Space Time Matter inflation

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Abstract

We study a model of power-law inflationary inflation using the Space-Time-Matter (STM) theory of gravity for a five dimensional (5D) canonical metric that describes an apparent vacuum. In this approach the expansion is governed by a single scalar (neutral) quantum field. In particular, we study the case where the power of expansion of the universe is $p \gg 1$. This kind of model is more successful than others in accounting for galaxy formation.

I. INTRODUCTION

A standard mechanism for galaxy formation is the amplification of primordial fluctuations by the evolutionary dynamics of spacetime. The inflationary cosmology is based on the dynamics of a quantum field undergoing a phase transition [1]. The exponential expansion of the scale parameter naturally gives a scale-invariant spectrum on cosmological scales, in agreement with experimental data. This is one of the many attractive features of the inflationary universe, particularly in regard to the galaxy formation problem [2] and it arises from the fluctuations of the inflaton, the quantum field which induces inflation. This field can be semiclassically expanded in terms of its expectation value plus other field, which describes the quantum fluctuations [3]. The quantum to classical transition of quantum fluctuations has been studied in thoroughly [4]. The infrared matter field fluctuations are classical and can be described by a coarse-grained field which takes into account only wavelengths larger than the Hubble radius. The dynamics of this coarse-grained field is described by a second order stochastic equation, which can be treated using the Fokker-Planck formalism. This issue has been the subject of intense work during the last two decades [5]. Because of the sucess of this theory to explain the large-scale structure formation, inflation has nowaday become a standard ingredient for the description of the early universe. In fact, it is the unique that solves some of the problems of the standard big-bang scenario and also makes predictions about Cosmic Microwave Background (CMBR) anisotropies, which are being measured with increasing precision.

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On the other hand, recently, extra dimensional theories of gravity have received much interest, mainly sparked by works in string [6] and supergravity theories [7]. For the most part, four-dimensional (4D) space-time has been extended by the addition of several extra spatial dimensions, usually taken to be compact. Other very interesting approach, developed by Wesson and co-workers [8] have given new impetus to the study of 5D gravity. None of the standard dimensional reduction techniques imposed to reduce the number of space-time dimensions to four, are adhered to in their approach; indeed, the extra spatial dimension is not necessarily assumed to be compact. The main question they address in whether the 4D properties of matter can be viewed as being purely geometrical in origin. This idea is not new, and was originally introduced by Einstein [10].

In this work we are interested in studying the early inflationary dynamics of the universe from the STM theory of gravity. In particular, we are aimed to study power-law inflation, where the scale factor of the universe growth as $a \sim t^p$, being p > 1 the power of expansion during inflation. To do this, we use the Ponce de Leon metric [11], in the limital case where the power of expansion is $p \gg 1$, which describe an asymptotic de Sitter expansion of the universe.

II. BASIC STM EQUATIONS

Following the idea suggested by Wesson and co-workers [8,9], in this section we develope the induced 4D equation of state from the 5D vacuum field equations, $G_{AB}=0$ (A,B=0,1,2,3,4), which give the 4D Einstein equations $G_{\mu\nu}=8\pi G~T_{\mu\nu}~(\mu,\nu=0,1,2,3)$. In particular, we consider a 5D spatially isotropic and 3D flat spherically-symmetric line element

$$dS^{2} = e^{\alpha(\psi,t)}dt^{2} - e^{\beta(\psi,t)}dr^{2} - e^{\gamma(\psi,t)}d\psi^{2},$$
(1)

where $dr^2 = dx^2 + dy^2 + dz^2$ and ψ is the fifth coordinate. We assume that e^{α} , e^{β} and e^{γ} are separable functions of the variables ψ and t. The equations for the relevant Einstein tensor elements are

$$G^{0}_{0} = -e^{-\alpha} \left[\frac{3\dot{\beta}^{2}}{4} + \frac{3\dot{\beta}\dot{\gamma}}{4} \right] + e^{-\gamma} \left[\frac{3\dot{\beta}^{*}}{2} + \frac{3\dot{\beta}^{2}}{2} - \frac{3\dot{\gamma}\dot{\beta}}{4} \right], \tag{2}$$

$$G_{4}^{0} = e^{-\alpha} \left[\frac{3 \stackrel{\star}{\beta}}{2} + \frac{3 \stackrel{\star}{\beta} \stackrel{\star}{\beta}}{4} - \frac{3 \stackrel{\star}{\beta} \stackrel{\star}{\alpha}}{4} - \frac{3 \stackrel{\star}{\gamma} \stackrel{\star}{\gamma}}{4} \right], \tag{3}$$

$$G^{i}_{i} = -e^{-\alpha} \left[\ddot{\beta} + \frac{3\dot{\beta}^{2}}{4} + \frac{\ddot{\gamma}}{2} + \frac{\dot{\gamma}^{2}}{4} + \frac{\dot{\beta}\dot{\gamma}}{2} - \frac{\dot{\alpha}\dot{\beta}}{2} - \frac{\dot{\alpha}\dot{\gamma}}{4} \right]$$

$$+ e^{-\gamma} \left[\stackrel{\star\star}{\beta} + \frac{3 \stackrel{\star}{\beta}^2}{4} + \frac{\stackrel{\star\star}{\alpha}}{2} + \frac{\stackrel{\star\star}{\alpha}}{2} + \frac{\stackrel{\star}{\alpha}^2}{4} + \frac{\stackrel{\star}{\beta} \stackrel{\star}{\alpha}}{2} - \frac{\stackrel{\star}{\gamma} \stackrel{\star}{\beta}}{2} - \frac{\stackrel{\star}{\alpha} \stackrel{\star}{\gamma}}{4} \right], \tag{4}$$

$$G_{4}^{4} = e^{-\alpha} \left[\frac{3\ddot{\beta}}{2} + \frac{3\dot{\beta}^{2}}{2} - \frac{3\dot{\alpha}\dot{\beta}}{4} \right] + e^{-\gamma} \left[\frac{3\overset{\star}{\beta}^{2}}{4} + \frac{3\overset{\star}{\beta}\overset{\star}{\alpha}}{4} \right], \tag{5}$$

where the overstar and the overdot denote respectively $\frac{\partial}{\partial \psi}$ and $\frac{\partial}{\partial t}$, and i = 1, 2, 3. We shall use the signature (+, -, -, -) for the 4D metric, such that we define $T_0^0 = \rho_t$ and $T_1^1 = -p$, where ρ_t is the total energy density and p is the pressure. The 5D-vacuum conditions $(G_B^A = 0)$ are given by [12]

$$8\pi G \rho_t = \frac{3}{4} e^{-\alpha} \dot{\beta}^2,\tag{6}$$

$$8\pi G \mathbf{p} = e^{-\alpha} \left[\frac{\dot{\alpha}\dot{\beta}}{2} - \ddot{\beta} - \frac{3\dot{\beta}^2}{4} \right],\tag{7}$$

$$e^{\alpha} \left[\frac{3 \stackrel{\star}{\beta}^2}{4} + \frac{3 \stackrel{\star}{\beta} \stackrel{\star}{\alpha}}{4} \right] = e^{\gamma} \left[\frac{\ddot{\beta}}{2} + \frac{3 \dot{\beta}^2}{2} - \frac{\dot{\alpha} \dot{\beta}}{4} \right]. \tag{8}$$

Hence, from eqs. (6) and (7) and taking $\dot{\alpha} = 0$, we obtain the equation of state for the induced matter

$$p = -\left(\frac{4}{3}\frac{\ddot{\beta}}{\dot{\beta}^2} + 1\right)\rho_t. \tag{9}$$

Notice that for $\ddot{\beta}/\dot{\beta}^2 \leq 0$ and $\left| \ddot{\beta}/\dot{\beta}^2 \right| \ll 1$ (or zero), this equation describes an inflationary universe. The particular case $\ddot{\beta}/\dot{\beta}^2 = 0$ corresponds to a 4D de Sitter expansion for the universe.

As in a previous paper [13], we shall consider power-law inflation, which can be obtained from the metric (1), when α , β and γ are functions ob the following coordinates:

$$\alpha \equiv \alpha(\psi); \quad \beta \equiv \beta(\psi, t); \quad \gamma \equiv \gamma(t).$$
 (10)

Here, e^{β} is a separable function of ψ and t. The conditions (10) imply that $\dot{\alpha} = \stackrel{\star}{\gamma} = 0$. Furthermore, we shall consider the case where all the coordinates are independent. The choice (10) implies that only the spatial sphere and the fifth coordinate have squared sizes e^{β} and e^{γ} , respectively, that evolve with time.

III. THE MODEL

In this paper we shall consider the particular case of choosing for the metric (1):

$$e^{\alpha} = \psi^2, \quad e^{\beta} = \left(\frac{t}{t_0}\right)^{2p} \psi^{\frac{2p}{p-1}}, \quad e^{\gamma} = \frac{t^2}{(p-1)^2},$$
 (11)

which corresponds to the Ponce de Leon metric [11]

$$dS^{2} = \psi^{2} dt^{2} - \left(\frac{t}{t_{0}}\right)^{2p} \psi^{\frac{2p}{p-1}} dr^{2} - \frac{t^{2}}{(p-1)^{2}} d\psi^{2}, \tag{12}$$

for which the absolute value for the determinant of the metric tensor g_{AB} is

$$|^{(5)}g| = \left[\frac{t^{3p+1}\psi^{\frac{4p-1}{p-1}}}{(p-1)t_0^{3p}}\right]^2.$$

Furthermore, we shall consider an action that describes a free scalar field minimally coupled to gravity

$$I = -\int d^4x \ d\psi \sqrt{\left|\frac{^{(5)}g}{^{(5)}g_0}\right|} \left[\frac{^{(5)}R}{16\pi G} + ^{(5)}\mathcal{L}(\varphi,\varphi_{,A})\right],\tag{13}$$

with a Lagrangian

$$L = \sqrt{\frac{^{(5)}g}{^{(5)}g_0}} \mathcal{L}(\varphi, \varphi_{,A}), \tag{14}$$

and the Lagrangian density

$$\mathcal{L} = \frac{1}{2} g^{AB} \varphi_{,A} \varphi_{,B}. \tag{15}$$

The scalar $^{(0)}g_0$ in the action (13) is given by $^{(0)}g\big|_{t=t_0,\psi=\psi_0}$, such that

$$|^{(5)}g_0| = \left(\frac{t_0\psi_0^{\frac{4p-1}{p-1}}}{(p-1)}\right)^2,$$

where t_0 and ψ_0 are constants. Note that $|^{(5)}g|$ and $|^{(5)}g_0|$ are not well defined for p=1. The Lagrange equation is given by

$$\ddot{\varphi} + \frac{3p+1}{t}\dot{\varphi} - \left(\frac{t_0^p \psi^{\frac{1}{1-p}}}{t^p}\right)^2 \nabla^2 \varphi - \psi \frac{(p-1)(4p-1)}{t^2} \varphi_{,\psi} - \psi^2 \frac{(p-1)^2}{t^2} \varphi_{,\psi\psi} = 0, \tag{16}$$

where the overdot denotes the derivative with respect to the time and $\varphi_{,\psi} = \frac{\partial \varphi}{\partial \psi}$. In order to simplify the structure of this equation we propose the transformation $\varphi = \left(\frac{t_0}{t}\right)^{\frac{3p+1}{2}} \left(\frac{\psi_0}{\psi}\right)^{\frac{4p-1}{2(p-1)}} \chi$, such that the equation of motion for χ is

$$\ddot{\chi} - \psi^{\frac{2}{1-p}} \left(\frac{t_0}{t}\right)^{2p} \nabla^2 \chi - \psi^2 \frac{(p-1)^2}{t^2} \chi_{,\psi\psi} + \frac{(31p^2 - 14p + 2)}{4t^2} \chi = 0.$$
 (17)

We propose the following Fourier's expansion for χ

$$\chi(t, \vec{r}, \psi) = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \int dk_\psi \left[a_{k_r k_\psi} e^{i(\vec{k_r} \cdot \vec{r} + \vec{k_\psi} \cdot \psi)} \xi_{k_r k_\psi} + a^{\dagger}_{k_r k_\psi} e^{-i(\vec{k_r} \cdot \vec{r} + \vec{k_\psi} \cdot \psi)} \xi^*_{k_r k_\psi} \right], \quad (18)$$

where the operators $a_{k_rk_\psi}$ and $a_{k_rk_\psi}^\dagger$ describe the algebra

$$\left[a_{k_r k_{\psi}}, a_{k'_r k'_{\psi}}^{\dagger}\right] = \delta^{(3)} \left(\vec{k}_r - \vec{k}'_r\right) \delta \left(\vec{k}_{\psi} - \vec{k}'_{\psi}\right), \quad \left[a_{k_r k_{\psi}}, a_{k'_r k'_{\psi}}\right] = \left[a_{k_r k_{\psi}}^{\dagger}, a_{k'_r k'_{\psi}}^{\dagger}\right] = 0.$$

The dynamics of the time dependent modes $\xi_{k_r k_{\psi}}$ is given by

$$\ddot{\xi}_{k_r k_\psi} + \left[\psi^{\frac{2}{1-p}} \left(\frac{t_0}{t} \right)^{2p} k_r^2 + \frac{\psi^2 (p-1)^2}{t^2} \left(k_\psi^2 + \frac{(31p^2 - 14p + 2)}{4(p-1)^2 \psi^2} - 2ik_\psi \frac{\partial}{\partial \psi} - \frac{\partial^2}{\partial \psi^2} \right) \right] \xi_{k_r k_\psi} = 0.$$
(19)

The commutator between χ and $\dot{\chi}$ is

$$[\chi(t, \vec{r}, \psi), \dot{\chi}(t, \vec{r}', \psi')] = i\delta^{(3)}(\vec{r} - \vec{r}')\delta(\psi - \psi'), \tag{20}$$

which complies for $\xi_{k_r k_\psi}(t, \psi) \dot{\xi}_{k_r k_\psi}^*(t, \psi') - \xi_{k_r k_\psi}^*(t, \psi') \dot{\xi}_{k_r k_\psi}(t, \psi) = i$, that guarantizes the normalization of $\xi_{k_r k_\psi}$. Furthermore, if we make the transformation $\xi_{k_r k_\psi} = e^{-i\vec{k}_\psi \cdot \vec{\psi}} \tilde{\xi}_{k_r k_\psi}$, we obtain from eq. (19) the equation of motion for $\tilde{\xi}_{k_r k_\psi}$

$$\ddot{\tilde{\xi}}_{k_r k_\psi} - \frac{\psi^2}{t^2} (p-1)^2 \frac{\partial^2 \tilde{\xi}_{k_r k_\psi}}{\partial \psi^2} + \left[\psi^{\frac{2}{1-p}} \left(\frac{t_0}{t} \right)^{2p} k_r^2 + \frac{(31p^2 - 14p + 1)}{4t^2} \right] \tilde{\xi}_{k_r k_\psi} = 0, \tag{21}$$

and the expansion (18) for χ can be rewritten as

$$\chi(t, \vec{r}, \psi) = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \int dk_\psi \left[a_{k_r k_\psi} e^{i\vec{k}_r \cdot \vec{r}} \tilde{\xi}_{k_r k_\psi} + a^{\dagger}_{k_r k_\psi} e^{-i\vec{k}_r \cdot \vec{r}} \tilde{\xi}_{k_r k_\psi}^* \right]. \tag{22}$$

A. 4D dynamics of the inflaton field

Now we consider a foliation (choice of a hypersurface) $\psi = \psi_0$ on the metric (12). On this foliation, the effective 4D line element is

$$dS^{2} \to ds^{2} = \psi_{0}^{2} dt^{2} - \left(\frac{t}{t_{0}}\right)^{2p} \psi_{0}^{\frac{2p}{p-1}} dr^{2}, \tag{23}$$

where ψ_0 is a dimensionless constant and the scale factor of the universe is given by $a=a_0\left(\frac{t}{t_0}\right)^p$. During inflation $p\gg 1$, such that $\ddot{a}>0$ and, for $\dot{\beta}=H=\dot{a}/a$ in eq. (9), the equation of state describes a quasi de Sitter (quasi vacuum) expansion: $p\simeq\left(\frac{4}{3}\frac{\dot{H}}{H^2}+1\right)\rho_t\simeq-\rho_t$. The Lagrangian density (15) can be expanded in the following manner: $^{(5)}\mathcal{L}=\frac{1}{2}\left[g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}+g^{\psi\psi}\varphi_{,\psi}\varphi_{,\psi}\right]$, such that the effective 4D Lagrangian density for the inflaton field on the foliation $\psi=\psi_0$ can be written as

$$^{(4)}\mathcal{L} = \frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - V(\varphi)\bigg|_{\psi=\psi_0}, \qquad (24)$$

where $V(\varphi)$ is given by

$$V(\varphi) = -\frac{1}{2}g^{\psi\psi}\varphi_{,\psi}\varphi_{,\psi}\bigg|_{\psi=\psi_0} = \frac{(p-1)^2}{2t^2} \left(\frac{\partial\varphi}{\partial\psi}\right)^2\bigg|_{\psi=\psi_0}.$$
 (25)

On the other hand, for a foliation $\psi = \psi_0$, the dynamics of the inflaton field holds

$$\ddot{\varphi} + \frac{(3p+1)}{t}\dot{\varphi} - \left(\frac{t_0}{t}\right)^{2p}\psi^{\frac{2}{1-p}}\nabla^2\varphi - \left[\frac{\psi(p-1)(4p-1)}{t^2}\varphi_{,\psi} + \frac{\psi^2(p-1)^2}{t^2}\varphi_{,\psi\psi}\right]\Big|_{\psi=\psi_0} = 0,$$
(26)

so that we can make the following identification:

$$V'(\varphi) = \left[\frac{\dot{\varphi}}{t} - \frac{\psi(p-1)(4p-1)}{t^2} \varphi_{,\psi} - \frac{\psi^2(p-1)^2}{t^2} \varphi_{,\psi\psi} \right] \Big|_{\psi=\psi_0}.$$
 (27)

Furthermore, the scalar field χ in eq. (22) on the foliation $\psi = \psi_0$ now holds

$$\chi(t, \vec{r}, \psi = \psi_0) = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \int dk_\psi \left[a_{k_r k_\psi} e^{i\vec{k}_r \cdot \vec{r}} \tilde{\xi}_{k_r k_\psi} + c.c. \right] \delta(k_\psi - k_{\psi_0}). \tag{28}$$

Note that the equation (26) is not separable on the coordinate ψ . However, in the limit $p \gg 1$, which is relevant to study the inflationary expansion of the early universe, the term $\left(\frac{t_0}{t}\right)^{2p} \psi^{\frac{2}{1-p}} \nabla^2 \varphi$ in eq. (26) tends asymptotically to $\left(\frac{t_0}{t}\right)^{2p} \nabla^2 \varphi$ as $p \to \infty$. Hence, in this limital case the dynamics for φ is governed by the equation

$$\ddot{\varphi} + \frac{(3p+1)}{t}\dot{\varphi} - \left(\frac{t_0}{t}\right)^{2p} \nabla^2 \varphi - \left[\frac{\psi(p-1)(4p-1)}{t^2} \varphi_{,\psi} + \frac{\psi^2(p-1)^2}{t^2} \varphi_{,\psi\psi}\right]\Big|_{\psi=\psi_0} = 0, \quad (29)$$

which is separable on the variable ψ and more easily workable. This particular case will be studied in the next section.

IV. INFLATIONARY EXPANSION $(P \gg 1)$

We consider the case where $p \gg 1$, which describes an inflationary power-law expansion in the very early universe. In this case the equation for the modes (21), can be approximated to

$$\ddot{\tilde{\xi}}_{k_r k_\psi} - \frac{\psi^2 (p-1)^2}{t^2} \frac{\partial^2 \tilde{\xi}_{k_r k_\psi}}{\partial \psi^2} + \left[k_r^2 \left(\frac{t_0}{t} \right)^{2p} + \frac{(31p^2 - 14p + 2)}{4t^2} \right] \tilde{\xi}_{k_r k_\psi} \simeq 0.$$
 (30)

The general solution for this equation on the foliation $\psi = \psi_0$ is

$$\tilde{\xi}_{k_r k_\psi}(t, \psi) = \alpha \sqrt{\frac{t}{t_0}} \mathcal{H}_{\nu}^{(1)}[x(t)], \tag{31}$$

where
$$\nu = \sqrt{\frac{1+4C_1}{2(p-1)}}$$
, $x(t) = k_r t_0^p t^{1-p}/(p-1)$, $\alpha = A_1 \left[M_1 \psi_0^{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{C}{(p-1)^2}}} + M_2 \psi_0^{\frac{1}{2} - \sqrt{\frac{1}{4} + \frac{C}{(p-1)^2}}} \right]$

 $C = C_1 + \frac{(31p^2 - 14p + 2)}{4}$ and C_1 , A_1 , M_1 and M_2 are constants of integration. Furthermore, the normalization condition implies that

$$\frac{4(p-1)}{t_0\pi} |\alpha|^2 = 1. {32}$$

To calculate the inflaton field fluctuations on the infrared sector, which is relevant for super Hubble scales during inflation, we can make use of the asymptotic representations of the first kind Hankel function $\mathcal{H}_{\nu}^{(1)}[x]$ for $x \ll 1$

$$\mathcal{H}_{\nu}^{(1)}[x] \simeq \frac{1}{\Gamma(\nu+1)} \left(\frac{x}{2}\right)^{\nu} + \frac{i}{\pi} \Gamma(\nu) \left(\frac{x}{2}\right)^{-\nu}.$$

With this representation the squared χ fluctuations on cosmological scales are $\langle \chi^2 \rangle|_{IR} = \frac{1}{2\pi^2} \int_0^{\epsilon k_0} dk_r k_r^2 \tilde{\xi}_{k_r k_{\psi_0}} \tilde{\xi}_{k_r k_{\psi_0}}^*$, where $k_0(t) = \frac{\sqrt{31p^2 - 14p + 2}}{2t_0^p} t^{p-1}$ is the wavenumber that separates the infrared (IR) and ultraviolet sectors. Making the calculation, we obtain

$$\langle \chi^2 \rangle \Big|_{IR} = \frac{t}{8\pi(p-1)} \left[\frac{1}{\Gamma^2(\nu+1)} \left(\frac{\epsilon^3 (31p^2 - 14p + 2)t^{2(p-1)}}{16(p-1)t_0^{2p}} \right)^{2\nu} \frac{1}{(2\nu+3)} - \frac{\Gamma^2(\nu)}{\pi^2(3-2\nu)} \left(\frac{16(p-1)t_0^{2p}t^{-2(p-1)}}{\epsilon^3(31p^2 - 14p + 2)} \right)^{2\nu} \right],$$
(33)

and the squared φ fluctuations, which are the relevant for us, are

$$\langle \varphi^2 \rangle \Big|_{IR} = \frac{t^{-3p} t_0^{(3p+1)}}{8\pi (p-1)} \left[\frac{1}{\Gamma^2 (\nu+1)} \left(\frac{\epsilon^3 (31p^2 - 14p + 2)t^{2(p-1)}}{16(p-1)t_0^{2p}} \right)^{2\nu} \frac{1}{(2\nu+3)} - \frac{\Gamma^2 (\nu)}{\pi^2 (3-2\nu)} \left(\frac{16(p-1)t_0^{2p} t^{-2(p-1)}}{\epsilon^3 (31p^2 - 14p + 2)} \right)^{2\nu} \right],$$
(34)

which decrease with time as $\langle \varphi^2 \rangle|_{IR} \sim t^{-3p}$. The power spectrum of these fluctuations goes as $\mathcal{P}(k_r) \sim k_r^{3-2\nu}$ which is scale invariant for $C_1 = \frac{9(p-1)-2}{8}$ and hence for $C = \frac{61p^2-(19p+7)}{8}$. This result is very interesting because (with an adequate choice of constant values) we can obtain scale invariance for the power spectrum of $\langle \varphi^2 \rangle$ for any $p \gg 1$, independently of some particular value for it.

V. FINAL COMMENTS

In this paper we have studied power-law inflation in the limital case $p \gg 1$ from a STM theory of gravity using the Ponce de Leon metric. In this approach, the inflationary expansion is governed by a single scalar field, that, on a foliation $\psi = \psi_0$ in the 5D metric (12) can be identified as the inflaton field evolving on the effective 4D Friedmann-Robertson-Walker metric (23). In the Wesson's theory [called Space-Time-Matter (STM) theory], the extra dimension is not assumed to be compactified, which is a major departure from earlier multidimensional theories where the cylindricity conditions was imposed. In this theory, the original motivation for assuming the existence of a large extra dimension was to achieve the unification of matter and geometry, i.e. to obtain the properties of matter as a consequence

of the extra dimensions. A very important fact in our approach is that the effective potential $V(\varphi) = -\frac{1}{2}g^{\psi\psi}\left(\frac{\partial\varphi}{\partial\psi}\right)^2\Big|_{\psi=\psi_0}$, has a geometrical origin.

In this paper we have studied with major detail the case $p \gg 1$, because is the only where the field $\varphi(t, \vec{r}, t)$ is separable on the variable ψ and can be more easily treated. However, for cases where the power p is of the order of the unity, the fifth coordinate ψ could play an more important role in the spectrum of the squared φ fluctuations.

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