ being the angle of rotation of the apparatus, $\hat{B}(t) \equiv 2B(t)$ and $\hat{C}(t) \equiv 2C(t)$ so that one finds an experimental amplitude

$$A^{\exp}(t) = \nu \sqrt{\hat{B}^2(t) + \hat{C}^2(t)}$$
(26)

Let us first consider the average signal detected in Ref.[3] where the relevant value is $\nu \sim 2.8 \cdot 10^{14}$ Hz. In this case, the experimental results obtained around February 6th, 2005, namely $\langle \hat{B}\nu \rangle \sim 2.8$ Hz and $\langle \hat{C}\nu \rangle \sim -3.3$ Hz, correspond to an average amplitude

$$\langle A^{\exp} \rangle \sim 4.3 \text{ Hz}$$
 (27)

This should be compared with the value of Eq.(24), for $(1/2-\beta+\delta) \sim 14 \cdot 10^{-10}$ and a reference value v = 300 km/s, $\langle A \rangle \sim 0.4$ Hz. Therefore, as suggested by the same authors of Ref.[3], for a meaningful comparison, we shall not consider the mean value (which likely contains systematic effects of thermal origin [3]) and restrict the analysis to the time modulations of the signal.

In Ref.[3], these were parameterized as $(\omega_{\rm sid} = \frac{2\pi}{23^{h}56'})$

$$\hat{C}(t) = C_0 + C_1 \sin(\omega_{\rm sid}t) + C_2 \cos(\omega_{\rm sid}t) + C_3 \sin(2\omega_{\rm sid}t) + C_4 \cos(2\omega_{\rm sid}t)$$
(28)

with an analogous expression for the $\hat{B}(t)$ amplitude. The experimental results (obtained around February 6th, 2005) can be cast into the form

$$C(\omega_{\rm sid}) \equiv \sqrt{C_1^2 + C_2^2} \sim (11 \pm 2) \cdot 10^{-16}$$
(29)

and

$$C(2\omega_{\rm sid}) \equiv \sqrt{C_3^2 + C_4^2} \sim (1 \pm 2) \cdot 10^{-16}$$
(30)

To compare with the cosmic motion defined by the CMB it is convenient to use the relations [3] obtained from Ref.[16]

$$C(\omega_{\rm sid}) = \frac{1}{2}(1/2 - \beta + \delta)\frac{V_{\rm sun}^2}{c^2} |\sin 2\Theta| \sin 2\chi$$
(31)

and

$$C(2\omega_{\rm sid}) = \frac{1}{2}(1/2 - \beta + \delta)\frac{V_{\rm sun}^2}{c^2}\cos^2\Theta(1 + \sin^2\chi)$$
(32)

In the above equations, $V_{\rm sun} \sim 369$ km/s and $\Theta \sim -6^{\circ}$ indicate the magnitude and the declination of the solar motion relatively to the CMB while χ is the colatitude of the laboratory (for Düsseldorf $\chi \sim 39^{\circ}$).

In this way, one obtains two very different estimates of the RMS parameter. In fact, on the one hand, the value $C(\omega_{\rm sid}) \sim (11 \pm 2) \cdot 10^{-16}$ implies $(1/2 - \beta + \delta) \sim (71 \pm 13) \cdot 10^{-10}$. On the other hand, from the analogous result $C(2\omega_{\rm sid}) \sim (1 \pm 2) \cdot 10^{-16}$, one finds $(1/2 - \beta + \delta) \sim (1 \pm 2) \cdot 10^{-10}$.

Of course, the value $(1/2 - \beta + \delta) = (-0.5 \pm 3) \cdot 10^{-10}$ was obtained in Ref.[3] from a global fit where also the data for the amplitudes $\hat{B}(t)$ were included. However, these other data are constrained by the same type of relations (see note [20] of Ref.[3]) and, therefore, the global fit reflects the same type of tension between the very different modulations at $\omega_{\rm sid}$ and $2\omega_{\rm sid}$.

As far as we can see, both determinations of $(1/2-\beta+\delta)$ are likely affected by a systematic uncertainty of theoretical nature. In fact, if we consider the relative weight (for the latitude of Düsseldorf)

$$R \equiv \frac{C(2\omega_{\rm sid})}{C(\omega_{\rm sid})} \sim \frac{0.7}{|\tan\Theta|}$$
(33)

its present experimental value (in February)

$$R_{\rm feb}^{\rm EXP} \sim 0.09^{+0.18}_{-0.09} \tag{34}$$

is very far from its theoretical prediction for the cosmic motion relatively to the CMB, namely

$$R^{\rm CMB} \sim 6.8 \tag{35}$$

Therefore, to explain the observed daily modulations embodied in $C(\omega_{\rm sid})$, one has to consider some other type of cosmic motion and replace the CMB with another possible choice of preferred frame. In this case, the experimental determination of the RMS parameter will likely be affected as well. To address the problem from a general point of view, let us first return to Eq.(24) and introduce the time-dependent amplitude of the ether-drift effect

$$A(t) = v^2(t)X\tag{36}$$

in terms of the Earth's velocity in the plane of the interferometer v(t) and of the correct unknown normalization of the experiment X. The main point is that the relative variations of the signal depend only on the kinematic details of the given cosmic motion and, as such, can be predicted independently of the knowledge of X. To predict the variations of v(t), we shall use the expressions given by Nassau and Morse [17]. These have the advantage of being fully model-independent and extremely easy to handle. Their simplicity depends on the introduction of a cosmic Earth's velocity

$$\mathbf{V} = \mathbf{V}_{\rm sun} + \mathbf{v}_{\rm orb} \tag{37}$$

that, in addition to the genuine cosmic motion of the solar system defined by \mathbf{V}_{sun} , includes the effect of the Earth's orbital motion around the Sun described by \mathbf{v}_{orb} . To a very good approximation, \mathbf{V} can be taken to be constant within short observation periods of 2-3 days. Therefore, by introducing the latitude of the laboratory ϕ , the right ascension $\tilde{\Phi}$ and the declination $\tilde{\Theta}$ associated with the vector \mathbf{V} , the magnitude of the Earth's velocity in the plane of the interferometer is defined by the two equations [17]

$$\cos z(t) = \sin \tilde{\Theta} \sin \phi + \cos \tilde{\Theta} \cos \phi \cos(\lambda) \tag{38}$$

and

$$v(t) = V \sin z(t) \tag{39}$$

z = z(t) being the zenithal distance of **V**. Here, we have introduced the time $\lambda \equiv \tau - \tau_o - \tilde{\Phi}$, $\tau = \omega_{\rm sid}t$ being the sidereal time of the observation in degrees and τ_o being an offset that, in general, has to be introduced to compare with the definition of sidereal time adopted in Ref.[3].

Now, operation of the interferometer provides the minimum and maximum daily values of the amplitude and, as such, the values v_{\min} and v_{\max} corresponding to $|\cos(\lambda)| = 1$. In this way, using the above relations one can determine the pair of values $(\tilde{\Phi}_i, \tilde{\Theta}_i), i = 1, 2, ...n$, for each of the *n* short periods of observations taken during the year, and thus plot the direction of the vectors \mathbf{V}_i on the celestial sphere. Actually, since the ether-drift is a second-harmonic effect in the rotation angle of the interferometer, a single observation is unable to distinguish the pair $(\tilde{\Phi}_i, \tilde{\Theta}_i)$ from the pair $(\tilde{\Phi}_i + 180^\circ, -\tilde{\Theta}_i)$. Only repeating the observations in different epochs of the year one can resolve the ambiguity. Any meaningful ether-drift, in fact, has to correspond to pairs $(\tilde{\Phi}_i, \tilde{\Theta}_i)$ lying on an 'aberration circle', defined by the Earth's orbital motion, whose center (Φ, Θ) defines the right ascension and the declination of the genuine cosmic motion of the solar system associated with \mathbf{V}_{sun} . If such a consistency is found, using the triangle law, one can finally determine the magnitude $|\mathbf{V}_{sun}|$ starting from the known values of $(\tilde{\Phi}_i, \tilde{\Theta}_i), (\Phi, \Theta)$ and the value $|\mathbf{v}_{orb}| \sim 30$ km/s.

We emphasize that the basic pairs of values $(\tilde{\Phi}_i, \tilde{\Theta}_i)$ determined in this way only depend on the *relative* magnitude of the ether drift effect, namely on the ratio $\frac{v_{\min}}{v_{\max}}$, in the various periods. As such, they are insensitive to any possible theoretical and/or experimental uncertainty that can affect multiplicatively the *absolute* normalization of the signal.

For instance, suppose one measures a relative frequency shift $\delta\nu/\nu = \mathcal{O}(10^{-15})$. Assuming a value $(1/2 - \beta + \delta) \sim 10 \cdot 10^{-10}$ in Eq.(24), this would be interpreted in terms of a velocity $v \sim 300$ km/s. Within Galileian relativity, where one predicts the same expressions by simply replacing $(1/2 - \beta + \delta) \rightarrow 1/2$, the same frequency shift would be interpreted in terms of a velocity $v \sim 14$ m/s. Nevertheless, from the *relative* variations of the ether-drift effect one would deduce the same pairs $(\tilde{\Phi}_i, \tilde{\Theta}_i)$ and, as such, exactly the same type of cosmic motion. Just for this reason, Miller's determinations with this method, namely [18] $\mathbf{V}_{sun} \sim 210$ km/s, $\Phi \sim 74^o$ and $\Theta \sim -70^o$, should be taken seriously.

We are aware that Miller's observations have been considered spurious by the authors of Ref.[19] as partly due to statistical fluctuations and/or thermal fluctuations. However, to a closer look (see the discussion given in Ref.[15]) the arguments of Ref.[19] are not so solid as they appear by reading the abstract of that paper. Moreover, Miller's solution is *doubly* internally consistent since the aberration circle due to the Earth's orbital motion was obtained in two different and independent ways (see Fig. 23 of Ref.[18]). In fact, one can determine the basic pairs ($\tilde{\Phi}_i, \tilde{\Theta}_i$) either using the daily variations of the magnitude of the ether-drift effect or using the daily variations of its apparent direction $\theta_0(t)$ (the 'azimuth') defined, in terms of Eq.(25), through the relation $\theta_0(t) = 1/2 \tan^{-1}(\frac{\hat{B}(t)}{\hat{C}(t)})$. Since the two methods were found to give consistent results, in addition to the standard choice of preferred frame represented by the CMB, it might be worth to consider the predictions associated with the cosmic motion deduced by Miller.

Replacing Eq. (39) into Eq. (24) and adopting a notation of the type introduced in Ref. [16],

we can express the theoretical amplitude of the signal as

$$\frac{A(t)}{\nu} = A_0 + A_1 \sin \tau + A_2 \cos \tau + A_3 \sin(2\tau) + A_4 \cos(2\tau)$$
(40)

where

$$A_0 = (1/2 - \beta + \delta) \frac{V^2}{c^2} (1 - \sin^2 \tilde{\Theta} \cos^2 \chi - \frac{1}{2} \cos^2 \tilde{\Theta} \sin^2 \chi)$$
(41)

$$A_1 = -\frac{1}{2}(1/2 - \beta + \delta)\frac{V^2}{c^2}\sin 2\tilde{\Theta}\sin(\tilde{\Phi} + \tau_o)\sin 2\chi$$
(42)

$$A_2 = -\frac{1}{2}(1/2 - \beta + \delta)\frac{V^2}{c^2}\sin 2\tilde{\Theta}\cos(\tilde{\Phi} + \tau_o)\sin 2\chi$$
(43)

$$A_{3} = -\frac{1}{2}(1/2 - \beta + \delta)\frac{V^{2}}{c^{2}}\cos^{2}\tilde{\Theta}\sin[2(\tilde{\Phi} + \tau_{o})]\sin^{2}\chi$$
(44)

$$A_4 = -\frac{1}{2}(1/2 - \beta + \delta)\frac{V^2}{c^2}\cos^2\tilde{\Theta}\cos[2(\tilde{\Phi} + \tau_o)]\sin^2\chi$$
(45)

Recall that V, $\tilde{\Theta}$ and $\tilde{\Phi}$ indicate respectively the magnitude, the declination and the right ascension of the velocity defined in Eq.(37). As such, they change during the year. Also, Eqs.(41)-(45) enter the *full* amplitude of the signal $A = \nu \sqrt{\hat{C}^2 + \hat{B}^2}$. Therefore, it is not so simple to express the coefficients A_0, A_1, A_2, A_3, A_4 in terms of the analogous coefficients entering $\hat{C}(\tau)$ and $\hat{B}(\tau)$.

As explained above, with an appropriate data taking in different epochs of the year, Eqs.(40)-(45) can be used to deduce the basic parameters of the Earth's cosmic motion from the daily variations of the measured frequency shifts. Here we shall follow the other way around and explore the implications of Miller's cosmic solution for the experiment of Ref.[3]. To this end, we shall start from observations performed around February 6th-8th using the entries reported in Tables I and II of Ref.[18]. In this case, by restricting to the southern apex pairs $\tilde{\Phi}_{\rm feb} \sim 90^{\circ}$ and $\tilde{\Theta}_{\rm feb} \sim -77^{\circ}$, for the latitude of Düsseldorf $\phi \sim 51^{\circ}$, one predicts a minimum velocity $v_{\rm min} \sim 0.44 V_{\rm feb}$ and a maximum velocity $v_{\rm max} \sim 0.79 V_{\rm feb}$. Therefore, introducing the unknown normalization in February, say $X_{\rm feb}$, such that $A_{\rm min} \sim (0.44)^2 X_{\rm feb}$ and $A_{\rm max} \sim (0.79)^2 X_{\rm feb}$, we obtain a mean theoretical value

$$\langle A \rangle \equiv \frac{1}{2} (A_{\min} + A_{\max}) \sim 0.41 \ X_{\text{feb}}$$

$$\tag{46}$$

and a daily modulation

$$(\Delta A)_{\rm feb} = \pm (\langle A \rangle - A_{\rm min}) \sim \pm 0.22 X_{\rm feb}$$
(47)

In this way, if one could subtract out from the data of Ref.[3] the spurious systematic component and obtain the true experimental signal $\langle A^{\exp} \rangle_{\text{true}}$ in terms of the correct normalization X_{feb} , the above relations amount to predict a daily modulation

$$(\Delta A)_{\rm feb} \sim \pm 0.53 \langle A^{\rm exp} \rangle_{\rm true}$$
 (48)

By repeating the same analysis for observations performed around September 15th, where the relevant values found by Miller were $\tilde{\Phi}_{\text{sept}} \sim 75^{\circ}$ and $\tilde{\Theta}_{\text{sept}} \sim -62^{\circ}$ one predicts, again for the latitude of Düsseldorf, $v_{\min} \sim 0.19 V_{\text{sept}}$ and a maximum velocity $v_{\max} \sim 0.92 V_{\text{sept}}$. Therefore, in terms of the arbitrary normalization in September, one finds $A_{\min} \sim (0.19)^2 X_{\text{sept}}$ and $A_{\max} \sim (0.92)^2 X_{\text{sept}}$, with a mean theoretical value

$$\langle A \rangle \equiv \frac{1}{2} (A_{\min} + A_{\max}) \sim 0.44 \ X_{sept}$$
 (49)

and the considerably larger daily modulation

$$(\Delta A)_{\text{sept}} \sim \pm 0.91 \langle A^{\exp} \rangle_{\text{true}}$$
 (50)

This represents a ~ +70% increase [20] with respect to the February value in Eq.(48). In this way, neglecting the small modulation at $2\omega_{\rm sid}$, and comparing Eqs.(47) and (50), one predicts an increase of the parameter associated with the daily modulation $C(\omega_{\rm sid}) \sim (19 \pm 2) \cdot 10^{-16}$ around September 15th, starting from its February value $C(\omega_{\rm sid}) \sim (11 \pm 2) \cdot 10^{-16}$ (within the present normalization of the experiment).

Here, we are assuming that the *central* value of the ether-drift effect, namely the quantity $\langle A^{\exp} \rangle_{\text{true}}$, does not change too much during the year. This assumption is motivated by the modest difference between the average values in Eqs.(46) and (49). It is also consistent with the re-analysis of Miller's data performed in Ref.[19] where it was found that the average magnitudes of the second-harmonic components were only slightly changing from one epoch to the other (see page 170 of Ref.[19]).

Let us now consider the equivalent of the relative weight defined in Eq.(33)

$$\tilde{R} \equiv \frac{A(2\omega_{\rm sid})}{A(\omega_{\rm sid})} \tag{51}$$

where

$$A(\omega_{\rm sid}) \equiv \sqrt{A_1^2 + A_2^2} \tag{52}$$

and

$$A(2\omega_{\rm sid}) \equiv \sqrt{A_3^2 + A_4^2} \tag{53}$$

Although \tilde{R} is not immediately readable from the numbers reported in Ref.[3], its estimate through the approximate relation (for the latitude of Düsseldorf)

$$\tilde{R} \sim \frac{0.2}{|\tan\tilde{\Theta}|} \tag{54}$$

shows that for Miller's solution the weight of the modulation at $2\omega_{\rm sid}$ in the overall daily change remains small. In fact, \tilde{R} evolves from ~ 0.05 to ~ 0.11 when $\tilde{\Theta}$ changes from $\tilde{\Theta} \sim -77^{\circ}$ in February to $\tilde{\Theta} \sim -62^{\circ}$ in September. By comparing with Eq.(33), this confirms that replacing the CMB with another cosmic solution that exhibits $|\Theta| \sim 70^{\circ}$, one can obtain a small value of

$$R \sim 3.5\tilde{R} \sim 0.25\tag{55}$$

in Eq.(33) and thus consistent estimates of $(1/2 - \beta + \delta)$ from the two sets of observables at $\omega_{\rm sid}$ and $2\omega_{\rm sid}$.

4. Summary and outlook

In this paper we have explored some phenomenological consequences of assuming the existence of a preferred frame. This scenario, that on the one hand leads us back to the old Lorentzian version of relativity, is on the other hand favoured by present models with extra space-time dimensions where the interactions with the gravitons change the vacuum into a physical medium with a non-trivial refractive index.

Our point is that, besides the effect of graviton loops considered so far, one should also take into account the existence of the *background* gravitational fields. In fact, they produce exactly the same effect transforming the local vacuum into a physical medium whose refractive index can be easily computed from the weak-field isotropic form of the metric. Thus, if there were a preferred frame Σ where light is seen isotropic, one should be able to detect some effect with the new generation of precise ether-drift experiments using rotating cryogenic optical resonators. In particular, one should look for periodic modulations of the signal that might be associated with the Earth's rotation and its orbital motion around the Sun.

When comparing with the experimental results of Ref.[3] (obtained around February 6th, 2005) we can draw the following conclusions. The data exhibit a clear modulation at the Earth's rotation frequency embodied in the coefficient

$$C(\omega_{\rm sid}) \sim (11 \pm 2) \cdot 10^{-16}$$
 (56)

that might represent the first modern experimental evidence for a preferred frame. At the same time, the signal at $2\omega_{sid}$

$$C(2\omega_{\rm sid}) \sim (1\pm 2) \cdot 10^{-16}$$
 (57)

is very weak. Thus, the experimental value of the ratio

$$R_{\rm feb}^{\rm EXP} \equiv \frac{C(2\omega_{\rm sid})}{C(\omega_{\rm sid})} \sim 0.09^{+0.18}_{-0.09}$$
(58)

is very far from the expected theoretical value for the Earth's motion relatively to the CMB

$$R^{\rm CMB} \sim 6.8 \tag{59}$$

For this reason, to explain the daily modulations, one should consider some other type of cosmic motion. As an example, we have explored the implications of the cosmic motion deduced from Miller's ether-drift observations. In this framework, one expects a modest daily modulation at $2\omega_{\rm sid}$ in all periods of the year. This prediction is in good agreement with the present experimental value Eq.(58) and will be tested with future measurements.

Within Miller's cosmic solution, one also predicts a ~ +70% increase of the daily modulation, from its February value $C(\omega_{\rm sid}) \sim (11 \pm 2) \cdot 10^{-16}$ up to $C(\omega_{\rm sid}) \sim (19 \pm 2) \cdot 10^{-16}$ in September (within the present normalization of the experiment).

This other prediction will also be tested with experimental data collected in the next few months and, whenever confirmed, would represent clean experimental evidence for the existence of a preferred frame, a result with far-reaching implications for both particle physics and cosmology.

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