Baryon Masses in Lattice QCD with Exact Chiral Symmetry*

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We investigate the baryon mass spectrum in quenched lattice QCD with exact chiral symmetry. For 100 gauge configurations generated with Wilson gauge action at $\beta = 6.1$ on the $20^3 \times 40$ lattice, we compute (point-to-point) quark propagators for 30 quark masses in the range 67 MeV $\leq m_q \leq$ 1790 MeV. For baryons only composed of strange and charm quarks, their masses are extracted directly from the time correlation functions, while for those containing $u(d)$ light quarks, their masses are obtained by chiral extrapolation to $m_\pi = 135$ MeV. Our results of baryon masses are in good agreement with experimental values, except for the negative parity states of Λ and Λ_c. Further, our results of charmed (including doubly-charmed and triply-charmed) baryons can serve as predictions of QCD.

1. Introduction

One of the basic objectives of lattice QCD is to compute hadron masses nonperturbatively from the first principles. For hadrons only composed of strange and/or charm quarks (e.g., Ω_c^0 in Fig. 1), their masses (in quenched approximation) can be measured directly with presently accessible lattice sizes. However, for hadrons containing u, d light quarks, the performance of the present generation of computers is still inadequate for computing their masses at the physical scale ($m_\pi \simeq 135$ MeV), on a lattice with enough number of sites in each direction such that the discretization errors and the finite volume effects both are negligible comparing to the statistical ones. Nevertheless, even with lattices of moderate sizes, lattice QCD can determine the parameters of the hadron mass formulas in the (quenched) chiral perturbation theory. Then one can use these formulas to evaluate the hadron masses at the physical scale. In this paper, we extrapolate the baryon mass linearly in m_π^2 (e.g., Ξ_{cc}^+ in Fig. 2).

Here the quark fields are formulated in the framework of optimal domain-wall fermion [1] such that the quark propagator and its fermion determinant can attain the maximal chiral symmetry for any finite N_s. For 100 gauge configurations generated with Wilson gauge action at $\beta = 6.1$ on the $20^3 \times 40$ lattice, we compute (point-to-point) quark propagators for 30 quark masses in the range $0.03 \leq m_q a \leq 0.8$ [2]. Then we determine the inverse lattice spacing $a^{-1} = 2.237(76)$ GeV from the pion correlation function, with the experimental input of pion decay constant $f_\pi = 132$ MeV. The strange quark bare

*This work was supported in part by the National Science Council (ROC) under Grant No. NSC93-2112-M002-016, and by the National Science Foundation under Grant No. PHY99-07949.

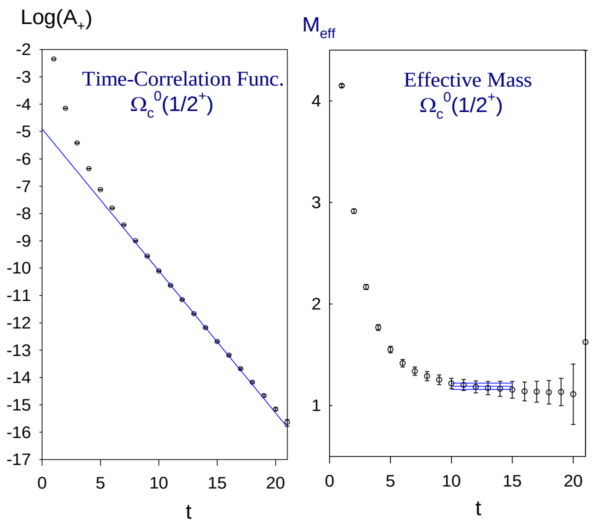

Figure 1. The even parity amplitude $A_+(t)$ of the time correlation function of Ω_c^0 is plotted versus the time slice. The solid line is the single exponential fit for $10 \le t \le 15$ where the effective mass attains a plateau, and it gives $M_{\Omega_c^0(1/2^+)} = 2677(30)$ MeV, in good agreement with the mass of $\Omega_c^0(2697)$.

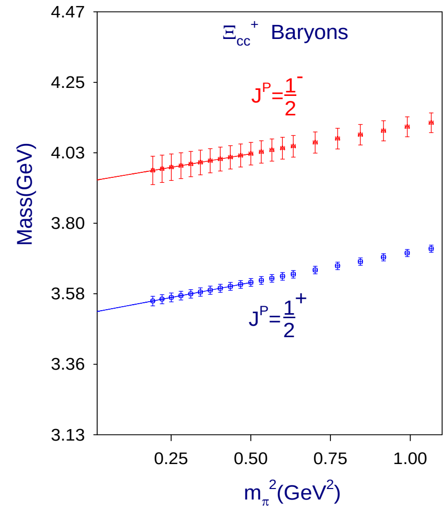

Figure 2. The mass of the doubly charmed Ξ_{cc}^+ baryon versus the pion mass square, for $J^P = 1/2^\pm$ states respectively. The solid lines are linear fits using the smallest 11 masses with $0.03 \le m_d a \le 0.08$. The even parity state gives $M_{\Xi_{cc}^+(1/2^+)} = 3522(16)$ MeV, in good agreement with $\Xi_{cc}(3520)$ observed by SELEX [3].

mass $m_s a = 0.08$ and the charm quark bare mass $m_c a = 0.8$ are fixed such that the masses extracted from the vector meson time-correlation function are in close agreement with the experimental masses of $\phi(1020)$ and $J/\psi(3010)$ respectively. Then *the masses of any other hadrons containing c, s, u and d quarks*[2] *are predictions of QCD from the first principles.*

2. Baryon interpolating operator and time-correlation fnnction

First we define the notation for the "diquark" operator

$$[\mathbf{q^A}\Gamma\mathbf{q^B}]_{xa} \equiv \epsilon_{abc}\Gamma_{\alpha\beta}(\mathbf{q^A}_{x\alpha b}\mathbf{q^B}_{x\beta c} - \mathbf{q^B}_{x\alpha b}\mathbf{q^A}_{x\beta c})$$

where $\mathbf{q^A}$ and $\mathbf{q^B}$ denote quark fields of flavors A and B; ϵ_{abc} is the completely antisymmetric tensor; x, $\{a, b, c\}$ and $\{\alpha, \beta, \gamma\}$ denote the lattice site, color, and Dirac indices respectively. Here $\Gamma_{\alpha\beta} = -\Gamma_{\beta\alpha}$ such that the diquark transforms like a spin singlet. Further, we define

$$(\mathbf{q^A}\Gamma\mathbf{q^B})_{xa} \equiv \epsilon_{abc}\Gamma_{\alpha\beta}\mathbf{q^A}_{x\alpha b}\mathbf{q^B}_{x\beta c}$$

[2] In this paper, we work in the isospin limit $m_u = m_d$.

Then the interpolating operators for N and Δ can be written as

$$N_{x\gamma} = [\mathbf{u}(C\gamma_5)\mathbf{d}]_{xa}\mathbf{u}_{x\gamma a}, \quad \Delta_{x\gamma} = (\mathbf{d}C\gamma_\mu \mathbf{d})_{xa}\mathbf{d}_{x\gamma a}$$

where C is the charge conjugation satisfying $C\gamma_\mu C^{-1} = -\gamma_\mu^T$ and $(C\gamma_5)^T = -C\gamma_5$.

Now suppressing the Dirac and site indices, the interpolating operators for baryons with $J = 1/2$ can be written as

$$\begin{aligned} N &= [\mathbf{u}(C\gamma_5)\mathbf{d}]\mathbf{u}, \quad \Xi^0 = [\mathbf{u}(C\gamma_5)\mathbf{s}]\mathbf{s}, \quad \Sigma^+ = [\mathbf{u}(C\gamma_5)\mathbf{s}]\mathbf{u} \\ \Lambda^0 &= [\mathbf{d}(C\gamma_5)\mathbf{s}]\mathbf{u} + [\mathbf{s}(C\gamma_5)\mathbf{u}]\mathbf{d} - 2[\mathbf{u}(C\gamma_5)\mathbf{d}]\mathbf{s}, \end{aligned}$$

while the interpolating operators for baryons with $J = 3/2$ can be written as

$$\begin{aligned} \Delta^- &= (\mathbf{d}C\gamma_\mu \mathbf{d})\mathbf{d}, \quad \Omega^- = (\mathbf{s}C\gamma_\mu \mathbf{s})\mathbf{s}, \quad \Sigma^+ = (\mathbf{u}C\gamma_\mu \mathbf{s})\mathbf{u} + (\mathbf{s}C\gamma_\mu \mathbf{u})\mathbf{u} + (\mathbf{u}C\gamma_\mu \mathbf{u})\mathbf{s}, \\ \Xi^0 &= (\mathbf{u}C\gamma_\mu \mathbf{s})\mathbf{s} + (\mathbf{s}C\gamma_\mu \mathbf{s})\mathbf{u} + (\mathbf{s}C\gamma_\mu \mathbf{u})\mathbf{s} \end{aligned}$$

Similarly, the operators for charmed baryons can be constructed, e.g., $\Omega_c^0 = [\mathbf{c}(C\gamma_5)\mathbf{s}]\mathbf{s}$, and $\Xi_{cc}^+ = [\mathbf{c}(C\gamma_5)\mathbf{d}]\mathbf{c}$.

The time-correlation function of any baryon operator B is defined as $C_{\alpha\beta}(t) = \sum_{\vec{x}} \langle B_{x\alpha} \bar{B}_{0\beta} \rangle$ which can be expressed in terms of point-to-point quark propagators from $y = (\vec{0}, 0)$ to $x = (\vec{x}, t)$. Its average over gauge configurations is fitted by the usual formula

$$\langle C(t) \rangle = \frac{1+\gamma_4}{2}(Z_+ e^{-m_+ at} - Z_- e^{-m_- a(T-t)}) + \frac{1-\gamma_4}{2}(Z_+ e^{-m_+ a(T-t)} - Z_- e^{-m_- at})$$

where $m_\pm$ are the masses of even and odd parity states. Thus, one can use parity projector $(1 \pm \gamma_4)/2$ to project out two amplitudes,

$$A_+(t) \equiv Z_+ e^{-m_+ at} - Z_- e^{-m_- a(T-t)}, \;\; A_-(t) \equiv Z_+ e^{-m_+ a(T-t)} - Z_- e^{-m_- at}.$$

Now the problem is how to extract $m_\pm$ from $A_\pm$ respectively. Obviously, for sufficiently large T, there exists a range of t such that, in $A_\pm$, the contributions due to the opposite parity state are negligible. Then $m_\pm$ can be extracted by a single exponential fit to $A_\pm$, for the range of t in which the effective mass $m_{eff}(t) = \ln(A_\pm(t)/A_\pm(t+1))$ attains a plateau. On the other hand, if T is not so large, then it may turn out that the heavier mass, say m_- (assuming $m_- > m_+$), could not be easily extracted from A_- due to the non-negligible contributions of the (lowest lying) even parity state. For our lattice with $T = 40$, it is sufficiently large to extract the mass of the lowest lying state for the entire range of $0.03 \le m_q a \le 0.8$. However, the excited state seems to suffer from the (backward propagating) contribution of the lowest-lying state, especially for $m_q a \le 0.05$, which yields relatively larger error in extracting the mass of the negative parity state.

3. Summary and Concluding Remarks

Our results of baryon mass spectra are summarized in Tables 1 and 2, along with the experimental mass spectra listed by the Particle Data Group [4]. The empty entries in the last column are baryons which have not been discovered in experiments. Thus the baryon masses obtained in this paper can serve as theoretical predictions of lattice QCD, in particular, the singly-charmed, doubly-charmed, and triply-charmed baryons. Details of our results including meson mass spectra will be presented elsewhere [5]. Our results of baryon masses are in good agreement with experimental values, except for some of the negative parity baryons, in particular, $\Lambda(1405)$ and $\Lambda_c(2593)$. This certainly warrants further studies on these excited baryons (e.g., with higher statistics, larger lattices, smaller $u(d)$ quark masses, and incorporation of dynamical fermions).

Table 1
Baryon mass spectra obtained in this paper [with $m_s a = 0.08$, $m_c a = 0.80$ and $a^{-1} =$ 2237(76) MeV]. The last column is from the listings of Particle Data Group [4], where J^P has not been measured for all entries.

Baryon	J^P	Mass	Expt
N	$1/2^+$	958(36)	939
N	$1/2^-$	1603(150)	1535
Δ	$3/2^+$	1243(74)	1232
Δ	$3/2^-$	1764(105)	1700
Λ	$1/2^+$	1119(38)	1116
Λ	$1/2^-$	1854(47)	1405
Σ	$1/2^+$	1211(43)	1189
Σ	$1/2^-$	1872(118)	1750
Ξ	$1/2^+$	1326(32)	1315
Ξ	$1/2^-$	1935(110)	1950
Σ	$3/2^+$	1376(61)	1385
Σ	$3/2^-$	1931(93)	1940
Ξ	$3/2^+$	1539(42)	1530
Ξ	$3/2^-$	2039(95)	2030
Ω	$3/2^+$	1668(29)	1672
Ω	$3/2^-$	2229(50)	2250
Ω_{ccc}	$3/2^+$	4681(28)	
Ω_{ccc}	$3/2^-$	5066(48)	

Table 2
Continuation of Table 1.

Baryon	J^P	Mass	Expt.
Λ_c	$1/2^+$	2275(27)	2285
Λ_c	$1/2^-$	3027(63)	2593
Σ_c	$1/2^+$	2458(24)	2455
Σ_c	$1/2^-$	2882(40)	
Ξ_c	$1/2^+$	2478(27)	2466
Ξ_c	$1/2^-$	2793(68)	2790
Ω_c	$1/2^+$	2677(30)	2697
Ω_c	$1/2^-$	3100(130)	
Σ_c	$3/2^+$	2522(30)	2520
Σ_c	$3/2^-$	3007(47)	
Ξ_c	$3/2^+$	2646(36)	2645
Ξ_c	$3/2^-$	2954(94)	2815
Ω_c	$3/2^+$	2756(32)	
Ω_c	$3/2^-$	3224(62)	
Ξ_{cc}	$1/2^+$	3522(16)	3520
Ξ_{cc}	$1/2^-$	3940(47)	
Ω_{cc}	$1/2^+$	3637(23)	
Ω_{cc}	$1/2^-$	4053(57)	
Ξ_{cc}	$3/2^+$	3655(20)	
Ξ_{cc}	$3/2^-$	4043(22)	
Ω_{cc}	$3/2^+$	3762(17)	
Ω_{cc}	$3/2^-$	4147(31)	

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